THE COURNOT AND BERTRAND OUTCOMES AS EQUILIBRIA IN A STRATEGIC METAGAME*

Huw Dixon

It has long been recognised that the flexibility of production lies at the heart of the distinction between Bertrand and Cournot models. The most natural application of the Cournot model would seem to be in the case where output is fixed in the short run. The Bertrand framework rests on the fact that output is fully variable in the short run so that if one firm is undercut by another, the lower-priced firm can expand output to serve all the demand. It is this basic insight that we explore. We present a general model in which the flexibility of production is endogenous, and which embraces both the Cournot and Bertrand outcomes as possibilities. This enables us to see which outcomes will emerge from firms' strategic decisions, rather than presupposing either.

In this paper, the flexibility of production is determined by the factors of production that the firm precommits.

Output is produced by two factors of production – capital and labour. There are two stages to the model. In the first 'strategic' stage, the firm will precommit one, both, or neither of its factors of production. In the second 'market' stage, a competitive equilibrium occurs (the price clears the market given firms' supply functions). Thus the firm's decision in the strategic stage determines the supply function that the firm has in the market stage.

There are three possible types of precommitment for the firm: (*a*) total precommitment – the firm precommits both inputs (and hence capacity); (*b*) strategic investment – the firm precommits *one* factor of production (capital) leaving the other (labour) freely variable in the market stage; (*c*) no precommitment – the firm is free to vary the level of both factors of production in the market stage. In the case of total precommitment the firm will have a supply function that is vertical at capacity; if the firm precommits only capital it will have a standard upward-sloping supply function; if the firm precommits neither factor, in firm will have a 'flat' supply correspondence.

In Section II we explore the case of imposed precommitment, where the firm has a given type of precommitment. If all firms precommit both factors of production, then the resultant Nash equilibrium in supply functions is the Cournot outcome. If all firms precommit only capital, then the resultant equilibrium is the model of strategic investment explored in Dixon (1985*a*). If all firms are uncommitted, then the Bertrand outcome occurs.

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In Section III we expand the firm's strategy set to include the type of precommitment, so that its choice embraces all three types of supply functions.[1] When firms' strategy sets are expanded in this way there are two types of equilibria (see Theorem). One equilibrium occurs when all firms precommit both factors of production, yielding the Cournot outcome. Another equilibrium occurs if one or more firms precommit neither factor of production, when the equilibrium price equals minimum average cost, the Bertrand outcome.

In Section IV we briefly discuss the impact of uncertainty, imperfect competition in the market stage, and entry. The result that the Cournot outcome is a metagame equilibrium is fairly robust with respect to non-competitive assumption about the market stage (i.e. Cournot, conjectural variations).

I. PRECOMMITMENT AND MARKET OUTCOME

We present the basic model, deriving the market outcome when the precommitment of firms is given. When a firm precommits a factor of production, it places an upper bound on the amount of the factor that it can employ in the market stage. The ith firm's capital and labour L_i, k_i are used to produce output x_i. Firms can be of three types depending on whether they precommit one, both, or neither factor of production. The precommitment type of the firm is represented by a discrete variable γ_i, which equals 0, 1, or 2 when the firm precommits neither, one, or both factors respectively. There are two stages to the model. In the first strategic stage firms precommit the relevant factor(s) of production, which determine(s) the firms' supply functions. Given these supply functions, in the market stage a competitive equilibrium occurs.

Throughout this paper we shall make the following primitive assumptions about technology and demand. All functions are assumed to be twice continuously differentiable.

A 21: *Technology*. Firms have the same constant returns production function $f(k_i, L_i)$ which is strictly concave in k_i and L_i.

A 22: Industry Demand. $F(p)$ is bounded from above, strictly decreasing when positive, and there exists price $p^* > 0$ such that $F(p) = 0$ when $p \geqslant p^*$.

Neither of these assumptions is as general as it could be, and they are chosen to keep the model simple. Under constant returns to scale there is an efficient least average cost of production, denoted hereafter as a. We assume that $p^* > a$. We shall now examine the firm's choice of supply function in the first stage for each of the three types of firm.

(*a*) *Total Precommitment*, $\gamma_i = 2$. When a firm precommits both factors in the strategic stage, it effectively chooses its capacity x_i^0, and is free to choose its output from the interval $[0, x_i^0]$ during the market stage. The costs of

[1] The model presented contrasts with other work on supply correspondence equilibria where the firm's choice of supply correspondence is very much wider, as in Grossman (1981), Hart (1982). In the strategic metagame the set of supply functions that the firm can have is directly related to the firm's cost structure in the market stage.

production are entirely fixed, with zero marginal cost up to capacity. The capacity can be expressed in terms of the capital stock chosen, since it will never pay a firm to precommit its capital and labour in any ratio other than the optimal labour–capital ratio (otherwise it could have the same capacity at a lower cost). The cost minimising output–capital ratio, denoted v, does not depend on output because of constant returns:

$$x_i^0 = f(k_i, L_i) = vk_i. \tag{1}$$

We define the firm's capacity-constrained supply function s^0 as:

$$s^0(p, k_i) = \max_{x_i} \arg\max_{\varepsilon[0,x_i^0]} [px_i] = vk_i \quad \text{for all} \quad p \geqslant 0 \tag{2}$$

where we ignore costs, since none are variable. For simplicity we assume that the firm prefers to produce the larger of two outputs with the same profits. Under total precommitment, the firm has a vertical supply function, the position of which is determined by its choice of capital and hence capacity. The indirect profit function corresponding to (2) if of course $(p - a)vk_i$

