

# Equilibrium Unemployment in a General Equilibrium Model with Taxes

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## Abstract

The ratio of unemployed to vacancies has risen sharply in the UK after the recession of 2008/09. How harmful is it for the long run growth, equity and efficiency and what sorts of long run cycles does it generate in the economy? With a dynamic computable general equilibrium model with Pissarides (1979, 2011) and Mortensen and Pissarides (1994) type equilibrium unemployment, impacts of tax-transfer programmes are assessed for the UK. The model contains more desirable structure of households and production sectors and includes more type of shocks in preferences, technology, trade and policy instruments for stochastic analyses than is usual in DSGE models. It assesses growth and cycles as well as equity and efficiency effects of policies simultaneously. Improvements in the matching technology lowers the equilibrium unemployment and raises the long-run growth rate and life time utilities of households and reduces long run cycles. Matching could be made more efficient by influencing the relative price system by optimal set of tax and transfer instruments. Better matching techniques can make transition of job-seekers to employment more efficient so that the intertemporal labour-leisure and consumption-saving decisions have greater impacts on growth and redistribution reducing fluctuations in the economy.

Keywords: Equilibrium Unemployment, general equilibrium, growth, business cycles

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# 1 Introduction

Market clearing is central to the fundamental theorems of welfare analysis and Pareto optimality conditions for determining the efficient allocation of resources and distribution of income in the classical Walrasian general equilibrium model. However, the existence of unemployed job-seekers (even their *persistent* unemployment) is a fact of real life in almost all economies around the world. Ignoring millions of unemployed job seekers and overlooking the consequences for welfare goes against the principle of choosing the best policy alternative which maximizes social welfare. There can be little debate that the process of job creation and destruction and the hiring of workers through the matching process between the skills of job seekers and requirement of employers (job creators) should be an integral part of any dynamic general equilibrium analysis if we want to shed light on the adjustment process required for the long run efficiency and growth in modern economies.

Economists have incorporated unemployment as a feature of the equilibrium process in the modelling of an economy for some time. Job matching and search models developed by Pissarides (1979) and Mortensen and Pissarides (1994) significantly contributed to the analysis of unemployment dynamics based on bargaining and matching for wages and jobs by workers and firms. Pissarides (2011) provides intuitive reasoning behind the matching function (Beveridge curve) and wage curves in the unemployment and vacancy  $(u, v)$  space to compare equilibrium positions in the expansionary, normal and recessionary phases in the UK and US labour markets. Pissarides (2013) illustrates how the UK, USA and OECD labour markets moved along the Beveridge curve between 2007 and 2009 but were unable to achieve the efficient equilibrium points after 2008 recession because of macroeconomic rigidities in Europe and microeconomic rigidities in the US. In the last decade, the new Keynesian DSGE models have been extended to assess the impacts of various measures designed to reduce the labour market frictions in the dynamics of inflation and its volatility with both firm-union bargaining (Zanetti (2007), Gertler and Trigari (2009)) and search-matching a la Mortensen and Pissarides (see for example Kraus and Lubik (2007), Zanetti (2011), Faccini, Millard and Zanetti (2013)). While Thomas and Zanetti (2009) found insignificant impacts of reducing unemployment benefits and firing costs on inflation volatility in the Euro area, Campolmi and Faia (2011) measure the welfare losses due to such volatility. Thus these measures designed to reduce the labour market frictions did not cause more inflation volatility and did not have welfare costs more than 0.3 percent of GDP. Michailat (2012) argued that in recessions job-rationing is more important than matching frictions: however, Pissarides (2013) strongly favours the search and matching model than the job rationing model for analysis of recessions as it is not only simple and more appealing but can also explain the emerging trends across countries.

Whilst these recent non-Walrasian developments are very much to be welcomed, in a standard DSGE model the focus is exclusively on short run "business cycle" fluctuations around a steady-state

in a representative household setting. We believe that the DGSE framework is lacking in certain key dimensions. First, these models shed no light on the long run general equilibrium impacts of tax-transfer policies on equilibrium unemployment, growth, capital accumulation among various sectors of the economy and the utility, wages and labour supply of households into the future. Secondly, the micro-foundations are very simple and abstract from diversity across households and production sectors, with no role for relative prices and wages by sectors of production and skill categories of labour (see Merz (1995), Hutton and Ruocco (1999), Ljungqvist and Sargent (2007)).

In this paper, we seek to extend the computable general equilibrium (CGE) framework to allow a full analysis of equilibrium unemployment in a dynamic general equilibrium model with heterogeneous households and firms, providing the structural details required for more realistic analysis of the allocation mechanism in the economy. It aims to contribute to the existing literature in these two directions with a focus on its application to the UK economy. The framework we develop provides a *medium and long-term* framework for evaluating and understanding economic policy as opposed to the short-run focus of standard new Keynesian DGSE models. The main contribution of this paper lies in providing greater degree of micro-foundations for a dynamic general equilibrium model in which relative prices of commodities and wage rates across skill categories and marginal productivities across firms, tax and transfer policies feature in determining the path of equilibrium unemployment. It considers the full impacts of the equilibrium rate of unemployment on labour supply, consumption and saving behavior by deciles of households in three different state of labour market and inequality in earnings and income emerging from the competitive equilibrium process in the economy. It also shows how the job market matching evolves alongside the investment and capital accumulation behavior of firms and the resulting relative prices of commodities and factors of production in the broader economy.

The job search and matching component is put in the dynamic equilibrium framework developed by Bhattacharai (2007a) to assess impacts of various fiscal policy measures including VAT, income and production taxes and transfers in the system of relative prices and allocations of resources as well as on the evolution of the economy. This issue is investigated here by comparing results of the equilibrium unemployment dynamic computable general equilibrium annual model (EUDCGEM) under the tax reform schemes as proposed after the current recession. The model is benchmarked to the 2009 micro-consistent input-output data of the UK economy. The reference dynamic path of the model is computed for a long horizon of 2100 in order to see the long run cycle and growth simultaneously and to study the evolution of economy over time and to know what options future generations will have if the current policies and institutions were to continue. The model with heterogeneity among households and firms finds the dynamically efficient and optimal path of the price and wage system of the economy given the preferences and technologies of households and firms who interact with fiscal policy choices of the government. Results of inflows in and outflows from the unemployment give features of negatively sloped Beveridge-UV curve (Fig. 5) and positively

sloped wage curves. Empirical analysis shows that more vacancies occur when unemployment is lower. Positively sloped wage-VS curve (Fig. 6) are observable when the sectoral job mismatches cause a rise in the number of vacancies despite larger numbers of unemployed job seekers. This is the first paper to model the general equilibrium impacts of such phenomenon with heterogeneous firms and skills for the UK economy.

## 2 Stylized facts of UK Labour Market

Since the focus of the current paper is to study the implications of equilibrium unemployment in the economy, four stylized facts, based on time series data on unemployment, vacancies and redundancies in the UK from the Office of the National Statistics (ONS) are worth considering.

Fact 1: Inflow and outflow rates have changed over time with resultant changes in the equilibrium unemployment rate. Literature suggests that outflow rates were more important determinants of unemployment than inflow rates till 1990 but the job separation rates have become more important after the recent recession. These two factors explain how the overall rate of unemployment had gone up from around 4 percent in early 1970s up to 12 percent in around 1984 and remained at fairly higher level till mid 1990s and declining gradually towards 5 percent around 2007 (Fig. 1). Recent studies including Pissarides (2013), Smith (2011), Petrongolo and Pissarides (2008) have attributed these pre-2007 trends to less frictions in the labour market due to structural reforms on benefits, taxes and trainings under the Thatcherian or the New Deal programmes in the last three decades. However, after the financial crisis in 2008, the unemployment rate rose to 8.4 percent in November 2011 (2.6 million) because of higher separation rates triggered by the recession against lower outflow rates as vacancies were less than 0.5 million (Fig. 2). There was a massive rise in the ratio of unemployed to vacancies from 2.2 to 5.8 (Fig. 4). Despite continued and unprecedented fiscal and monetary stimulus from 2009 to 2014, as reflected in budget deficits up to 11 percent of GDP and base interest at 0.5 for more than five years, the private sector enterprises did not create vacancies to match the growing pool of unemployed. Unemployment rate should have been less than 7 percent now because of these stimulants. From our model results summarised in figures 19 to 22 in section 5, we expect that the economy eventually will return to its equilibrium unemployment rate (natural rate of unemployment) bringing inflows and outflows consistent to the steady state growth path as the heterogeneity in preferences of households and technologies of firms, fundamental drivers of demand and supply in the economy have time to adjust to those policy measures after full rounds of economy-wide income and substitution effects in general equilibrium settings over time.

