Macroeconomic policy in a large unionised economy*

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This paper models an economy with many sectors. In each sector a monopoly union sets the nominal wage, and then firms choose output. The economy is large in that all agents treat the general price level as fixed. The paper focusses on policy effects on employment. An increase in government expenditure in one sector will increase (decrease) total employment as sectoral employment is above (below) average. A general increase in government expenditure in fixed proportions will have a positive employment multiplier below unity. Policy which increases employment will lead to higher welfare.

1. Introduction

This paper seeks to explore two issues. Firstly, the implications of imperfect competition in the labour and product markets for macroeconomic equilibrium; second, its implications for the effectiveness of macroeconomic policy. It has long been recognised that imperfect competition can provide a coherent non-Walrasian equilibrium in which there is some form of unemployment. Monopolists in the product market, and unions in the labour market, might restrict output and employment below the market clearing level. Imperfect competition thus provides one important explanation of unemployment. On the practical level, casual empiricism also suggests that industrial concentration and high levels of unionisation characterise many sectors of modern industrial economies. Whilst imperfect competition may provide an interesting alternative approach to macroeconomic equilibrium, the question still remains as to whether it provides any results which are qualitatively different to competitive models, or at least give different insights into the workings of policy? The model presented in this paper gives an affirmative answer to this question. In a multi-sector unionised economy,

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there is not only scope for government to influence aggregate employment, but also to increase welfare.

The present paper develops and extends the approach in Dixon (1988), presenting a simple general equilibrium model of a unionised economy. Whilst the economy presented does not pretend to generality, it should be seen as a not atypical example. In section 2 of the paper the components of the model are set out. There are assumed to be many sectors in the economy, and in each sector a monopoly union sets the nominal wage, and output prices are determined through Cournot competition between firms who treat wages as given. When agents set prices or wages they treat the general price level as given, which seems appropriate in a large economy with many sectors. The basic macroeconomic equilibrium can be conceived as a two-stage perfect equilibrium for the private sector given government policy. In the second stage, outlined in section 2, firms choose outputs given the wage. In the first stage, outlined in section 3, unions choose nominal wages, given the 'objective' demand they face, the second-stage equilibrium relationship between wages and unemployment. Whilst unions have no direct interest in the wages set in other sectors (no 'envy' effect), the general level of wages determines the price level (the 'mark-up' effect). Thus we are able to model the first stage as a Nash equilibrium between unions: Each sectoral union choosing its wage given the wages set in the other sectors. The division of private sector wage and price decisions into two stages reflects the view that wages are less flexible than prices, in the sense that firms' outputs vary over a much shorter period than do wages. The equilibrium level of employment in each sector depends on the general degree of monopoly in the economy, union preferences, and the strength of sectoral demand relative to the general level of demand in the economy. Under the specific assumptions made, there is a log-linear relationship between sectoral employment and the ratio of sectoral expenditure to the geometric mean of sectional expenditures.

In section 4 we explore the ability of government to influence aggregate employment. As in Dixon (1988), we find that whilst for each government policy there is a unique private sector equilibrium and corresponding employment level, there exists a continuum of feasible employment levels that can be attained – a Natural Range of employment. The impact of macroeconomic policy is explored through a series of propositions. First we consider the effect of an increase in direct government expenditure in one sector – this will raise the level of wages in all sectors, raising employment in the chosen sector, reducing it in others. In Proposition 1 we show that the overall effect can be positive or negative, that it will be more positive the larger is employment in the sector, and that if the sector is below average employment there will be a reduction in aggregate employment. The method of financing expenditure influences the effectiveness of fiscal policy: Unlike the Walrasian model, tax financed expenditure increase have a larger (more
positive) multiplier than money financed increases (Proposition 2); the level of employment is higher the larger the proportion of expenditure which is tax financed (Proposition 3). An increase in the general level of expenditure always has an expansionary effect on employment (Proposition 4), and the fiscal multiplier is between zero and one (Propositions 5 and 6). There is thus crowding out in this model. Perhaps the most important result of the paper refers to the welfare effects of policy. Even though government expenditure is assumed to be 'waste', and despite the crowding out of consumption, in Theorem 2 we show that a government policy which increases total employment will increase total welfare (the sum of household utilities). This result is possible because (unlike the Walrasian case) the equilibrium is not Pareto-optimal, and unions ensure that employed households earn a surplus over the disutility of labour.

The model presented is of course specific, and the results relating to particular policy effects are not general. We would, however, argue that the model is of interest even if only regarded as an example: the positive and normative analysis of policy differ significantly from competitive models. Furthermore, we would argue that some general properties of this model will be common in multisector unionised economies. As shown in Dixon (1988, pp. 1140–1143), the Natural Range property is far from the exception. More importantly, we believe the welfare analysis of such models will reveal a far greater scope for welfare-improving government policy than in Walrasian models, if only because the equilibrium is not in general efficient, and the real wage will in general exceed the shadow price of labour.

2. The model

There are $n$ sectors $i=1,\ldots,n$. Each sector consists of a labour market and a product market. In the labour market there is a monopoly union which unilaterally sets the nominal wage. Labour is supplied through the union closed shop by households, who supply one unit of labour with a fixed disutility $d$. There is a single economy-wide labour market with perfect labour mobility.

In the product market there are $m$ firms who behave as Cournot oligopolists, choosing outputs given the market wage. We will now outline the specific assumptions made about the various agents in the economy – households, firms, government. Union behaviour is outlined in section 3.