(*b*) *Non-commitment*, $\gamma_i = 0$. Being unable to precommit either factor, the uncommitted firm has no choice in the strategic stage. Assuming that firms prefer to produce the larger of two outputs yielding the same profits, the Bertrand supply function is for our purposes defined using some large upper bound on output, $\Xi \geqslant F(0)$ say:

$$b(p) = \max_{x_i} \arg\max_{\varepsilon[0,\Xi]} \quad x_i(p - a). \tag{3}$$

Hence $$b(p) = \Xi \quad \text{for } p \geqslant a, 0 \quad \text{for} \quad p < a. \tag{4}$$

(*c*) *Partial Precommitment*, $\gamma_i = 1$. The case where the firm precommits only one factor (capital) has been explored in some detail in Dixon (1985*a*), so that treatment here will be brief. Consider the firm's supply function given its choice of capital in the strategic stage. The supply at a given price solves the following programme:

$$\max_{x,L} px - wL - rk \quad \text{subject to } x \leqslant f(k, L). \tag{5}$$

Since we have assumed constant returns, we can write the solution output to (5) in terms of the supply-per-unit-capital function $s(.)$:

$$ks(p). \tag{6}$$

If we look at the dual of (5), the minimisation of the cost of producing output x given k (the 'short-run' cost function), we have:

$$kc(x/k) \tag{7}$$

where $c(.)$ is the cost-per-unit capital of producing a given output-per-unit capital. Note that average cost $kc(x/k)/x$ is minimised at the efficient output – capital ratio $x/k = v$, and that $s(a) = v$. For $p > a$, $s(p) > v$, so that the firm's production is undercapitalised, the output–capital ratio being too large. When $p \neq a$, the firm will not be on its long-run cost function, although it will be on the short-run cost function (7). The indirect profit function from

(5) is:

$$k\pi(p) \tag{8}$$

where $\pi(p) \equiv ps(p) - c[s(p)]$.

From (5), clearly $\pi(p) \leqslant 0$ for $p \leqslant a$, and $\pi(p) > 0$ for $p > a$. If the firm earns positive profits, its production will be under capitalised.

Whereas in the case of total precommitment the firm chooses a vertical supply function, in the strategic investment case the supply function under A1 will be upward sloping, its position determined by the capital stock of the firm. There are thus three types of supply function that the firm might have, depending on its precommitment type γ_i.

There are n firms with given precommitment types γ_i. For firms which precommit one or both factors, the choice of supply function can be represented by their choice of capital stock in the strategic stage (see (2) and (6)). Uncommitted firms must have the supply function given by (4). The supply functions of all firms can be summarised by the n-vectors of firms' capital stocks $\mathbf{k}$ and precommitment types γ. In order to state the industry supply function, we first define the variable δ_{qi} where $q = 0, 1, 2$ and $i = 1, \dots n$: $\delta_{qi} = 1$ if $\gamma_i = q$, 0 if $\gamma_i \neq q$. Summing over all firms, industry supply will depend on p, $\mathbf{k}$, and γ:

$$S(p, \mathbf{k}, \gamma) = \sum [\delta_{2_i} v k_i + \delta_{1_i} k_i s(p) + \delta_{0i} b(p)]. \tag{9}$$

The price that results in the market stage given $\mathbf{k}, \gamma$ is defined by:

$$\theta(\mathbf{k}, \gamma) =_{\text{def}} \inf [p > 0 \colon F(p) \leqslant S(p, \mathbf{k}, \gamma)]. \tag{10}$$

If any firm is uncommitted, $\gamma_i = 0$, then $\theta \leqslant a$, since $b(a) > F(a)$ from (3). The firm's payoff function U_i gives profits as a function of $\mathbf{k}$ and γ:

$$U_i(\mathbf{k}, \gamma) = k_i [\delta_{2_i} v(\theta - a) + \delta_{1_i} \pi(\theta)], \tag{11}$$

where $\theta = \theta(\mathbf{k}, \gamma)$. We can omit the term for the ith firm's profits when $\gamma_i = 0$, since from (10) its profits will then be zero.

II. EQUILIBRIUM WITH IMPOSED PRECOMMITMENT

This section briefly explores the types of equilibrium which result when the *type* of precommitment is given, but the firm is free to choose the *level* of precommitment. The firms choose k_i, but not γ_i.