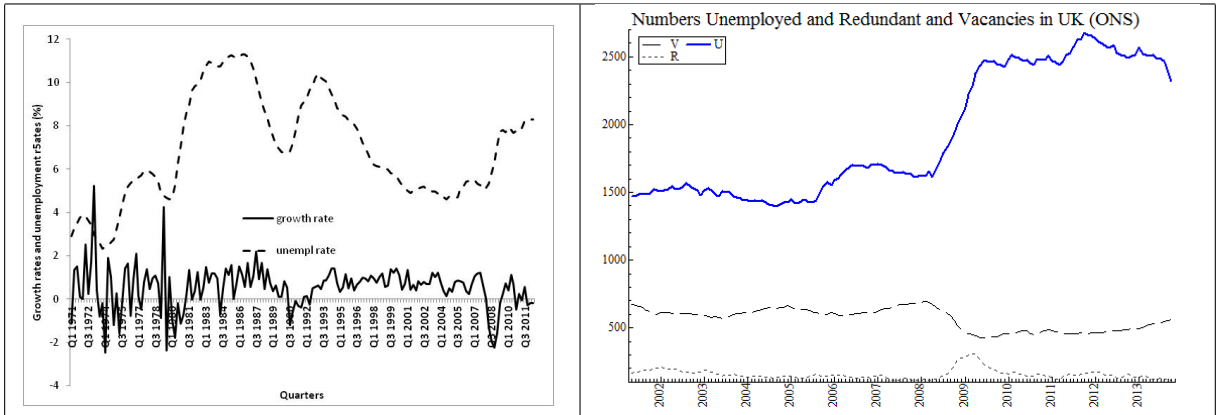


Fig. 1: Quarterly Growth Rates and Unemployment Rates of the UK

Fig. 2: Unemployed, Vacancies and Redundancies in the UK

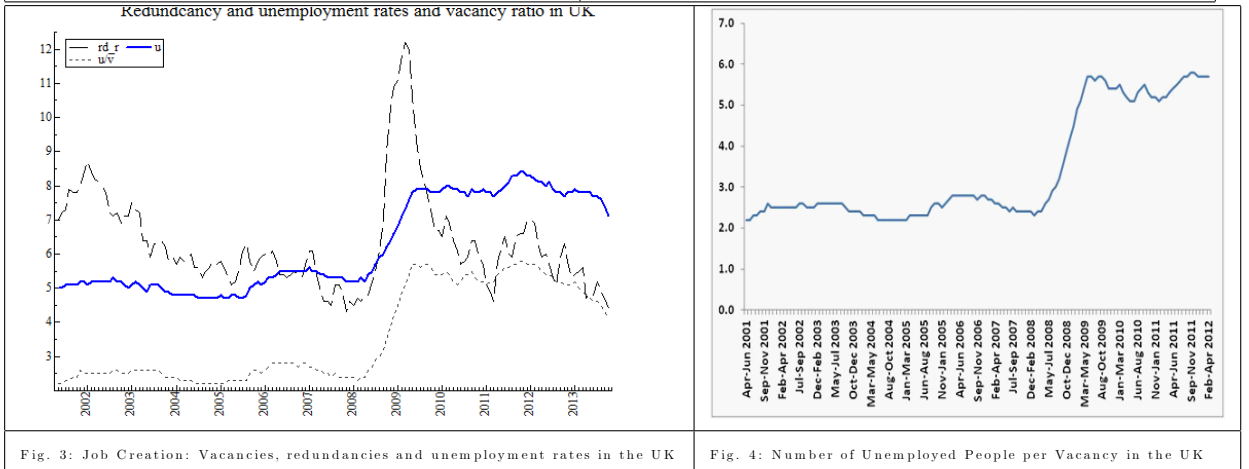


Fig. 3: Job Creation: Vacancies, redundancies and unemployment rates in the UK

Fig. 4: Number of Unemployed People per Vacancy in the UK

Fact 2: There is an evidence of sectoral shift in the structure of employment and causes of unemployment. Vacancies have mainly been in the distribution, finance and education sectors while redundancies have been higher than vacancies in the manufacturing, transportation, construction and other sectors. It is important to consider household and sectoral composition of the economy for clear understanding of inflows to and outflows from the unemployment. Model results summarised in figures 23 and 24 show that education, health, government and professional services will contribute significantly to employment creation in coming years.

Fact 3: Evolving trends of productivity, wages, income distribution and sectoral compositions of the economy influence not only the transitions between inflows and outflows but also the dynamic efficiency and competitiveness of the economy. There are concerns about the growing rate of earning and income inequality in the UK in recent years as the Gini coefficient has increased from around 28 percent in 1980 to 39 percent in 2011. Inclusion of heterogenous households in the model allows us to assess redistribution issues particularly in relation to skill and wage differences among household groups over the model horizon. This itself is a key policy issue of immense interest. Unemployment

can be very costly if it is centred at certain groups of workers such as low income households. The burden of unemployment can be reduced by designing policies assisting firms to hire more or helping job seekers in their search for employment. Sectoral composition as well as short or long term employment matter. Higher rate of equilibrium unemployment lowers the rate of economic growth and reduces the level of utility and life time income for all categories of households. Income is more equally distributed with the policy reforms implemented in this model with the Gini coefficient at around 30.7 percent. The income distribution pattern by decile based on model results is shown in Figures in 8 and 10 along with welfare levels in Figures 7 and 9.

Fact 4: The growth in employment or fall in unemployment follows from the growth of the economy. The number of vacancies are far greater than the number of redundancies. This means the rate of job creation is higher than the rate of job destruction in the UK, as is evident from consistent gaps between vacancies and redundancies in the growing UK economy (Fig. 3). This fact matches well with a negatively sloped Beveridge curve for the UK as seen from our estimates<sup>1</sup>. A simple regression of the change in vacancy ( $\Delta v$ ) on change in unemployment rate ( $\Delta u$ ) in UK for 2001:5 to 2012:2 yields:

$$\begin{aligned} \Delta v &= -0.288 - 0.152\Delta u \\ t &: \quad (-0.366) \quad (-5.70) \\ R^2 &= 0.20, F(1, 128) = 32.4 (0.00), T = 130 \end{aligned}$$

During the same period regressing the change in vacancy on inflation ( $\pi$ ) in UK yields:

$$\begin{aligned} \Delta v &= 4.01 - 2.37\pi \\ t &: \quad (2.09) \quad (-3.25) \\ R^2 &= 0.08, F(1, 128) = 10.6 (0.00), T = 130 \end{aligned}$$

Unemployment is lower during the expansionary phase of business cycle when firms create more jobs and hire; during the contractionary phase when firms are not able to create more vacancies. We show long run business cycle effects based on model results in Figures 25 to 28.

A steady fall in the unemployment rate from 1990 to 2007 was possible because of output growth that balanced the number of vacancies to that of redundancies. In contrast the increase in unemployment rate from 5 percent in 2007 to 8.1 percent in 2011 was due to the recession. Pissarides (2013) attributes 19 percent fall in the unemployment to the growth rate of the economy. In the monthly data from 2001 to 2008, the three percent gap between the unemployment rate and vacancy rate is an indication to the natural rate of unemployment. The net vacancy had remained about 400 thousands each year and with 1.6 million people unemployed but the vacancy unemployment gap increased substantially after the recession in 2008 (Fig. 2). Vacancies per 100 jobs has reduced from 2.5 to 1.5 in the current recession.

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<sup>1</sup>see Chesher and Lancaster (1983) and Lancaster (1979) for more sophisticated econometric methods based on hazard functions and duration analysis.

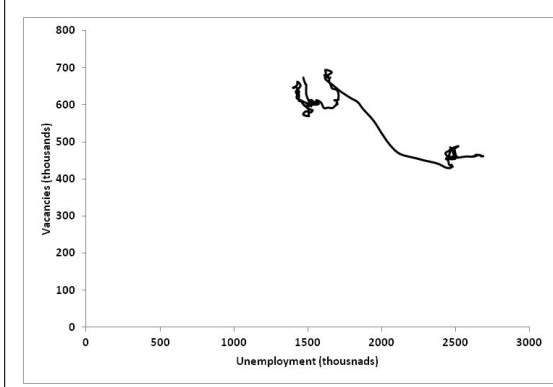


Fig. 5: Beveridge Curve for the UK

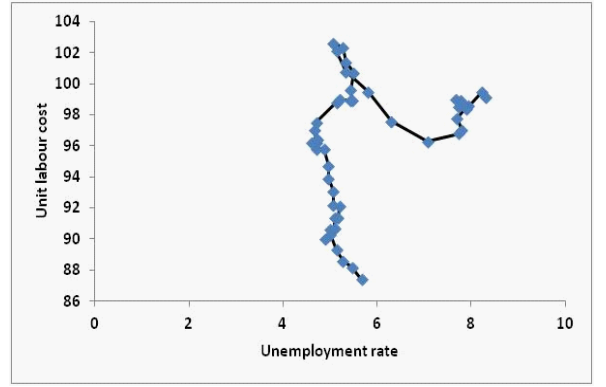


Fig. 6: Wage Curve for the UK

Next section introduces a general equilibrium model and concepts of unemployment rate, wage rate and wage curve as found in the equilibrium unemployment and job search literature. The calibration and computation procedure of the model are presented in section 4 with analysis of results in section 5 and conclusions in section 6 followed by the list of references at the end.

### 3 General Equilibrium Model with Unemployment

The economy consists of households  $h_1, \dots, h_{10}$  indexed according to their levels of income<sup>2</sup> and firms  $i_1$  to  $i_N$  representing the major industries<sup>3</sup> that supply goods and services produced using labour and capital inputs. Both of them solve an infinite horizon problem,  $\{t\}_0^\infty$  approximated here by a terminal period  $t = T$  which is the final year of the 21st century. The major objective of households is to maximise lifetime expected utility ( $EU$ ) against their life time expected budget constraints ( $EBC$ ). Firms are primarily concerned in maximising expected profits over the model horizon. The government sector provides public goods and transfers, financing them by taxes on income, consumption and production. The savings of households are channeled into investment driving capital accumulation across sectors. Growth of the capital stock and the labour force along with continuous adjustments in the prices of goods and services, drive the dynamics of the economy. In contrast to full employment general equilibrium models, this model allows long term equilibrium unemployment rate determined by inflows into and outflows from unemployment as given by transition probabilities from employment to unemployment (separation rate) or from unemployment to employment (job finding rate). The under-utilisation of labour force available for production in this manner results in the loss of output to the economy and loss of utility to households but it cannot be avoided as it is the main feature of the dynamic process of a modern

<sup>2</sup>Derived from the ONS' Reference Tables "The effects of taxes and benefits on household income, 2009/10."

<sup>3</sup>Agriculture, Production, Construction, Distribution, Information and communication, Finance and insurance, Real estate, Professional and support activities, Government, Health and Education, Other Services.

economy. This is the reason why it is necessary to integrate the equilibrium unemployment in line of Pissarides (1979), Mortensen and Pissarides (1994) and Pissarides (2000, 2011) into the general equilibrium model for evaluating impacts of equilibrium unemployment in the economy<sup>4</sup>.