2.1. Households

There are $H$ households $h$. Households may be employed in a particular sector $i$, to which they supply one unit of labour with disutility $d$ and receive nominal wage $W_i$: if unemployed they receive unemployment benefit $b$. 

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Household $h$ has initial money balances $M_h^0$ and pays lump-sum taxes $t_h$. Each household also receives some proportion $\phi_h$ aggregate profits $\Pi$ so that profit income is $\phi_h \Pi$. All profits are assumed distributed, so that $\sum \phi_h = 1$.

All households have symmetric Cobb–Douglas utility functions defined on household consumption of each output $X_{ih}$ (purchased at price $P_i$), and real money balances $M_h/P$ where $P$ is the cost-of-living (money can be seen as providing some kind of liquidity services or as being the only store of value as in an indirect utility function). The household then maximises utility subject to the budget constraint:

$$\max_{X_{ih}, M_h} \sum_{i=1}^{n} c \log X_{ih} + (1-c) \log (M_h/P),$$

subject to

$$\sum P_i X_{ih} + M_h + t_h \leq M_h^0 + W_i + \phi_h \Pi,$$  \hspace{1cm} (2.2a)

or

$$\sum P_i X_{ih} + M_h + t_h \leq M_h^0 + \phi_h \Pi + b.$$  \hspace{1cm} (2.2b)

(2.2a) is the appropriate constraint if the budget constraint of the household is employed, (2.2b) if unemployed. Clearly, given $d$ the disutility of work, the household will only work if:

$$\frac{(W_i - b)}{P} \geq d.$$  \hspace{1cm} (2.3)

where $P$ is the true cost-of-living index for consumption:

$$P = \prod_{i=1}^{n} P_i^{1/n}.$$  \hspace{1cm} (2.4a)

Since all households have identical homothetic preferences over consumption, we can aggregate over households, expressing aggregate consumption of each output and savings as a function of aggregate income. With Cobb–Douglas preferences, expenditure shares are constant, so that a proportion $c/n$ of income is spent on each output and proportion $1-c$ is 'saved' to accumulate money balances. For obvious reasons, we interpret $c$ as the marginal propensity to consume. Aggregating over income we have:

$$P_i X_i = (c/n) \left( M_0^0 - T + \sum_{i=1}^{n} N_i W_i + \left( H - \sum_{i=1}^{n} N_i \right) b + \Pi \right),$$  \hspace{1cm} (2.4a)

$$M = (1-c)(M_0^0 - T + \sum N_i W_i + (H - \sum N_i) b + \Pi),$$  \hspace{1cm} (2.4b)
where $N_i$ is employment in sector $i$ (to be determined), $M^0$ and $T$ aggregate (initial) money balance and lump-sum taxes.

2.2. The government

The government spends money and raises taxes. Since the distribution of lump-sum taxes is not relevant, we will only consider aggregate tax $T$. The government chooses nominal government expenditure $G_i$ in each sector $i=1\ldots n$, the $n$-vector of expenditure being $G$. The government also pays unemployment benefit to the $(H-N)$ unemployed households in each sector $i$ (where $N$ is total employment, $\sum N_i$). The government’s budget constraint is then:

$$M-M^0 = \sum_{i=1}^{n} G_i + (H - \sum_{i=1}^{n} N_i)b - T,$$

where of course $M$ is aggregate end of period money balances. For simplicity, we will assume that the government divides its taxation into two parts: the decision to finance unemployment benefit, and the decision to finance direct expenditure. The government always funds unemployment benefit through taxation, but may decide to finance proportion $(1-\tau)$ of direct expenditure through money creation. Thus:

$$T = \left(H - \sum_{i=1}^{n} N_i\right)b + \tau \sum_{i=1}^{n} G_i,$$  \hspace{1cm} (2.5)

$$M-M^0 = (1-\tau) \sum_{i=1}^{n} G_i.$$  \hspace{1cm} (2.6)

Substituting (2.5) into the household expenditure eq. (2.4) we have total expenditure in each sector:

$$P_iX_i = G_i + \frac{c}{n} \left[ M^0 + II + \sum_{j=1}^{n} W_j N_j - \tau \sum_{j=1}^{n} G_j \right].$$  \hspace{1cm} (2.7)

2.3. Firms

There are $m_i$ firms in sector $i$, who choose outputs given wages $W_i$. There are constant returns to scale, and without loss of generality:
Since we are considering a large economy, it is reasonable to assume that firms treat the price level $P$ as exogenous, and maximise nominal profits. Thus firms ignore the very small effect of own-price on the general price level; nominal and real profit maximization are (almost) the same.

Industry demand in each sector is given by adding nominal government expenditure $G_i$ to personal consumption (2.7). Since both are unit elastic, so is industry demand. The Cournot equilibrium with $m_i$ firms, constant returns with marginal cost $W_i$ and unit elastic demand is:

$$\mu_i = \frac{(P_i - W_i)}{P_i} = 1/m_i,$$  \hspace{1cm} (2.9)

where $\mu_i$ is the price-cost margin or Lerner's 'degree of monopoly'. There is thus a constant markup of prices over wages which depends on how competitive the market is, which with Cournot competition depends on the number of firms. Clearly, as $m_i$ tends to infinity, price tends to marginal cost, so $\mu_i$ tends to 0. Because of constant returns, $\mu_i$ is also the profit-to-sales ratio for the industry, so that industry profits and wages are:

$$\Pi_i = \mu_i P_i X_i; \quad K_i = W_i N_i = (1 - \mu_i) P_i X_i.$$  \hspace{1cm} (2.10)