### 3.1 Heterogeneous Household preferences and demand for goods.

Individuals in type  $h$  household are endowed with  $\bar{L}_0^h$  amount of active time and  $K_0^h$  of capital stock in the initial period  $t = 0$  (for simplicity it is assumed that a household type  $h$  represents all types of individuals in that category from now on). While the total time endowment has an exogenous growth process as  $\bar{L}_t^h = \bar{L}_0^h e^{nt}$  with growth rate  $n$ , capital accumulation results from the consumption saving decision of households and investment decision of firms and given by the accumulation equation in (15). In general the intertemporal optimisation process guides the process of capital accumulation and labour supply but there are further issues and complications in the job market because of three states. At each period  $t$ , each individual is either employed ( $SE_t^h$ ) or unemployed ( $SU_t^h$ ) or inactive ( $SI_t^h$ ). Those who are employed are in  $SE_t^h$  state and allocate time between work ( $LS_t^h$ ) and leisure ( $l_t^h$ ) as guided by the real wage rates in relation to the real price system of the economy. Work generates wage income ( $w_t^h LS_t^h$ ) which is normally more than the job-seeker allowances ( $R_t^h$ ) that these individuals would receive if unemployed in  $SU_t^h$  state or basic need related transfers ( $R_t^h$ ) if in the  $SI_t^h$  state. Those who are unemployed divide time into leisure ( $l_t^h$ ) and job search ( $JS_t^h$ ) activities. How intensive is their search efforts then depends not only on its contribution to job finding rate ( $f_t^h$ ) but also on the amount of transfer ( $R_t^h$ ) they could claim by remaining in unemployment state. Uncertainties in job markets implies that with probability  $s_t^h$  each worker may lose the current job and transit from  $SE_t^h$  either to  $SU_t^h$  becoming unemployed or get out of the labour force to state  $SI_t^h$  with probability  $ss_t^h$ . Each unemployed person can get a job with probability  $f_t^h$  as a result of economy wide matching process between job seekers and firms with vacancies or transit to inactive state  $SI_t^h$  with probability  $ff_t^h$ . On the other hand households in the inactive state can enter into employment or unemployment by transition rates  $ie_t^h$  and  $iu_t^h$ . Thus the matching function in this paper takes account of these transitions emerging from heterogeneity in preferences of households and in technology of firms. This set up is consistent to the micro-foundations implied in Pissarides (1979 and 2011) where the gains from job creation are split between workers and firms according to their bargaining power in it. Some estimates for these parameters are found in the literature. The labour market survey (ONS) shows that on average 76.8 percent of 16-64 population was economically active in the UK in the last

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<sup>4</sup>This integration can be seen as the partial fulfilment of Milton Friedman's research program set out in his American Economic Association presidential address in 1968, when he defined "...The 'natural rate of unemployment' ... is the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is embedded in them the actual structural characteristics of the labour and commodity markets, including market imperfections, stochastic variability in demands and supplies, the costs of gathering information about job vacancies, and labor availabilities, the costs of mobility, and so on."



two decades. This implies inactivity rate of 23.2 percent. Petrongolo and Pissarides (2008) from the LFS and Smith (2011) from the BHPS found  $s_t^h$  to vary between 0.35 to 0.41,  $f_t^h$  between 0.36 and 0.31,  $iu_t^h$  between 0.133 and 0.016 and  $ss_t^h$  between 0.151 and 0.09 in the UK. Pissarides (2013) provides a very clear intuition why these rates vary over time as an economy close to the full employment suddenly enters into recession precipitating the fall in demand for labour and increase in unemployment. He matches movement of equilibrium along or outside the Beveridge curve with empirical evidence in explaining cyclical, structural and institutional problems relating to job search and matching processes and public policies in the UK and Germany in comparison to other states of EU and the USA. With these understandings on the underlying process we now proceed to specify the general equilibrium model with unemployment.

Households care for the expected life time utility ( $EU_0^h$ ) from consumption of goods ( $C_{i,t}^h$ ) and leisure ( $l_t^h$ ). They receive income supplying labour ( $LS_t^h$ ) and capital services ( $K_t^h$ ) to firms which pay them according to their marginal productivities. Lower income households receive transfer ( $R_t^h$ ) from the government that is financed by the public revenue collected by taxing high income households. Given the subjective discount factor ( $0 < \beta^h < 1$ ) each  $h$  type of household maximizes utility subject to its budget constraint that sets present value of its expected income and expenditure. Thus the demand side problem of households facing above three labour market states,  $S = \{SE_t^h, SU_t^h, SI_t^h\}$ , could be stated as:

$$\max EU_0^h = \sum_{t=0}^{\infty} \sum_{s=1}^S \beta^{ht} U_{t,s}^h (C_{t,s}^h, l_{t,s}^h) \pi_{t,s}^h \quad (1)$$

subject to the life time expected budget constraint (EBC):

$$\sum_{t=0}^{\infty} \sum_{s=1}^S \sum_{i=1}^N P_{i,t} (1 + tc_i^h) C_{i,t,s}^h \pi_{t,s}^h = \sum_{t=0}^{\infty} \sum_{s=1}^S (1 - t_h) [w_t^h (1 - tw^h) LS_t^h + r_t (1 - tk) K_t^h + R_t^h] \pi_{t,s}^h \quad (2)$$

Here  $P_{i,t}$ ,  $w_t^h$ ,  $r_t$  are the prices of commodities, wage of  $h$  type of labour and rental rate of capital;  $tw^h$  is labour input tax rate,  $tk_i$  is capital input tax rate and  $t_h$  is tax rate on household income. Then  $\pi_{t,s}^h$  is the probability of being in state  $s$  at time  $t$  with the base year actual proportions for the UK being the initial probabilities  $\pi_{0,s}^h$  in the model implementation. As each household  $h$  is certainly in one of these three states in each time  $t$  and these probabilities sum to one,  $\sum_s \pi_{t,s}^h = 1$  for each  $t$ . Each household with time endowment,  $\bar{L}_t^h = \bar{L}_0^h e^{nt}$  which grows by  $n$  between  $t$  and  $t - 1$ , divides it into the labour supply ( $LS_t^h$ ) and leisure ( $l_t^h$ ) if it belongs to the working state, between job search ( $JS_t^h$ ) and leisure ( $l_t^h$ ) if unemployed and puts all in leisure,  $l_{t,si}^h = \bar{L}_{t,si}^h$ , if in the inactive state. These inactive individuals rely on capital income and non-jobseeker benefit

payments to meet their expenses on consumption. Utility maximisation decision leads to the choice of  $C_{i,t,se}^h$ ,  $l_t^h$  and  $LS_t^h = \bar{L}_t^h - l_t^h$  for households in status  $SE_t^h$ . Those who are unemployed (in status  $SU_t^h$ ), get income from capital ( $r_t(1 - tk)K_t^h$ ) or transfers ( $R_t^h$ ) and choose  $C_{i,t,su}^h$ , leisure ( $l_t^h$ ) and hours in job search ( $JS_t^h$ ). There is a logit relation between job finding rate ( $f_t^h$ ) to job search efforts,  $f_t^h = \frac{1}{1 + e^{-JS_t^h}}$ . For those who are trapped in benefit,  $R_t^h$  in the (2) is the form of benefits for the fulfillment of basic needs paid by the state (e.g. allowances for sickness, disability, war veterans, widower or state pensions). The balance in the labour market in these three labour market conditions are:

$$LS_{t,se}^h + l_{t,se}^h = \bar{L}_t^h; \quad JS_{t,su}^h + l_{t,su}^h = \bar{L}_t^h; \quad l_{t,si}^h = \bar{L}_{t,si}^h; \quad \bar{L}_t^h = \bar{L}_0^h e^{nt} \quad (3)$$

As a household  $h$  can transit from one state to another it is important to evaluate all these three states for each household while evaluating their life time EBC (2). In a dynamic labour market the probability of remaining in a particular state  $\pi_{t,s}^h$  itself is an autoregressive process and function of inflows and outflows among states,  $S$ . If a person is in the employment state with probability  $\pi_{t-1,se}^h$  in period  $t - 1$  and this probability is reduced by transitions to unemployment ( $SU_t^h$ ) and inactive ( $SI_t^h$ ) states roughly by  $\pi_{t-1,s}^h (s_t^h + ss_t^h)$  but rises as individuals transit from unemployment and inactive state to employment state by  $f_t^h \pi_{t,su}^h + ie_t^h \pi_{t,si}^h$ . Thus the probability of being in a particular labour market condition itself varies over time. While a particular household is in one particular state  $s$  at a given point of time but it transits to different labour market states as time progresses. Probability of being in employed state is:

$$\pi_{t,se}^h = \pi_{t-1,se}^h - \pi_{t-1,se}^h (s_t^h + ss_t^h) + f_t^h \pi_{t-1,su}^h + ie_t^h \pi_{t-1,si}^h \quad (4)$$

Probability of being in unemployment state is:

$$\pi_{t,su}^h = \pi_{t-1,su}^h - \pi_{t-1,su}^h (f_t^h + ff_t^h) + s_t^h \pi_{t-1,se}^h + iu_t^h \pi_{t-1,si}^h \quad (5)$$

and probability of being in the inactive state is:

$$\pi_{t,si}^h = \pi_{t-1,si}^h - \pi_{t-1,si}^h (ie_t^h + iu_t^h) + ss_t^h \pi_{t-1,se}^h + f_t^h \pi_{t-1,su}^h. \quad (6)$$

Policy makers influence allocation of resources by choosing the VAT rate on commodities ( $tc_i^h$ ), labour input tax rate ( $tw^h$ ), capital input tax rate ( $tk_i$ ) and the tax rate on household income ( $th$ ) hence determining the relative prices of commodities and factors ( $P_{i,t}, w_t^h, r_t$ ). The policy measures eventually will influence the movement across states with probabilities given in (4), (5) and (6). These policy rates are assumed to remain at the base year rates in model implementation.

### 3.2 Production technology and supply of goods

The production technology of firms is nested and differs across sectors. They operate either the constant elasticity of substitution (CES) or Cobb-Douglas production technology depending on substitutability of inputs according to their prices. They hire workers and capital stocks from households. The objective of a firm in the  $j$ th sector of the economy is to maximise the present value of profits subject to production technology constraints. The unit revenue function is a constant elasticity transformation (CET) composite of the unit price of domestic sales and the unit price of exports. The unit costs are divided between value-added (payments to labour and capital) and intermediate inputs (domestic and imported) as in Bhattarai (2007b):

$$\max \Pi_{j,t} = \left( (1 - \delta_i) PD_{j,t}^{\frac{\sigma_y - 1}{\sigma_y}} + \delta_i PE_{j,t}^{\frac{\sigma_y - 1}{\sigma_y}} \right)^{\frac{1}{\sigma_y - 1}} - \theta_i PY_{j,t} - \theta_i^d \sum_{t=0}^{\infty} a_{i,j} P_{i,t} - \theta_i^m \sum_{t=0}^{\infty} a_{i,j}^m P_{i,t} \quad (7)$$

where:  $\Pi_{j,t}$  is the unit profit of activity in sector  $j$ ;  $PE_{j,t}$  is the export price of good  $j$ ,  $PD_{j,t}$  is the domestic price of commodity  $j$ ;  $PY_{j,t}$  is the price of value added per unit of output in activity  $j$ ;  $Y_{j,t} = \Psi_i [\gamma_i L_{i,t}^{\rho_i} + (1 - \gamma_i) K_{i,t}^{\rho_i}]^{\frac{1}{\rho_i}}$ ,  $\sigma_y$  is a transformation elasticity parameter;  $P_{i,t}$  is the price of final goods used as intermediate goods;  $\delta_i$  is the share parameter for exports in total production;  $\theta_i$  is the share of costs paid to labour and capital;  $\theta_i^d$  is the cost share of domestic intermediate inputs;  $\theta_i^m$  is the cost share of imported intermediate inputs;  $a_{i,j}$  are input-output coefficients for domestic supply of intermediate goods;  $a_{i,j}^m$  are input-output coefficients for imported supply of intermediate goods. Their investment activities, which depend on sector specific profitability conditions result in accumulation of sector specific capital net of depreciation. A greater amount of capital enhances productivity of labour and raises the wage rate. Employment in each sector is the CES aggregation over all skill categories  $h$  according to  $L_{i,t} = \sum_{h=1}^H \left[ \zeta LS_{i,t}^h \frac{\sigma_{lh} - 1}{\sigma_{lh} - 1} \right]^{\frac{1}{\sigma_{lh} - 1}}$  where the supply of households match the employment across industries,  $LS_t^h = \sum_{i=1}^N LS_{i,t}^h$ . The size of employment in industry  $i$ ,  $L_{i,t}$  is the result of matching process between existing workers and new entrants and is subject to fluctuations due to job separation or finding rates across sectors for each type of skill  $h$ .