Note that the own-product wage in sector $i$ is determined purely by the nature of product market competition from (2.9):

$$\frac{W_i}{P_i} = 1 - \mu_i.$$  \hspace{1cm} (2.11)

We can now substitute the profits and wage-bill eqs. (2.10) into the personal consumption eq. (2.7) which now reads:

$$P_i X_i = G_i + \frac{c}{n} \left[ \sum_{j=1}^{n} P_j X_j - \tau \cdot \sum_{j=1}^{n} G_j + M^0 \right] \quad i = 1 \ldots n.$$  \hspace{1cm} (2.12)

2.4. Labour demand

We can now solve the demand system (2.12) for final expenditures in each sector. If we define final expenditure in sector $i$ $E_i \equiv P_i X_i$, and $\gamma = \sum G_i/n : m^0 = M^0/n$ and $E$ the $n$-vector of $E_i$, we can write the system (2.12) in matrix notation:

$$E = G + I(c/n)(m^0 - \tau \cdot \gamma) + (c/n)[1]E,$$  \hspace{1cm} (2.13)
where \( I \) and \([1]\) are the unit \( n \)-vector and \( n \)-matrix respectively. Solving for \( E \) we have:

\[
E = \left( I - \frac{c}{n} [1] \right)^{-1} \left[ G + c(m^0 - \tau y)I \right].
\] (2.14)

It is easiest to evaluate the inverse by the power expansion rule, which yields:

\[
E_i = G_i + \left( c/(1-c) \right)(m^0 + \gamma(1-\tau)).
\] (2.15)

Eq. (2.15) is a 'reduced form' equation taking account of all of the income-expenditure feedbacks in the system. Final expenditure in sector \( i \) depends upon direct government expenditure in the sector \( G_i \), plus the aggregate level of government expenditure represented by \( \gamma \), plus aggregate wealth represented by initial average money balances \( m^0 \) less lump sum taxes \( \tau y \).

The reduced-form sectoral demand eq. (2.15) defines the demand-curve that the union faces. Since final expenditures \( E_i \) are constant in this Cobb-Douglas system, so is the wage bill \( K_i = (1-\mu_i)E_i \). Employment is then given by:

\[
N_i = K_i/W_i.
\] (2.16)

Throughout the paper we assume that the full-employment constraint is non-binding, i.e. \( \sum N_i \leq H \). Eq. (2.16) defines the objective demand curve for labour in each sector. It tells the monopoly union what level of employment will occur if it sets a particular nominal wage.

3. Wage determination

We now move to the first stage of the model, representing the 'long run' in which wages are determined. In each sector there is a monopoly union which is the unique seller of labour to firms in that sector, and which has the power to unilaterally set the nominal wage for labour in that sector. This is of course an extreme assumption: a more general version of bilateral bargaining over the wage (as in the 'right to manage' model), or sequential bargaining over wages and employment as in Manning (1987) would be more satisfactory. However, we leave this for subsequent work, and merely point out that despite its faults the monopoly union has a long pedigree and is widely used.

Each union sets the nominal wage \( W_i \) in its sector, treating the price level \( P \) as given through the large economy argument. As in Dixon (1988) we assume that the union has a utility function defined on real wages and...
employment, we adopt the standard Stone–Geary specification to parameterize it, so union utility is given by:

\[(N_i - F)((W_i/P) - \theta).\]  

(3.1)

The parameters \(F\) and \(\theta\) are open to various interpretations [see Dertouzos and Pencavel (1984)]. Most importantly, \(\theta\) can be interpreted in at least two ways: as a parameter capturing relative wage ‘envy’ effects; or as being the disutility of work. In terms of our assumptions, the latter interpretation is:

\[\theta = B + d,\]  

(3.2)

where \(B = b/P\) is the real benefit level. If a worker becomes employed his ‘net wage’ is \(W_i\) less unemployment benefits and a sum compensating him for the disutility of work, as in (3.2).

The union chooses \(W_i\) to maximize its utility, given the ‘objective’ demand curve it faces (2.15), and \(P\):

\[
\max_{W_i} (N_i - F)((W_i/P) - \theta),
\]

s.t.

\[N_i = K_i/W_i.\]  

(3.3)

To avoid trivialities in what follows, we shall assume that the optimal solution to (3.3) involves the unions setting wages above the full employment level. Solving (3.3), we obtain the unions’ ‘best-response’ given the level of sectoral demand \(K_i\) and the price level \(P\):

\[
\log W_i = \frac{1}{2} \log \theta/F + \frac{1}{2} \log K_i + \frac{1}{2} \log P.\]  

(3.4)

The general price level is of course determined by wages and the markup equation:

\[
\log P = \sum_{i=1}^{n} \frac{1}{n} \log P_i = \sum_{i=1}^{n} (1/n) \log W_i - \sum_{i=1}^{n} (1/n) \log (1 - \mu_i).\]  

(3.5)

If we define \(1 - \mu\) as the geometric mean \((1 - \mu) = \prod_{i=1}^{n} (1 - \mu_i)^{1/n}\) and substitute (3.5) into (3.4), we have the unions reaction function in terms of the general wage level and markup:
The Nash-equilibrium occurs when each union is on its reaction function. Writing the n-equation of reaction functions in matrix form we have, 

$$W = -\frac{1}{2} \cdot N_B + \frac{1}{2} \cdot K + \frac{1}{2n} \begin{bmatrix} 1 \end{bmatrix} W,$$  \hspace{1cm} (3.7)

where $W$ and $K$ are the $n$-vectors of log $W_i$ and log $K_i$, and $N_B$ is the $n$-vector in which all elements are log $F(1-\mu)/\theta$. Solving (2.7) we obtain the equilibrium wage vector as a function of $(\theta, F, m, K)$:

$$W^* = \left[ I - \frac{1}{2n} \begin{bmatrix} 1 \end{bmatrix} \right]^{-1} \left[ -N_B + K \right].$$  \hspace{1cm} (3.8)

Again, it is easiest to evaluate the inverse in (3.8) using the power expansion rule, which yields:

$$\log W^*_i = -\log N_B + \frac{1}{2} \log K_i - \frac{1}{2} \log K,$$  \hspace{1cm} (3.9)

where $K$ is the geometric mean of $K_j$: $K = \prod_{j=1}^n K_j^{1/n}$.

Substituting (3.9) into the sectoral labour demand eq. (2.15) we obtain equilibrium employment. In plain levels we have:

$$W^*_i = N_B^{-1} \cdot K_i^{1/2} \cdot K^{1/2},$$  \hspace{1cm} (3.10a)

$$N^*_i = N_B \cdot k_i^{1/2} = (1-\mu)^{F/\theta} k_i^{1/2} \text{ (where } k_i = K_i/K).$$  \hspace{1cm} (3.10b)

(3.10) gives the Nash-equilibrium wages and employment in each sector $i$ as a function of the general degree of monopoly $\mu$, union preferences $F/\theta$, sectoral demand $K_i$, and the general level of demand $K$. Note that what is important in determining equilibrium employment in each sector is the level of sectoral demand relative to the general level of demand, $k_i$. Very simply, this is because from (3.10a) a high level of general demand in the economy will lead to higher nominal wages, which in turn lead to a high nominal price level, which means that the union in a particular sector will want to set a higher nominal wage. If demand in that sector is relatively low, then employment will be relatively low, and vice versa. If $k_i = 1$ in all sectors, then $N_i = N_B$. Substituting for the equilibrium wage levels using (3.10) we have:
The real wage in sector $i$ depends upon the average degree of monopoly \((1-\mu)\), and the relative strength of final demands $k_i$ (the degree of monopoly determines the markup of prices over wages à la Kalecki: $k_i$ the level of relative wages).

Whilst for simplicity we have assumed union preferences to be identical across sectors, it is trivial to allow union preferences to differ. If we define union preferences in sector $i$ as $\xi = \theta/F_i$ and $\xi$ the geometric mean of preferences, we have:

\[
\frac{W_i}{p} = (1-\mu)k_i^{1/2}. \tag{3.11}
\]

\[
N_i = \left(\frac{\xi_i}{\xi}\right)^{1/2}k_i^{1/2}, \tag{3.12a}
\]

\[
N_i = \left(\frac{\xi_i}{\xi}\right)^{1/2}K_i^{1/2}. \tag{3.12b}
\]

Throughout this paper, we assume that sectoral wages and employment are given by (3.10). This is a simplification: we have ignored the households participation constraint (real wages exceeds benefit plus disutility of work), and the employment constraint. These issues are tangential to the main theme of the paper, which is the exploration of wage employment formation in a unionised economy. We have relegated the discussion and full characterisation of equilibrium to an appendix. Assuming that the union’s reservation wage $\theta$ is tied to benefit and disutility of labour $\theta = B + d$, we prove the following in the appendix:

**Theorem 1.** Let government policy be $\left(G, M^0, \tau, b\right)$ with resultant final demands $K_i$. There exists an equilibrium characterised by (3.10) if for all $i$

\[
k_i^{1/2} \leq \theta/(1-\mu) \quad \text{and} \quad \sum N_i \leq H.
\]

*If demand is relatively high in sector $i$, exceeding the upper bound in the theorem, wages and employment are determined by the full-employment constraint; if demand is low, the real wage equals $B + \theta$, and employment is demand determined.*

4. **Government policy**

The previous two sections have set up a simple framework in which both nominal wages and prices are determined by price-making agents. We are now in a position to explore the impact of government policy on the economy. Macroeconomic policy is primarily concerned with aggregates, so
that we shall concentrate on the impact of government policy on aggregate employment \( N \). From (3.10)

\[
N = N_B \sum_{i=1}^{n} k_i^{1/2}.
\]  

(4.1)

For a given policy \((G, M^0, \tau, b)\), there will be a corresponding vector of employment levels: without loss of generality let us rank the sectors in order of final expenditures \( K_i \) so that: \( K_1 \geq K_2 \cdots \geq K_n; \ W_1 \geq W_2 \cdots \geq W_n; \ N_1 \geq N_2 \cdots \geq N_n. \) What is the relationship between \( G_i \) and \( K_i \)? If there is a uniform degree of monopoly across sectors \((1 - \mu_i) = (1 - \mu) \ i = 1 \ldots n, \) then \( K_i \) will be ranked with \( G_i \) (so \( G_1 \geq G_2 \cdots \geq G_n \)). However, if \( \mu_i \) differ, then it is possible that the orderings of the \( G_i \)'s in magnitude will differ from the ordering of \( K_i \)'s. A \( \mu \)-symmetric vector \( G \) is a combination of government expenditure which given \( \mu \) yields the same final expenditure in each sector, so that \( k_i = 1 \ i = 1 \ldots n. \) If \( \mu \)'s are uniform, any balanced fiscal policy \( G_i = \gamma \) for all \( i \) is \( \mu \)-symmetric. If \( \mu \)'s differ across sectors, a \( \mu \)-symmetric vector will involve more monopolised sectors with higher \( \mu \)'s having larger \( G_i \).