### 3.3 Equilibrium unemployment (vacancies, job search and matching)

As stated above in section 3.1, the transition from employment to unemployment or to inactive states and from unemployment to employment or to inactive states depend very much on the heterogeneity of preferences and technology determined by demand and supply conditions in the economy. This process is shown in the equilibrium unemployment model where unemployed households do not contribute in production but take non-labour income and transfers for their consumption.

Inflows and outflows in unemployment depend on the generosity of benefits, the bargaining power of workers and unions, information about the vacancies and applicants, and the competitiveness of the economy.

The phenomenon of equilibrium unemployment results from the interaction among  $N$  number of firms and unions (representing  $H$  number of households) which bargain over wages and employment. Following the market signals of demand and relative prices and costs of inputs, profit maximising firms create vacancies for specific tasks and hire workers when they find suitable candidates for these jobs. Similarly there are workers seeking jobs that match their skills and others who quit jobs and join the pool of unemployed who may choose to quit jobs and become unemployed. Market specific idiosyncratic shocks cause such entries and exits in the labour market. Equilibrium unemployment and wage rates result from a Nash-bargain between workers and firms. Whether the rate of unemployment falls or rises depends on the relative proportion of entry and exit into the labour market.

Matching and bargaining functions across all  $N$  industries are key elements determining equilibrium unemployment and are modeled here following Pissarides (1979, 2000 and 2011). The Matching function (Beveridge curve) gives equilibrium conditions in the labour market balancing entry and exit from unemployment by aggregating sector and skill specific vacancies ( $V_{i,t}^h$ ) and unemployment ( $UN_{i,t}^h$ ) with job creation as:

$$M_t = M(V_t, UN_t) = V_t^{\gamma_t} UN_t^{(1-\gamma_t)} \quad (8)$$

where  $M_t$ ,  $V_t$  and  $UN_t$  denote the aggregate number of matching, vacancies and unemployment respectively among job seekers at time  $t$  and aggregate variables are geometric means of household level variables<sup>5</sup>. The matching parameter  $\gamma_t$  is between zero and one and varies over time. It can be adjusted for prosperous period when there are more vacancies than job seekers or in recession when there are more unemployed than vacancies. In steady state it should be about 0.5 to reflect the balance between job creation and job destruction. Heterogeneity in the labour market is reflected by sector and skill specific  $M_{i,t}^h$ ,  $V_{i,t}^h$  and  $UN_{i,t}^h$ . These capture the labour market conditions where production sectors suffer from shortages of certain skills while facing abundance of other skills. In each case job seekers and employers bargain over expected earnings by maximising the Nash-product ( $NP_{i,t}^h$ ) of the bargaining game over the difference between the earnings from work ( $W_{i,t}^h$ ) than in being unemployed ( $UN_{i,t}^h$ ) and earnings to firms from filled ( $J_{i,t}^h$ ) and vacant jobs ( $V_{i,t}^h$ ).

$$NP_{i,t}^h = (W_{i,t}^h - UN_{i,t}^h)^{\theta_b^h} (J_{i,t}^h - V_{i,t}^h)^{1-\theta_b^h} \quad (9)$$

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<sup>5</sup>  $V_t = \prod_{i=1}^N V_{i,t}^h$ ;  $UN_t = \prod_{i=1}^N UN_{i,t}^h$ ;  $M_t = \prod_{i=1}^N M_{i,t}^h = \prod_{i=1}^N M(V_{i,t}^h, UN_{i,t}^h)$ .

Market imperfections in the labour market create opportunity of gains from bargains which is divided between firms and workers as indicated by parameter  $\theta_b^h$  that can assume any value between zero and one, reflecting the relative strength of unions (workers) over firms in such bargains. Symmetric solution of this satisfies joint profit maximisation condition as:

$$(W_{i,t}^h - UN_{i,t}^h) = \theta_b^h (J_{i,t}^h + W_{i,t}^h - V_{i,t}^h - UN_{i,t}^h) \quad (10)$$

In aggregate the job search model can be explained using three simple equations as summarised by Pissarides (1979, 2000).

First, for each skill type  $h$  the dynamics of unemployment depends on the rate of job destruction,  $\lambda_t^h (1 - un_t^h)$ , and the rate of job creation,  $\theta_t^h q(\theta_t^h) un_t^h$  as  $\Delta un_t^h = \lambda_t^h (1 - un_t^h) - \theta_t^h q(\theta_t^h) un_t^h$ . The steady state equilibrium implied by this is:

$$un_t^h = \frac{\lambda_t^h}{\lambda_t^h + \theta_t^h q(\theta_t^h)}; \quad un_T = \frac{\lambda_T}{\lambda_T + \theta_T q(\theta_T)} \quad (11)$$

where  $\lambda_t^h$  is the rate of idiosyncratic shock of job destruction of household type  $h$  and  $\theta_t^h$  is the ratio of vacancy to the unemployment and  $q(\theta_t^h)$  is the probability of filling a job with a suitable candidate through the matching process explained in (8). Then  $un_T$  is the equilibrium unemployment rate average across all households expressed in terms of averages of  $\lambda_t^h$ ,  $\theta_t^h$  and  $q(\theta_T)$  given by  $\lambda_T$ ,  $\theta_T$  and  $q(\theta_T)$  respectively.

Secondly the upward sloping wage curve in  $(\theta_t^h, w_t^h)$  space shows positive links between the reservation wage ( $z_t^h$ ) the price of product  $p$  and cost of hiring ( $\theta_t^h c_t^h$ ) implying higher wage rates for tighter labour markets as:

$$w_{i,t}^h = z_t^h (1 - \theta_b^h) + \theta_b^h p_t (1 + \theta_t^h c_t^h) \quad (12)$$

Finally there is a downward sloping job creation curve  $w_t^h = p_t - (r_t + \lambda_t^h) \frac{p_t c_t^h}{q(\theta_t^h)}$ , where  $p_t$  is the price of product,  $w_t^h$  the wage rate, and  $(r_t + \lambda_t^h) \frac{p_t c_t^h}{q(\theta_t^h)}$ , is the cost of hiring and firing. It shows the possibility of job creation at lower wage rates and creation of fewer jobs at higher wage rates. The optimal job creation (demand for labour curve) occurs when firms balance the marginal revenue product of labour to wage and hiring and firing costs<sup>6</sup>.

Derivations of these equations from value functions of employed and unemployed workers and from occupied and non-occupied vacancies along with the union-firm wage bargaining is based on Pissarides (2000). It is important to consider the general equilibrium impacts of these vacancies,

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<sup>6</sup>See Nash (1953), Calvo (1979), Diamond and Maskin (1979), Nickell (1982), Lockwood and Manning (1989), Layard and Nickell (1990), Driffill and Schultz (1992), Blanchflower and Oswald (1994) and Dixon and Rankin (1994) for bargaining for wage and employment.

redundancies and unemployment on the growth of output, employment, relative prices, allocation of factors between private and public sectors and welfare of various categories of households in the economy. Whilst the frictional unemployment literature ( Smith (2011), Zannetti (2010), Krause et al. (2008)) suggests that some degree of unemployment can make an economy more flexible and allows a smooth process of adjustment, a high rate of persistent structural unemployment may result in the lower level of output and living standards. In general the preferences of households between the current and future consumption and leisure and labour supply determines the overall dynamics of the economy. The tax and transfer system, internal and external competition, tastes and technological factors influence on the structure of the economy.

### 3.4 Trade arrangements

The economy is open. Exports and imports are guided by the ratio of domestic to foreign prices and balance in the long-run. Total absorption in the economy is given by the standard Armington function aggregating domestic supplies and imports as:

$$A_{i,t} = \Psi \left( \delta_i^d D_{i,t}^{\frac{\sigma_y-1}{\sigma_y}} + \delta_i^m MM_{i,t}^{\frac{\sigma_m-1}{\sigma_m}} \right)^{\frac{\sigma_m}{\sigma_y-1}} \quad (13)$$

where  $A_{i,t}$  is the CES aggregate of domestic supplies  $D_{i,t}$  and import supplies  $MM_{i,t}$  for each sector,  $\delta_i^d$  is the share of domestically produced goods,  $\delta_i^m$  is the share of good  $i$  imports,  $\sigma_m$  is the elasticity of substitution in the aggregate supply function, and  $\Psi$  is the shift parameter of the aggregate supply function. The value of exports ( $PE_{i,t}E_{i,t}$ ) balances to the value of imports ( $PM_{i,t}MM_{i,t}$ ) over the model horizon, implying zero net capital inflows ( $\pm F = 0$ ) in the economy.

$$\sum_{t=0}^{\infty} \sum_{i=1}^N PE_{i,t}E_{i,t} - \sum_{t=0}^{\infty} \sum_{i=1}^N PM_{i,t}MM_{i,t} = \pm F = 0 \quad (14)$$

### 3.5 Drivers of the dynamics in the economy

Dynamics of the economy are driven by the accumulation of capital and growth in the labour supply subject to equilibrium unemployment. Capital stock evolves naturally with its initial and boundary conditions:

$$I_{i,t} = K_{i,t} - (1 - \delta) K_{i,t-1} \quad (15)$$

As stated above there is an exogenous process of growth of time endowment,  $\bar{L}_t^h$ ;  $\bar{L}_t^h = \bar{L}_0^h e^{nt}$ . Those who are in employment drive the production process of the economy. The labour market dynamics depends on six transition rates  $ss_t^h$  and  $s_t^h$  and  $ff_t^h$  and  $ff_t^h$  and  $ie_t^h$  and  $iu_t^h$  and transition probabilities as given by (4), (5) and (6) above. Changes in these six transition rate parameters,

because of the shocks in demand and supply situations in the public, private or the external sectors of the economy, are the major causes of fluctuations in labour supply and output, employment ( $SE_t^h$ ) and unemployment ( $SU_t^h$ ) and in and out of labour force or being in active or inactive states ( $SI_t^h$ ) in the economy.