Before evaluating the policy multipliers, we can explore the natural range property of the model as in Dixon (1988). For any given macroeconomic policy \((G, M^0, \tau, b)\) there will be a unique equilibrium in the private sector given by (3.10). However, the equilibrium levels of employment will vary with macroeconomic policy. This raises the question as to what levels of employment are attainable by varying the mix of macroeconomic policy? In the context of this model, the answer to this question is that there is a continuum of aggregate employment levels than can be obtained, which we denote the Natural Rate of Employment. From (4.1) if fiscal expenditures \( G \) are \( \mu \)-symmetric in each sector, then employment in each sector is at \( N_B \), the balanced rate of employment. This turns out to be the lowest level of employment attainable as an equilibrium in this economy. The government can increase the aggregate level of employment by adopting as asymmetric government fiscal policy, spending more on a few sectors (even one) than the rest. This can be illustrated geometrically if we note that the product of equilibrium employment levels is invariant with respect to government policy:

\[
\prod_{i=1}^{n} N_i - N_B^n \prod_{i=1}^{n} k_i^{1/2} - N_B^n.
\]

If we consider the set of achieved combinations of employment \( N \), then they must lie on an \( n \)-dimensional rectangular hyperbole, as illustrated for \( n = 2 \) in fig. 1. Of course since \( k_i \) are bounded (see theorem and appendix) only a
portion of the rectangular hyperbole can be achieved. Turning to aggregate employment $N$, iso-employment loci are simply hyperplanes, in the two dimensional case with slope $-1$, higher levels of employment being represented by hyperplanes further from the origin. In fig. 1, the Natural Range lies between the Balanced Rate of employment $2N_B$ and $\bar{N}$. By moving away from a $\mu$-symmetric policy, the government can increase aggregate employment. The reason behind this result is the log-linearity of sectoral employment equations (3.10b) in $k_i$. This log-linearity means that a proportionate increase in relative demand $k_i$ will have a larger absolute effect on $N_i$ the larger is $N_i$. Thus the government can increase aggregate employment by making sectoral employment levels more unequal.

Having given a geometric interpretation of the Natural Range result in this model, we can now go on to explore the effectiveness of government policy. Let us consider the effect of an increase in government expenditure in sector $i$ on employment in this economy. Clearly, an increase in $G_i$ will increase

![Fig. 1. The natural range of employment.](image-url)
employment in sector $i$, and decreases employment in all other sectors $j \neq i$. This occurs because an increase in $G_i$ increases the relative sectoral demand $k_i$, and decreases $k_j$. The interesting question is what will happen to aggregate employment $N$?

If we calculate the fiscal multiplier for an increase in nominal sectoral expenditure $G_i$, after some manipulation we find:

**Proposition 1. Sectoral employment expenditure multiplier.**

\[
\frac{dN}{dG_i} = \frac{1}{2K_i} \left( N_i - \bar{N} + \frac{c(1-\tau)}{n(1-c)} \sum_{j=1}^{n} \frac{(N_j - \bar{N})}{N_j^2} \right),
\]

where $\bar{N}$ is the arithmetic mean of sectoral employment ($\sum_{j=1}^{n} N_j/n$).

Turning to the RHS summation, note that

\[
\sum_{j=1}^{n} \frac{(N_j - \bar{N})}{N_j^2} \leq 0,
\]

with strict inequality unless $N_j = \bar{N} = N_B$ for all $j$, which occurs only with a $\mu$-symmetric-fiscal policy. Thus the second RHS group of terms in (4.2) is negative. The first RHS term will be positive or negative depending on whether employment in $i$ is greater or smaller than average. Although the sign of (4) is ambiguous we can certainly say that an increase in nominal government in sector $i$ will have an overall negative effect on employment in sector $i$ if it is at or below average employment $\bar{N}$.

We will now examine the impact of fiscal policy in this framework in a series of Propositions. The main point to remember is that in equilibrium, sectoral employment combinations $N_i$ lie on a rectangular hyperbole. Aggregate employment $N$ increases as sectoral demands $K_i$ become less symmetric, and we move away from the 'bottom' of the hyperbole.

**Proposition 2.** Suppose that we start from an initial equilibrium $N$, such that for some $j$ $N_j \neq N_B$ (i.e. fiscal policy is not $\mu$-symmetric). The balanced budget multiplier ($\tau = 1$) is larger than the multiplier with some money creation ($0 < \tau < 1$).

**Proof.** From (4.2) setting $\tau = 1$:

\[
\left. \frac{dN}{dG_i} \right|_{\tau=1} = \frac{1}{2K_i} \cdot (N_i - \bar{N}),
\]
which is strictly greater than (4.2) with a non $\mu$-symmetric policy (of course, the balanced budget multiplier will be negative when $N_i < \bar{N}$).

Proposition 3. Consider a non-$\mu$-symmetric government expenditure plan $G$. The level of aggregate employment is increasing in the proportion $\tau$ of expenditure that is tax-financed.

Proof.

$$\frac{dN}{dt} = \frac{c\tau}{n(1-c)} \sum_{j=1}^{n} \frac{(\bar{N} - N_j)}{K_j} > 0,$$

since in the summation larger $N_j$ occur with larger $K_j$.

The intuition behind these two results is the same. A higher degree of tax-finance $\tau$ means that money is taken from households who spend symmetrically: if government expenditures is not $\mu$-symmetric, this makes the final expenditures $K$ more unbalanced, thus increasing aggregate employment.

The multiplier (4.2) applies to an increase in expenditure in one sector. What happens if there is a general increase in government expenditure? To analyse this case, suppose that the government spends a fixed proportion $\delta_i$ of its total expenditure $G$ on sector $i$ (i.e. $G_i = \delta_i G$, $\sum \delta_i = 1$). For simplicity, let us take the case where $\mu$ is uniform across sectors, so that a $\mu$-symmetric policy involves setting $G_i$ equal across sectors. Proposition 4 tells us that with an unbalanced expenditure plan ($\delta_j \neq 1/n$ for some sector $j$) an increase in total expenditure $G$ will increase total employment $N$. The government's expenditure plan is summarised by $\delta$ and $G$ (where $\delta_1 > \delta_2 > \delta_n$).

Proposition 4. Suppose the division of total government expenditure $G$ between the $n$ sectors $\delta$ is fixed, and $\mu_i$ is uniform across sectors. If fiscal policy is unbalanced, then an increase in $G$ will increase employment.

Proof. For simplicity we take the balanced budget case, $\tau = 1$:

$$\frac{dN}{dG} = \sum_{j=1}^{n} (1-\mu_j) \frac{\delta_j}{2K_j} (N_j - \bar{N}) > 0,$$

since larger $N_j$ will be associated with larger $\delta_j/K_j$.

There are ultimately two sources of final demand in this model: $M^0$ which is symmetric across sectors, and $G$ which (by assumption), is unbalanced. The
degree of imbalance in final demands \( K \) depends on the relative size of \( G \) to \( M^0 \): an increase in \( G \) leads to more imbalance in final demands \( K \), and hence an increase in \( N \).

The previous propositions have dealt with the impact of changes in nominal expenditure on employment. Conventional macroeconomic analysis concentrates on real government expenditure multipliers. In the framework of this paper, real government expenditure is 'endogenous': the government chooses nominal expenditures, then the private sector arrives at equilibrium wages and price, which determine real expenditure. If we define real expenditure in sector \( i \) as \( g_i \):

\[
g_i = \frac{G_i}{P_i} = \frac{(1 - \mu_i)G_i}{W_i}, \tag{4.4}
\]

where \( W_i \) is the equilibrium wage given by (3.10a). Aggregating over sectors is not quite straightforward, since the output of sectors is heterogeneous. However, if we choose as our measure of total real expenditure \( g \) the total number of households employed in producing government output, we have \( g = \sum g_i \).

We will now explore two questions. First, what happens to total real expenditure \( g \) when total nominal expenditure \( G \) rises? Secondly what can we say about the value of the real expenditure multiplier – is there crowding out with a multiplier less than one? The algebra here is rather lengthy, and we have simplified our analysis by concentrating on the balanced budget multiplier \( \tau = 1 \), with a uniform degree of monopoly.

**Proposition 5.** The government has an unbalanced fiscal policy \((G, \delta)\), and there is a uniform degree of monopoly. With \( \tau = 1 \), total real expenditure \( g \) is increasing in total nominal expenditure \( G \).

**Proof.** Note that from (4.4)

\[
g_i = N_i \cdot \frac{G_i}{E_i} = N_i \cdot \frac{G_i}{E_i} = \frac{\delta_i G}{1 - c} M^0. \]

Hence:

\[
\frac{dg}{dG} = \sum_{i=1}^{n} \left( \frac{dG_i}{dG} \right) \frac{N_i G_i dN}{E_i dG}
\]
This is positive, since

\[ \sum_{i=1}^{n} \frac{\delta_i}{K_i} (N_i - \bar{N}) \text{ is non-negative.} \]

Clearly, Proposition 5 will also be true for \( \tau < 1 \), although the increase in \( G \) will then be more inflationary, leading to higher wages and prices, and hence lower real expenditure. Note that Proposition 5 holds even for a balanced fiscal policy.

Given that real government expenditure \( g \) is increasing in nominal expenditure \( G \), and that employment \( N \) is also increasing in \( G \) (if fiscal policy is unbalanced), what can we say about the value of the real expenditure multiplier in this model? It turns out that there is always crowding out in a unionised economy the multiplier being weakly positive but strictly less than unity.

**Proposition 6.** An increase in government expenditure \( G \) in fixed proportions \( \delta \) leads to crowding out.

**Proof.**

\[
\frac{dN}{dg} = \frac{dN}{dG} \cdot \frac{dG}{dg} = \frac{dN}{dG},
\]

from (4.3) and (4.4):

\[
\frac{dN}{dg} = \sum_{i=1}^{n} \frac{\delta_i}{K_i} [N_i - \bar{N}] / \sum_{i=1}^{n} \frac{\delta_i}{K_i} \left[ \frac{M^0}{1-c} \frac{C}{K_i} - \bar{N} \right],
\]

hence \( 0 \leq dN/dg < 1 \). A zero multiplier occurs if fiscal policy is \( \mu \)-symmetric, and hence \( N_i = N_{\mu} \) for all sectors.