### 3.6 Public sector

At every period government provides public services to households and pays for them using taxes. Such tax transfer system influences choices of households and creates distortions in the system. Government provides  $G$  amounts of public services, from ( $g_{i,t}$ ) subsectors including education, health, security, law and order, to households. Thus the government consumption is represented by:

$$G_t = \sum_{i=1}^N g_{i,t} \quad (16)$$

It collects revenue ( $RV_t$ ) from direct taxes on capital  $\left(\sum_{h=1}^H r_t \cdot tk \cdot K_t^h\right)$  and labour income  $\left(\sum_{h=1}^H w_t^h \cdot tw^h \cdot LS_t^h\right)$  and indirect taxes on consumption  $\left(\sum_{h=1}^H P_{i,t} \cdot tc_i^h \cdot C_{i,t}^h\right)$ .

$$RV_t = \sum_{h=1}^H P_{i,t} tc_i^h C_{i,t}^h + \sum_{h=1}^H r_t tk K_t^h + \sum_{h=1}^H w_t^h tw^h LS_t^h$$

Budget is balanced over the model horizon:

$$\sum_{t=1}^{\infty} G_t = \sum_{t=1}^{\infty} \left( RV_t + \sum_{h=1}^H R_t^h \right) \quad (17)$$

The optimal level of public sector balances benefits and costs from the public sector activities.

### 3.7 Conditions for the Intertemporal General Equilibrium

Given the initial endowments of labour and capital, the intertemporal dynamics in the economy requires fulfilling five equilibrium conditions. For each period there is a balance between demand and supply for each sector  $i$  as the output ( $Y_{i,t}$ ) equals to consumption  $\left(\sum_{h=1}^H C_{i,t}^h\right)$ , investment ( $I_{i,t}$ ), exports ( $E_{i,t}$ ), public consumption ( $g_{i,t}$ ) and resources spent on the matching process ( $\theta_i M_{i,t}$ ) as in Merz (1995):

$$Y_{i,t} = \sum_{h=1}^H C_{i,t}^h + I_{i,t} + E_{i,t} + g_{i,t} + \theta_i M_{i,t} \quad (18)$$

and for each type of labour and the capital stock.

These equilibrium conditions should be consistent to intertemporal constraints of households, firms, the government and the international balance of the economy. Each household can engage in intertemporal lending and borrowing but its present value of expenditure should equal to the present value of income as given in (2).

Similarly each firm can finance investment by retained earning, bonds or equities but the expected present value of revenue should equal expected present value of the cost

$$\sum_{t=0}^T P_{i,t} Y_{i,t} = \sum_{t=0}^T \left[ r_t (1 - t_k) K_{i,t} + \sum_{h=i}^H w_t^h L_{i,t}^h \right] \quad (19)$$

The government may have deficits or surplus in its budget but it must ensure that present value of public spending equals the present value of revenue as given in (17)

The economy may have balance of payment surplus or capital outflows or inflows but the present value of exports must equal the present value of imports ( $F = 0$ ) as given in (14). Exchange rate appreciation (depreciations) of the home currency makes this happen.

The economy will converge to the steady state equilibrium unemployment rate for aggregate (for each type  $h$  worker-household) at the terminal period as given in (11).

The capital stock at the terminal period will be:

$$I_{i,T} = (g_i + \delta_i) K_{i,T-1} \quad (20)$$

These conditions should fulfil the basic Euler equation of optimisation that states that marginal utility of consumption or marginal product of a factor between two periods should equal in present value terms  $[U_t^{h'}(C_t^h, l_t^h) = \beta^h U_{t+1}^{h'}(C_{t+1}^h, l_{t+1}^h)]$  and  $\frac{\partial Y_{j,t}}{\partial K_{j,t}} = \frac{\partial Y_{j,t+1}}{\partial K_{j,t+1}} \frac{1}{1+r_t}$  or  $\frac{\partial Y_{j,t}}{\partial L_{j,t}^h} = \frac{\partial Y_{j,t+1}}{\partial L_{j,t+1}^h} \frac{1}{1+w_t^h}$ . In addition these solutions should satisfy the initial (starting) and terminal (steady state) conditions. In general as Hicks (1939) had mentioned that the relative prices of commodities and factors of production keep adjusting until the demand and supply balance. For a model with  $T$  years and  $N$  sectors such equilibrium is given by the system of  $T.N(N-1)$  relative prices expressed in terms of preferences, technologies and policy instruments that clear all goods and factor markets. In the Walrasian system given the vector of prices,  $p_t = (p_{1,t}, p_{2,t}, \dots, p_{j,t} \dots p_{n,t})$  demand for commodities are expressed in terms of the price vector  $X_{j,t}^d = X_{j,t}^d(p_t) = X_j^d(p_{1,t}, p_{2,t}, \dots, p_{j,t} \dots p_{n,t})$  and supply functions defined similarly  $X_{j,t}^s = X_{j,t}^s(p_t) = X_j^s(p_{1,t}, p_{2,t}, \dots, p_{j,t} \dots p_{n,t})$ . The excess demand functions  $E(p_t) = X_{j,t}^d(p_t) - X_{j,t}^s(p_t)$  reflect the gap between demand and supply for each commodity for  $j = 1, 2, \dots, n$ . Economy has  $n$  excess demand functions. The general equilibrium



is a price vector,  $p_t^*$ , such that  $p_t^* \geq 0$ , if  $E(p_t^*) \leq 0$   $p_t^* = 0$ . The excess demand functions are single valued continuous functions, bounded from below  $E(p_t) \geq b_t$  for all  $p_t^*$  and are homogeneous of degree zero in all prices  $E(p_t) = p_t$ . Only relative prices matter to satisfy the Walras' law;  $p_t \cdot E(p_t) = \sum_{i=1}^n p_{i,t} \cdot E_i(p_t) = 0$  for all  $p_t \geq 0$ . If the excess demand functions satisfy above properties then, the existence of the general equilibrium is guaranteed by fixed point theorems. The fixed equilibrium point is found in Arrow-Debreu system by continuous transformation of the non-empty convex set onto itself  $p_t^* \rightarrow E(p_t^*) \rightarrow p_t^*$ . Given the linear homogeneity properties of demand and supply functions, equilibrium is stable and unique.

## 4 General equilibrium solution procedure

Optimal allocation of resources in the model economy requires fulfillment of the above intertemporal equilibrium conditions in goods and factor markets in addition to the general equilibrium in each period. Abstract theoretical proofs of existence, uniqueness and stability of the dynamic general equilibrium in the model discussed above requires use of Arrow-Debreu (1954) theorem or algorithm. Whalley (1977) had applied Scarf's simplex algorithm of the form  $By = \hat{w}$  to prove existence of equilibrium in the presence of taxes (here  $B$  is the coefficient matrix,  $y$  is a vector of endogenous variables and  $\hat{w}$  is a vector of endowments). Rutherford (1998) applied discrete approximation of infinite horizon approach to compute long-rung growth and welfare (see also Mercenier and Philippe (1994)). In this paper we compute the dynamic general equilibrium model with heterogeneous households and firms taking Pissarides (1979, 1985, 2011) and Mortensen and Pissarides (1994) type equilibrium unemployment as an outcome of matching of vacancies and unemployment and bargaining between unions and firms in each period over the model horizon.

The solution procedure of this model now can be characterised in terms of optimal quantities

$$Q : \{U_t, U_t^h, C_t^h, l_t^h, C_{i,t}^h, LS_{i,t}^h, \Pi_{i,t}, Y_{j,t}, V_{i,t}, K_{j,t}, L_{j,t}, I_{i,t}, D_{i,t}, E_{i,t}, M_{i,t}, A_{i,t}, RV_t, G_t\}_t^T \quad (21)$$

and the relative prices

$$P : \{P_{i,t}, PY_{i,t}, PD_{i,t}, PE_{i,t}, w_t^h, r_{i,t}, R_{i,t}, PE_{i,t}, PM_{i,t}\}_t^T \quad (22)$$

for each period  $t$  up to the model horizon  $T$ , and which are determined by the system of parameters describing the behavior, technology, policy and institutional parameters of the economy.

These prices are consistent to the optimal decisions of households and firms given the set of policies in operation and are determined by the parameters of preferences  $(\sigma_c^h, \sigma_l^h, \sigma_u^h, \delta_c^h, \delta_l^h, \sigma_u^h, \sigma_{ucl}^h)$ , gross production  $(\sigma_i^y, \psi_i, \theta_i, \theta_i^d, a_{i,t})$ , value added  $(\sigma_p, \delta_i^p, \delta_i)$ , trade  $(\delta_i^d, \delta_i^m, \sigma_m)$  and the tax trans-

fer policies  $(tc_i^h, tw_i^h, tk_i, T_t^h)$ . Economy starts with the initial capital stock  $(K_{i,0})$  and is subject to the exogenous process of evolution of total labour force  $(LF_t = \sum \bar{L}_{i,t}^h)$ , long run steady state conditions in terms for growth rate, depreciation and the real return on investment across sectors  $(g_i, \delta_i, r_i)$ . The working out of the dynamic equilibrium process through elimination of excess demands across all markets,  $\{Q\}_t^T : ED_{i,t} = D\{P\}_t^T - S\{P\}_t^T = 0$ , results in the system of relative prices being determined simultaneously with all these parameters taking supply, demand and policy side interactions in the economy.

Allocations of resources and welfare obtained through these price vectors are optimal in the sense that these meet optimal conditions for households and firms. Whilst long run cycles are generated because of shocks in any one or a subset of these parameters (see Table 1) the policy generated shocks are the ones that matter most in policy analyses.

The model is benchmarked to the reference path of the evolving economy for years,  $t = 2009$  to  $T = 2101$  with ten categories of households  $\{h_1, \dots, h_{10}\}$  and ten sectors  $\{i_1, \dots, i_{10}\}$  with the production and demand function parameters calibrated from the ten sector input-output table aggregated from the 111 sector input-output table available in the Input-output (IO) Table 2009 from the Office of the National Statistics (ONS) of the UK (Table 2).

The distribution of income among categories of households is obtained from the ONS Reference Tables-"The effects of taxes and benefits on household income, 2009/10." These income shares are also applied to decompose the consumption shares by sectors as shown in Tables 3 and 4. VAT rates, corporation tax rate and household income tax rate are taken from the current budget statements. Imports and exports by sectors as well as the production taxes are also taken from the IO-Table.

Table 1: Key parameters of general equilibrium model with equilibrium unemployment in the UK

Parameters	$\sigma_c^h$	$g_{i,t}$	$r$	$\sigma_{ucl}^h$	$\delta_{i,t}$	$\sigma_y$	$\sigma_k$	$\sigma_m$	$t_c^h$	$tw_i^h$
Values	1.55	0.02	0.03	0.98	0.02	2.0	1.5	3.0	0.2	0,0.32,0.4, 0.45

The model includes transfers from high income to the low income households in the form of conditional unemployment and benefit system as discussed in Mirrlees et al. (2010), Bhattarai and Whalley (2009), Blundell, Pistaferri, and Preston (2008), Blundell (2001) and Hutton and Roucoo (1999). Households consider intertemporal factors in labour supply with continuous rather than discrete choice of labour hours. Focus on equilibrium unemployment with heterogeneity of firms and labour provides deeper understanding of the economy than from single sector long run growth model such as Basu and Bhattarai (2011). The household side parameters of the model are given in Table 3.

The model is solved using the Newtonian Path-search Ferris algorithm in the GAMS/MPSGE (Rutherford, 1998) and has a long horizon of 2009-2101.

Table 2: Input-output coefficient matrix of UK, 2009 (based on the IO Table from the ONS)

	Agri	Prod	Constr	Dist	Infcom	Finns	Rlest	Prfspp	Ghlthd	Othrsrv
Agri	0.0589	0.0109	0.0011	0.0056	0.0001	0.0000	0.0000	0.0001	0.0002	0.0002
Prod	0.0199	0.2490	0.1577	0.2319	0.0787	0.0272	0.0035	0.0351	0.1549	0.0825
Constr	0.0117	0.0040	0.2583	0.0456	0.0099	0.0195	0.0550	0.0062	0.0147	0.0097
Dist	0.0268	0.0147	0.0156	0.2111	0.0245	0.0606	0.0039	0.0270	0.0316	0.0229
Infcom	0.0062	0.0054	0.0065	0.0638	0.0953	0.0596	0.0071	0.0295	0.0216	0.0431
Finins	0.0237	0.0127	0.0197	0.0392	0.0164	0.0940	0.0251	0.0176	0.0148	0.0152
Rlest	0.0038	0.0013	0.0089	0.0523	0.0070	0.0178	0.0064	0.0051	0.0117	0.0089
Prfspp	0.0217	0.0215	0.0757	0.1525	0.1190	0.1213	0.0256	0.2286	0.0722	0.1553
Ghlthd	0.0009	0.0021	0.0068	0.0184	0.0067	0.0116	0.0190	0.0270	0.1051	0.0097
Othrsrv	0.0021	0.0008	0.0005	0.0060	0.0186	0.0043	0.0003	0.0066	0.0106	0.0820
Ten sectors: Agri (Agriculture), prof (Production), Constr (Construction), Infcom (Information and communication),										
Finins (Finance and insurance), Rlest (Real estate), Prfspp (Professional and support activities), Dist (Distribution),										
Ghlthd (Government,Health and Education), Othrsrv (Other Services) ; Data source ONS. UK,										

## 5 Analysis of Results of EU Dynamic CGE Model

Policy experiments are conducted once the model is calibrated to the reference benchmark path up to 2100 starting from 2009, which assumes that the tax structure of 2009 continues over the model horizon. Then the tax reform scenarios are based on the reforms on the tax-transfer system after 2009 including 1) zero tax rate on income of households in the bottom decile, reduction of income tax rate to 45 percent for the households in top decile and 32 percent income tax rate for households in the middle income group (Table 3). It also includes transfers to household deciles as existed in 2011. 2) the corporate tax to 22 percent and the value added tax (VAT) increased from 17.5 percent to 20 percent; 3) production taxes on capital and labour input use are derived from production taxes and subsidies ( $\tau_s$ ) and margins ( $\frac{MR}{Y}$ ) entries of the input-output table from the ONS as given in Tables 4.

Model solutions provide a perspective on the evolution of the economy for the entire model horizon and can be seen as capturing both cycles and growth effects in the medium term (5-10 years) and longer terms (50 to 100 years). They are used to form scenarios of fiscal policies or to assess the robustness of the elasticities in consumption and production that reflect the reactions from the private sector to the shocks either from policy initiatives or from the technology. What happens to the economy if system of vacancies and job matching process becomes more efficient? Which of the several taxes is more efficient or which set of policy alternatives is better for the higher level of utilities and lifetime income of the households and for the growth of the economy? How does the equilibrium unemployment evolve under the current system and how does it compare to the counter-factual scenarios? How does the distribution of income evolve over time? How sensitive are the model solutions to the set of intertemporal substitution elasticities between consumption and

Table 3: Benchmark income and expenditure shares and income tax rates for households, 2009

	Income share	Consumption share	Income tax rate
Deciles	$\theta^h$	$\delta^h$	$tw_i^h$
$h_1$	0.0281	0.0627	0.0
$h_2$	0.0433	0.0552	0.32
$h_3$	0.0551	0.0624	0.32
$h_4$	0.0669	0.0850	0.32
$h_5$	0.0789	0.0966	0.32
$h_6$	0.0908	0.1067	0.32
$h_7$	0.1081	0.1078	0.32
$h_8$	0.1276	0.1323	0.32
$h_9$	0.1521	0.1409	0.40
$h_{10}$	0.2493	0.1945	0.45
Note: The lowest ( $h_1$ ) to the highest income ( $h_{10}$ ) categories.			
Household categories: Data source ONS, UK			

leisure or to the intra-temporal elasticities of substitution in consumption, production and trade? For above reasons this dynamic general equilibrium process contains very realistic micro-foundation to macro level policy analysis than is common in the DSGE models.

Only a few selective model scenarios are discussed below and details of model solutions are available upon request. In a nutshell the main results of this model could be summarised as follows:

1. The level of utility for each household  $h$  rises over the model horizon, *i.e.*  $U_1^h < U_2^h < \dots < U_T^h$ . Whilst the levels of utilities of all households rises over time, the utility of richer households rises faster than that of the poorer households as shown in the Figure 7. The impacts of redistributive tax-transfer policies are clear as shown by the lifetime income distribution among households, generating overall gini coefficient of 30.7 percent in counterfactual. Changes in income and position of households in deciles in counterfactual are illustrated in Figures 8 and 10. The overall welfare index in the counter-factual scenarios declines relative to its benchmark values because of distortions (Fig. 9).

Table 4: Benchmark production tax-subsidy rate, shares of labour and capital income, imports and margins in total output and sectoral shares of output

	$\tau_s$	$\frac{wL}{Y}$	$\frac{rK}{Y}$	$\frac{MM}{Y}$	$\frac{MR}{Y}$	$\frac{Y_i}{Y}$
Agri	-0.0900	0.0932	0.2175	0.2506	0.1463	0.0116
Prod	0.0035	0.0934	0.0656	0.2701	0.2449	0.3694
Constr	0.0037	0.2129	0.1771	0.0063	0.0491	0.0705
Dist	0.0348	0.5599	0.2203	0.1028	-0.7443	0.0915
Infcom	0.0066	0.2861	0.1724	0.0755	0.0833	0.0527
Finins	0.0116	0.2576	0.3054	0.0535	-0.0441	0.0701
Rlest	-0.0030	0.0383	0.4741	0.0051	0.1137	0.0564
Prfspp	0.0034	0.2849	0.1543	0.1008	0.0737	0.1066
Ghlthd	0.0008	0.4801	0.0710	0.0048	0.0060	0.1456
Othrsrv	0.0144	0.3653	0.1373	0.0710	-0.0177	0.0256

Note:  $\tau_s$ : tax and subsidy in production  $\frac{wL}{Y}$ : labour share  $\frac{rK}{Y}$ : capital share  $\frac{MM}{Y}$ : import GDP ratio  $\frac{MR}{Y}$ : ratio of trade and transport margin to GDP  $\frac{Y_i}{Y}$ : sectoral output shares. Data source ONS, UK

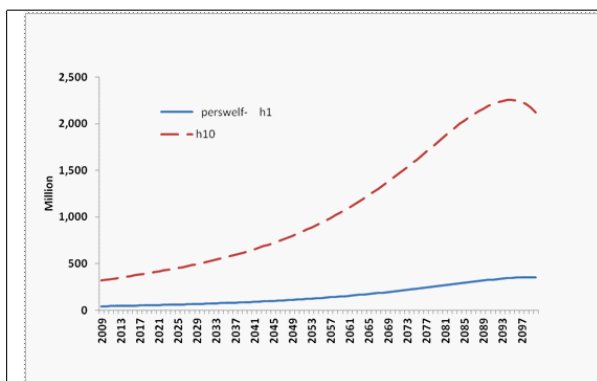


Fig. 7: Utility levels of households

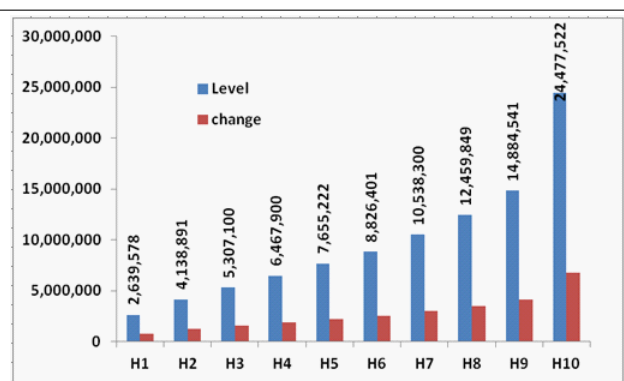


Fig. 8: Level of lifetime income of households ('000)