Whilst an increase in the general level of government can increase employment, it will crowd-out private consumption. Private consumption
comes from two sources: the employed and the unemployed. Since unemploy-
ment benefits are transfer payments, from the aggregate point of view
aggregate net income is their initial money balances. An increase in nominal
government expenditure will increase prices, and hence reduce the real
money balances of the unemployed, 'crowding-out' their consumption. As for
the employed, the same real balance effect will operate, whilst their real
wages will increase in above average employment sectors, and reduce in
below average employment sectors. Proposition 6 tells us that the net effect
of changes in private sector real income leads to a fall in private consump-
tion which is no greater than the increase in real government expenditure.

The unionised economy is similar to a standard Walrasian economy in
that it has a multiplier less than unity. The underlying reasons for crowding
out are, however, completely different. In the present model, the multiplier
emerges from an inherently inter-sectoral effect, the underlying wage-
determination process in a unionised economy with many sectors. In the
text-book Walrasian economy, the multiplier is between zero and one due to
the real balance effect in the supply of labour (if leisure is normal). In the
present model there is no real balance effect on household (or union) labour
supply (employment) decisions.

Whilst the multiplier is less than unity, the welfare effects of policy can still
be beneficial, in total contrast to the Walrasian case. The reason for this is
that unions raise the real wage above the disutility of labour, so that the
employed earn a 'surplus'. If macroeconomic policy increases total employ-
ment, then it can increase the total surplus in the economy. This argument is
reinforced if there are oligopolistic industries, since then shareholders will
also earn a surplus in the form of profits. In the Walrasian economy with
constant returns, there can be no surplus in the form of either profits or
factor rents. In this case an increase in wasteful government expenditure will
tend to reduce welfare.

In order to concentrate on the welfare analysis, we will consider policy in
a two sector model \( n = 2 \), with fixed total government expenditure \( G \), money
balances, zero unemployment benefit and a balanced budget. Furthermore,
we will take the case where \( \theta = d \). Policy here will then consist in altering the
proportion \( \delta \) of government expenditure spent on sector 1 \( (G_1 = \delta G, \ G_2 = (1 - \delta)G) \).
Furthermore, we will concentrate on the labour market imperfection, setting \( \mu_1 = 0 \) for both sectors. Under these simplifying assump-
tions, total nominal national income is fixed, and equals total labour income:

\[
Y = \sum K_i = G + \frac{c}{1-c} M^0. \tag{4.5}
\]

From (3.5) and (3.10a), the equilibrium cost-of-living index \( P \) is given by:
If we treat $K_i$ as free variables, we can construct iso-$P$ curves in $K$ space. Since $P$ is strictly concave in $K$, the upper-contour sets are strictly-convex as depicted in fig. 2, with $P$ increasing from the origin. Since we are holding nominal national income constant, the feasible $K$'s lie on the downward sloping line defined by (4.5). Clearly, as we move down away from the 45° line, the cost of living falls. The (indirect) utility from consumption is given by $(Y + M)/P$: hence utility from consumption is also increasing as $P$ falls. Total utility, however, also includes the disutility of work $\theta N$. The key question is whether the disutility of work as employment increases is greater than the increased consumption-utility as $P$ falls. To examine this question, we will first define our utilitarian social-welfare function:

$$SW = \frac{K + M^0}{P} - N\theta.$$
The first RHS term is the indirect utility from consumption; the second the total disutility of labour. After rearrangement we have:

\[ SW = \sum_{i} N_i \left[ \frac{W_i}{P} - \theta \right] + \frac{M^0}{P}. \tag{4.7} \]

Using the equilibrium relationships (3.10) to substitute for \( N_i, W_i \) and \( P \) yields:

\[ SW = \sum_{i} \left[ \left( F_i/\theta \right) N_i^2 - \theta N_i \right] + \left( M^0/P \right), \tag{4.8} \]

since \( W_i N_i/P = F k_i/\theta = FN_i^2/\theta \). The first RHS term in (4.7) and (4.8) is the total surplus earned by the employed, since the real wage exceeds the disutility of work (\( \theta = d \)). Note that (4.8) only holds when equilibrium conditions (3.10) are valid so that \( k_1^{1/2} \geq \theta \): this guarantees that there is some surplus in each sector, i.e. \( ((FN_i^2/\theta) - \theta N_i) > 0 \) for all \( i \). We will now consider what happens to \( SW \) as we vary \( \delta \).

**Theorem 2.** If the government alters the sectoral allocation of expenditure to increase total employment, then Social welfare increases.

**Proof.** First, note that in this symmetric economy, the government will only increase employment if it increases \( \delta \) when \( \delta > 1/2 \), or decreases it when \( \delta < 1/2 \). Without loss of generality, we will consider the case where \( \delta > 1/2 \). Hence, since \( N_1 N_2 = N_1^B \):

\[
(\partial N_1/\partial \delta) > 0 > (\partial N_2/\partial \delta) \quad \text{and} \quad (\partial N_1/\partial \delta) + (\partial N_2/\partial \delta) > 0.
\]

A shift of expenditure to sector 1 will increase \( N_1 \) and decrease \( N_2 \), but (since \( \delta > 1/2 \)) total employment will increase. Furthermore, consider the geometric average \( \bar{K} \): since total expenditure \( K \) is fixed, from (4.5) it follows that \( d\bar{K}/d\delta < 0 \). Hence, from (4.6) \( dP/d\delta < 0 \): when the government reallocates expenditure to the larger sector, the cost of living falls. Differentiation of (4.8) yields:

\[
\frac{dSW}{d\delta} = \sum \left[ \frac{2F_i}{\theta} N_i - \theta \right] \cdot \frac{\partial N_i}{\partial \delta} - \frac{M^0}{P^2} \cdot \frac{dP}{d\delta} > 0. \tag{4.9}
\]

The first RHS term is positive: the larger \( N_1 \) is multiplied by the larger positive \( \partial N_1/\partial \delta \). The second term is positive since \( dP/d\delta < 0 \).
Hence total welfare increases with employment. This is because the increase in employment in the high-wage sector is larger than the decrease in the low-wage sector, which leads to a rise in total surplus. In addition there is a fall in the cost-of-living.