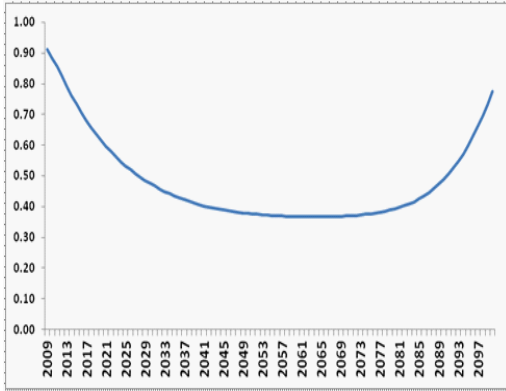


Fig. 9: Total welfare relative to the benchmark

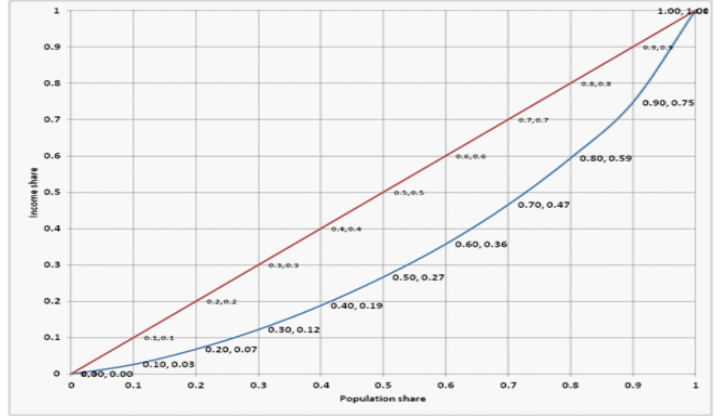


Fig. 10: Distribution of lifetime income implied by the model solutions

- The level of GDP rises steadily as a result of the steady increase in the labour supply and capital stock (Fig. 11-13), as does the aggregate consumption of households (Fig. 14). However it was noticed that the current reforms slightly distort the economy causing the counterfactual levels to be lower than in the benchmark reference path.

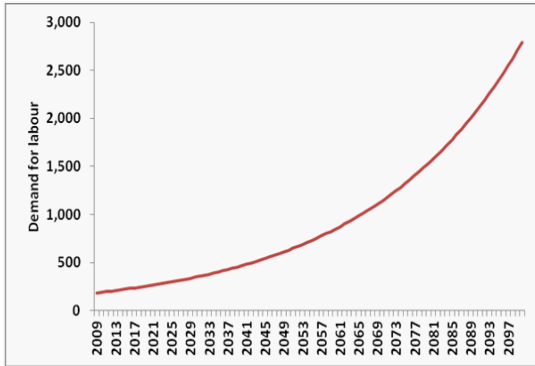


Fig. 11: Evolution of demand for labour

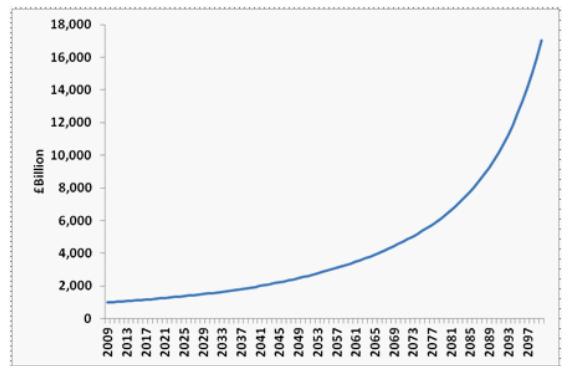


Fig. 12: Evolution of GDP

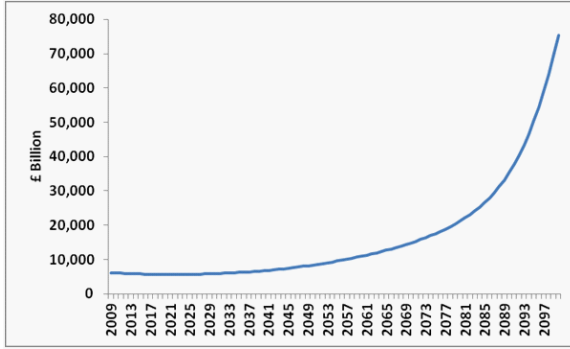


Fig. 13: Evolution of capital stock

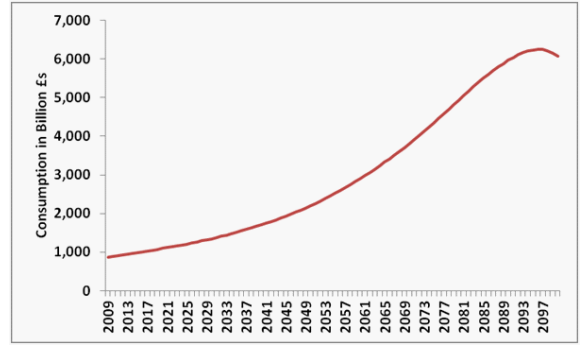


Fig. 14: Evolution of total private consumption

3. The public transfer programme requires revenue to be higher than public spending (Fig. 15). However, higher taxes result in the higher prices of goods and services (Fig. 16). The general price index rises for the economy (Fig. 17). These distortionary taxes raise the cost of production and lower the level of output in all 10 sectors, with the construction sector being hardest hit in this process. While maintaining transfer requires revenue to be higher than public spending it has negative impacts on prices and output. As discussed in the literature [Mirrlees et al. (2010), Bhattarai and Whalley (2009) and Blundell et al.(2008)], the economic costs of transfer programmes are very high.

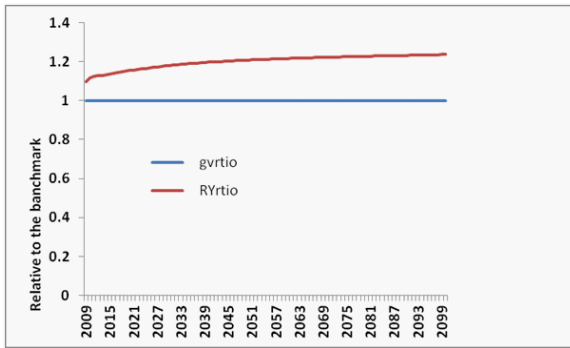


Fig. 15: Ratio of public spending, and revenue relative to the benchmark

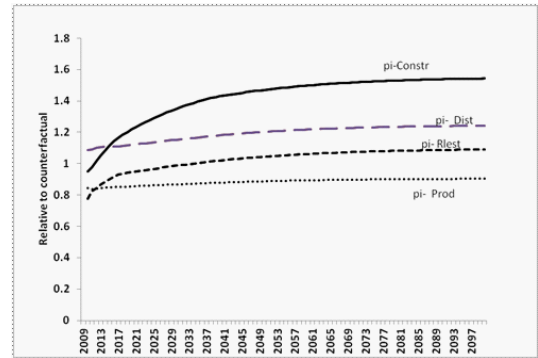


Fig. 16: Relative prices of goods and services relative to the benchmark

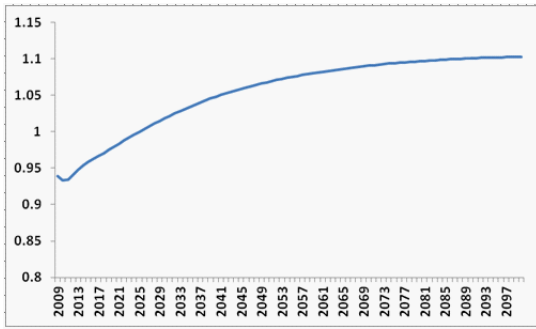


Fig. 17: Price index under reform relative to the benchmark

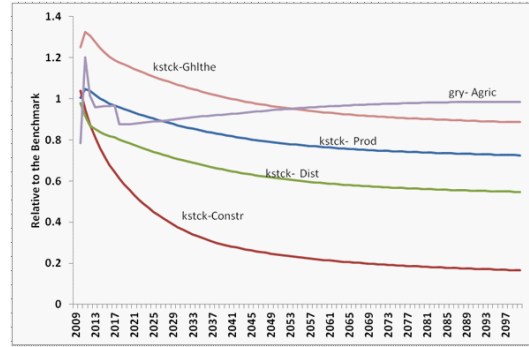


Fig. 18: Capital stock in counterfactual relative to the benchmark

4. The impact of the tax reforms on equilibrium unemployment are very small if the matching technology works well in the system (Fig. 21). Better information technology can make matching smoother so that number of matches increase according to number of vacancies and unemployment in the system. If such process could work in the economy the issue of equilibrium unemployment does not seem to matter much in the long run for the long run growth. Intertemporal choice between labour-leisure and consumption, and saving and investment, as well as the technological advancements are more important factors driving the long run growth of the production sectors and the economy. The evolution of matching and unemployment levels resulting from the complex mechanism of demand and supply processes in the economy are shown in Figures 19 and 20. Active policies for job search and matching result in lower equilibrium unemployment rate the matching ratio in the counterfactual (Fig. 21) but there are fluctuations in the matching ratios because of cyclical factors in the economy (Fig. 22). Model results summarised in figures 23 and 24 show that education, health, government and professional services will contribute significantly to employment creation in coming years.

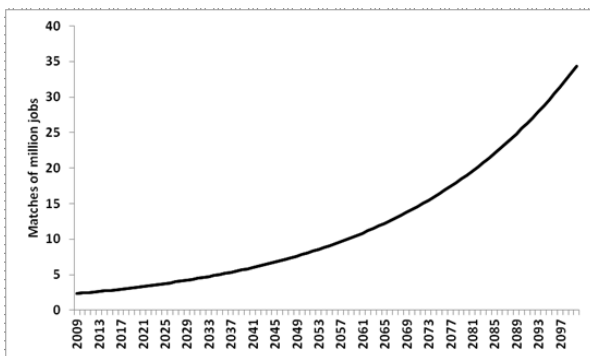


Fig. 19: Equilibrium matching of jobs

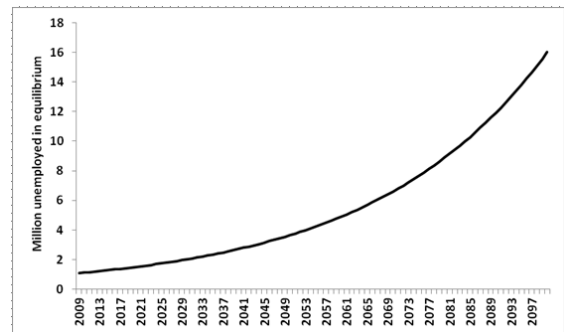


Fig. 20: Numbers unemployed in equilibrium



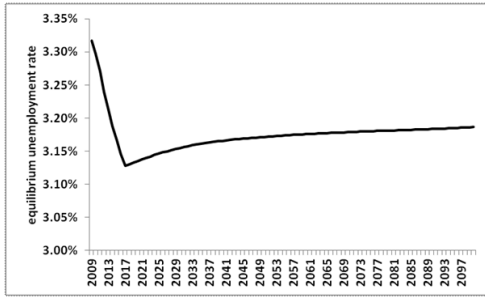


Fig. 21: Equilibrium unemployment rate

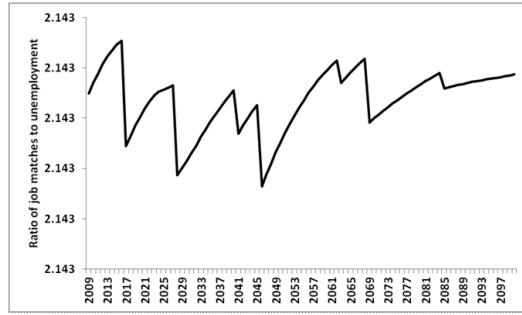


Fig. 22: Job matching unemployment ratio

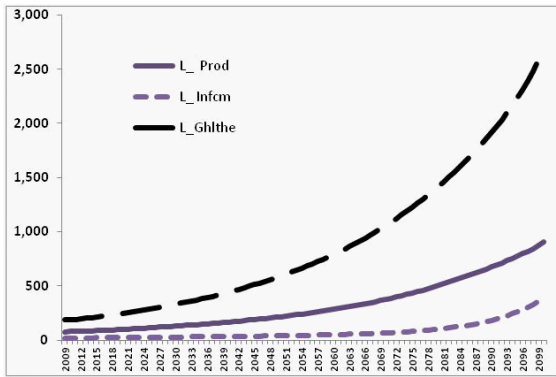


Fig. 23: Labour supply by sectors

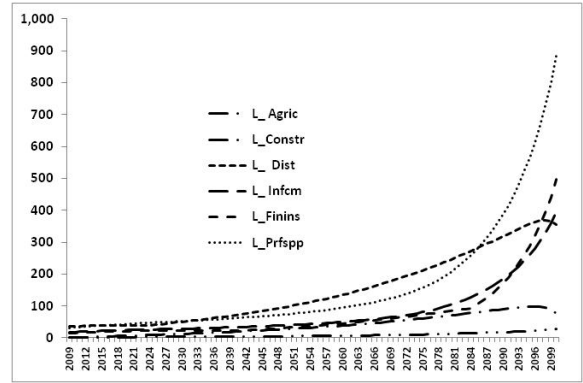


Fig. 24: Labour supply by sectors

- Long run business cycles caused by policy changes are represented in Figures 25-28. It is obvious that more shocks to model parameters, listed in Tables 1, 2, 3 and 4 in line of the DSGE models, will bring more of these cycles in output, consumption, labour supply and welfare and in patterns of income distribution. In our view the DSGE literature needs to move towards introducing heterogeneity among households and firms as in this paper to be more realistic in reflecting economic activity as a whole.

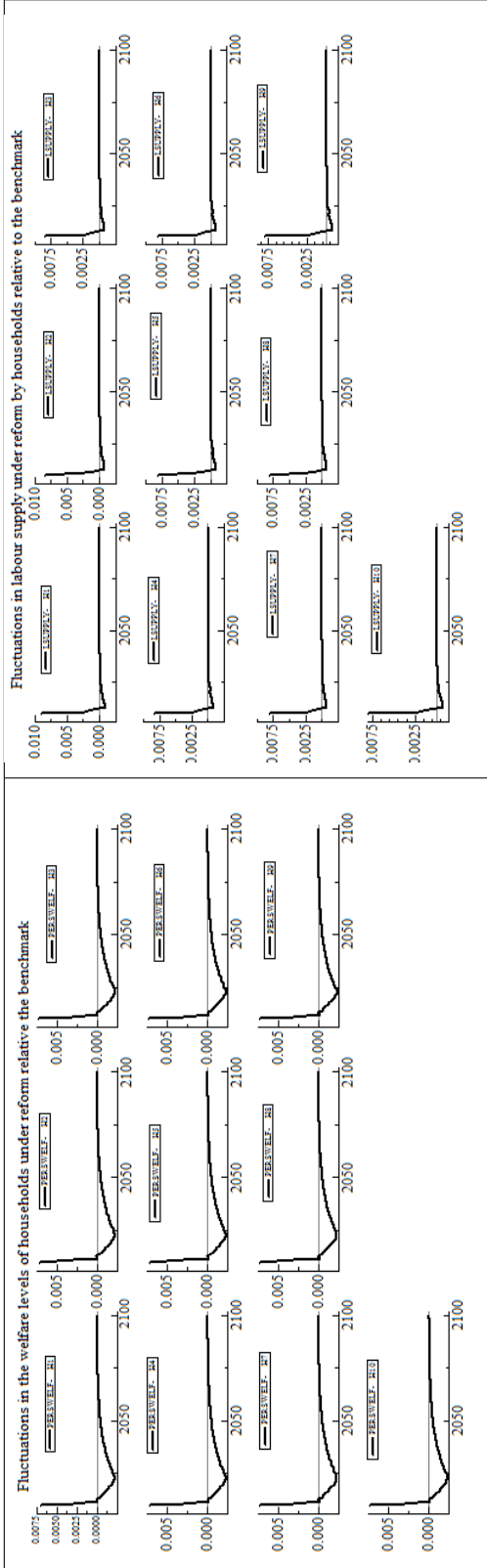


Fig. 25: Utility of Households in the Long Run in DCGE Model

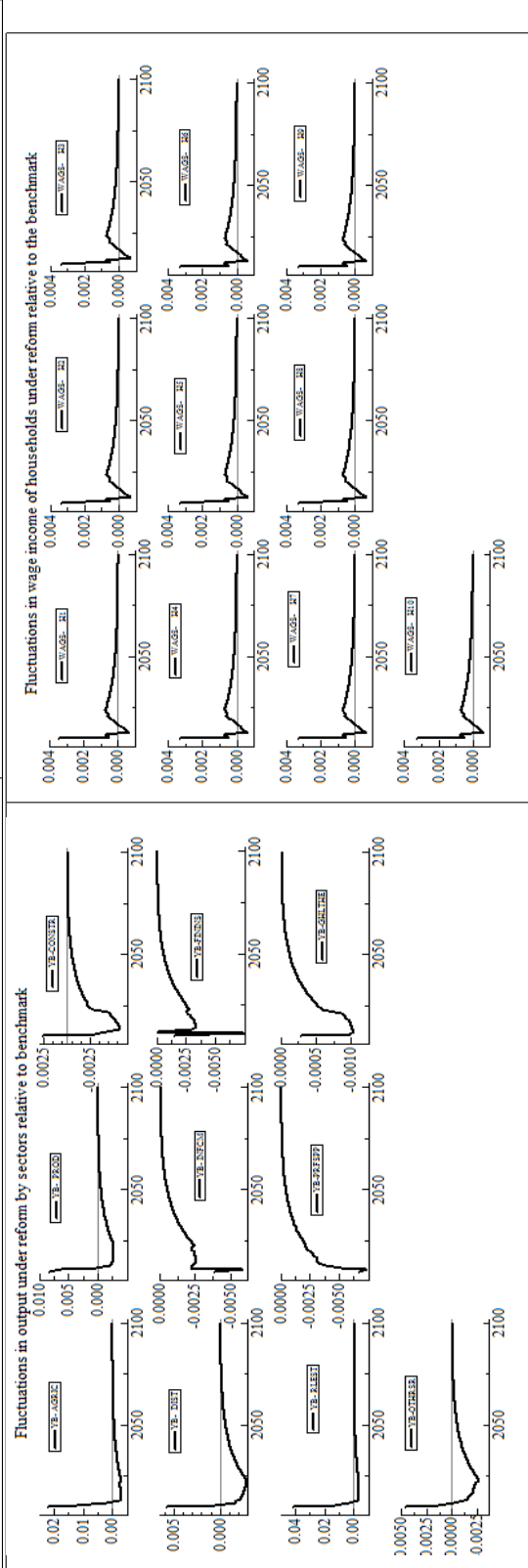


Fig. 26: Fluctuations in Labour Supply of Households in DCGE Model

Fig. 27: Cycles in the Level of Output by Sectors in DCGE Model

Fig. 28: Business Cycle Impact on Wages of Households in DCGE Model

The above results are indicative of the model solutions. Flexibility of goods and factor markets as reflected in the intra-temporal and inter-temporal elasticities of substitution are very important determinants for the time paths of variables. Knock-on effects of changes in one part of the system can have very extensive impacts in others. Great care is necessary using economic theories in setting up policies and in calibrating/choosing parameters before computing the model and interpreting the model solutions.

## 6 Conclusion

An attempt is made to incorporate the Pissarides (1979, 2011) and Mortensen and Pissarides (1994) type of equilibrium unemployment into a computable dynamic general equilibrium model to evaluate the impact of matching technology in the long-run growth and level of utilities and lifetime income of households in the UK economy. Dynamic interactions among heterogeneous consumers and producers generate interesting results. Utility of all households increase over time as levels of output, capital stock and labour supply rise over time. The model reproduces income distribution patterns and the Gini coefficients as one would expect from the analysis of the real data. Taxes create distortions and raise the prices of commodities in all sectors when taxes and transfers rise relative to the benchmark. The price index rises steadily, economy becomes more expensive, costs of production rise and output falls steadily relative to that in the benchmark economy in almost all sectors of the economy. The level of equilibrium unemployment rises but the job matching increases at a faster space in the policy reform scenario causing equilibrium unemployment to decline. Better matching technology lowers the rate of equilibrium unemployment rate as vacancies are filled more efficiently. Long run growth and redistribution are more sensitive to flexibility of markets as reflected in the intertemporal elasticity of substitution between leisure and consumption and substitutability of commodities in consumption and between capital and labour in production across sectors. More efficient matching technology improves efficiency in consumption and production, enhances growth and raises effectiveness in achieving the equity and growth objectives of the tax and transfer system existing in the UK. This is the first paper that includes the search and matching model and equilibrium unemployment with heterogeneous households and firms explicitly in this way in a dynamic computable general equilibrium model with taxes for the UK economy.

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