This analysis has shown that macroeconomic policy can not only increase total employment, but also Social Welfare. This is a very surprising result: not only is government expenditure treated as ‘waste’, but from Proposition 6 there is also crowding out of private expenditure. This result stands in complete contrast to the Walrasian case. With a competitive labour market, the real wage equals the disutility of labour, and hence there is no surplus earned by the employed: in terms of (4.6), the RHS summation would be zero. The only effect that policy can have is then via the price level. However, under the assumptions of this paper – identical CRTS technology – the labour theory of value holds, and relative output prices are fixed at unity. Altering the allocation of government expenditures across sectors can have no effect on total employment, and hence cannot influence the cost-of-living. There can therefore be no welfare gain in the model with a competitive labour market.

The results of this paper have been derived from a model of a unionised economy with oligopolistic product markets. Can similar results be derived in a multisector Walrasian economy under different assumptions? It is certainly true that in a multisector economy in which sectors have different technologies a reallocation of government expenditure will in general have some effect. Few would doubt that if the government switched expenditure from road-sweeping to nuclear submarine construction that total employment would decline. However, in this paper I have assumed identical technologies. The reason for the Natural Range result in this paper is that unions trade-off employment and real wages differently as demand shifts across sectors, so that relative wages change. If the economy-wide labour market is competitive, then wages in each sector are determined by the same marginal disutility of labour, so that relative wages are fixed across sectors (this argument applies equally to the case where the marginal disutility of labour varies with labour supplied). If nominal wages and prices are the same in each sector, then there can be no mechanism similar to the one in this paper. However, if there is imperfect labour mobility, then sectoral wages could differ, so that even with competitive labour sub-markets, relative wages could alter, giving rise to similar affects as in this paper. This paper obtains the result of a natural range in an economy with identical technologies and perfect labour mobility.

5. Conclusion

This paper provides a simple model of a large unionised economy which
enables us to examine both the nature of macroeconomic equilibrium, and the effect of macroeconomic policy on the equilibrium. What fundamentally differentiates this from the Walrasian approach is the process of wage and price determination underlying the equilibrium, which explains the response of wages, prices, and employment to policy. The imperfectly competitive framework provides a coherent theory of non-Walrasian equilibrium, in which wages and prices are set by agents in the model rather than by some auctioneer. The results of the paper suggest that both the comparative static results as well as the welfare properties of this type of economy can differ significantly from the Walrasian economy.

Appendix

*The existence and characterisation of equilibrium*

Throughout the paper, we have assumed that an equilibrium exists, and moreover that equilibrium wages and employment were given by (3.10). Two issues arise from the participation decision of workers, and the ability of unions to obtain non-negative utility in equilibrium. Turning first to the participation constraint, given that there is disutility of labour $d$ and unemployment benefits $B$, households will choose to supply labour only if the real wage is no less than $d + B$. From (3.10a) and the mark-up equation:

$$W/P = (1 - \mu)k_t^{1/2} \geq d + B \quad \text{so that} \quad k_t^{1/2} \geq (B + d)/(1 - \mu). \quad (A.1)$$

(A.1) gives us the labour force participation constraint on union wage setting, assuming that unions are unable to force unwilling members to work! The second issue is that the unions must achieve non-negative utility in equilibrium. This requires that:

$$W_i/P \geq \theta, \quad N_i \geq F. \quad (A.2)$$

Both inequalities (A.2) are satisfied if

$$k_t^{1/2} \geq \theta/(1 - \mu). \quad (A.3)$$

If we require $\theta = B + d$, i.e. the union is concerned about the 'surplus' real wage over the disutility of work plus benefit, then of course (A.1) and (A.3) are equivalent. However, $\theta$ may reflect other factors as well (e.g. relativities, militancy, etc.) so we state the two conditions separately. If, the union is unable to obtain positive utility due to adverse sectoral demand, then the union becomes irrelevant, and the lower bound on the wages (the real wage equals the disutility of labour plus benefits) takes over:
\[ W_i/P = B + d = \theta. \]  \hspace{1cm} (A.4)

Clearly, depending on the mix of government policy and resultant final demands \( K \), some sectors might be in an 'unionised' regime, with wages and employment determined by union reaction functions (3.10); some sectors with low demand may be in an 'underemployment' regime with the wage determined by the disutility of labour and benefit levels (A.4). Taking into account the two possible regimes, the general equation for each sector \( i \) is:

\[ \frac{W_i}{P} = \max \left[ d + B, \left( \frac{N_B \cdot \theta}{F} \cdot K_i \right)^{1/2} \right]. \]  \hspace{1cm} (A.5)

Since wage equations (A.5) are piecewise log-linear and continuous, we can solve the system for equilibrium wages and employment. Throughout the main body of the paper, we have concentrated only on the 'unionised regimes', which simplified the analysis and enabled us to derive explicit formulations for equilibrium employment.

References

Rankin, N., 1987, Monetary and fiscal policy in a Hartian model, Mimeo., Queen Mary College.