(*a*) $\gamma = \mathbf{2}$: *Nash Equilibrium in Capacities.* If all firms precommit both factors, then we have an industry where firms choose capacities in the strategic stage. In this case $S(p, \mathbf{k}, \mathbf{2}) = v \sum k_i$ from (9), and for

$$v \sum k_i < F(0), \theta(\mathbf{k}, \mathbf{2}) = F^{-1}(v \sum k_i)$$

from (10), so that:

$$U_i(\mathbf{k}, \mathbf{2}) = [F^{-1}(v \sum k_i) - a] v k_i. \tag{12}$$

But of course (12) is simply the Cournot payoff function where we have capacities vk_i rather than outputs. The Nash equilibrium vector of capital stocks $\mathbf{k}^c$ and the resultant price $\theta^c = \theta(\mathbf{k}^v, \mathbf{2})$ are therefore the same as occur in the Cournot model. Under A1–2 a Cournot equilibrium exists (Novshek, 1985), and has the following relevant properties:

Properties of $(\mathbf{k}^c, \theta^c)$(P1) Productive efficiency $x_i/k_i = v$,

$$\text{(P2)}\ \frac{\theta^c - a}{\theta^c} = \frac{-1}{n\varepsilon_d}, \quad \text{where} \quad \varepsilon_d = \frac{\theta\, dF}{F\, d\theta}.$$

(P2) is of course simply the equilibrium profit-to-sales ratio in a symmetric Cournot equilibrium with constant marginal cost a.

(*b*) $\gamma = \mathbf{1}$: *Strategic Investment*. In this case all firms precommit only capital. This has been explored in Dixon (1985*a*), to which the reader is referred for a more detailed analysis. When $\gamma = \mathbf{1}$, the industry supply function is $S(p, \mathbf{k}, \mathbf{1}) = s(p)\sum k_i$. The payoff function in the case of strategic investment is:

$$U_i(\mathbf{k}, \mathbf{1}) = k_i \pi[\theta(\mathbf{k}, \mathbf{1})]. \tag{13}$$

Should an equilibrium exist,[2] it will be symmetric under A1. The equilibrium capitals and price are denoted $(\mathbf{k}^s, \theta^s)$. The properties of interest in the strategic investment equilibrium are:

Properties of $(\mathbf{k}^s, \theta^s)$.

$$\text{(P3)}\ \frac{\pi(\theta^s)}{\theta^s s(\theta^s)} = \frac{1}{n(\varepsilon_s - \varepsilon_d)}, \quad \text{where} \quad \varepsilon_s = \frac{\theta\, ds}{s\, d\theta}$$

(P4) $\theta^s(\mathbf{k}^s, \mathbf{1}) > a$

(P5) Undercapitalisation, $x/k = s(\theta^s) > v$.

P3 is derived from setting $\partial U_i/\partial k_i = 0$. The LHS is the profit to sales ratio, which in equilibrium equals the reciprocal of the sum of demand and supply elasticities times the number of firms. The presence of supply elasticity ε_s here reflects the fact that as a firm increases k_i, hence shifting the industry supply function outwards, the consequent reduction in price reduces the quantities produced by all firms. This term is absent in the capacity model because outputs are invariant with respect to the market price. Property P4 follows trivially from P3, so long as the LHS of P3 is positive (recall that $\pi(p) > 0$ iff $p > a$). Property P5, undercapitalisation, follows from P4: $p > a$ implies $s(p) > v$. The reason for this 'strategic inefficiency' is that when the firm chooses its capital stock it takes into account the effect that this will have on profits' $\pi(\theta)$ via the market price θ.

(*c*) $\gamma = \mathbf{0}$, *Bertrand*. When all firms are totally uncommitted, the capital stocks precommitted are constrained to be zero. This is a degenerate case of a Nash equilibrium in supply functions, since firms' strategy sets consist of only one element, $b(p)$. The industry supply function is $S(p, \mathbf{0}, \mathbf{0}) = n\Xi > nF(0)$. From (10) the resultant price is $\theta = a$, and hence the firms earn zero profits. This is the familiar Bertrand outcome. Production will of course be efficient since capital and labour are chosen simultaneously. From (10) we will

[2] Existence is examined in Dixon (1984, chapter 5 pp. 163–9).

Table: *A Comparison of Equilibria with Imposed Precommitment*

Variables Precommitted	*Profit to sales*	*Equilibrium type*
Both	$\frac{\theta^c - a}{\theta^c} = \frac{-1}{n\varepsilon_d}$	Cournot
Capital	$\frac{\pi(\theta^s)}{\theta^s s(\theta^s)} = \frac{1}{n(\varepsilon_s - \varepsilon_d)}$	Strategic investment
Neither	$\theta - a/\theta = 0$	Bertrand

obtain the Bertrand outcome whenever one or more firms are totally uncommitted.

We have now explored three different Nash equilibria in supply functions when the precommitment type is exogenously given. Total precommitment of all firms yields the Cournot outcome. Precommitment of capital only yields the strategic investment equilibrium explored in Dixon (1985*a*). In an industry where all firms are perfectly flexible, the Bertrand outcome occurs. These results are summarised in Table 1.

III. VOLUNTARY PRECOMMITMENT IN THE STRATEGIC METAGAME

In this section we treat precommitment type γ_i as a decision variable of the firm. In essence, the firm chooses the type of supply function it will have by its decision of which factors it will precommit. The firm's strategy space is expanded to of the firms to A_i in R^2_+:

$$(k_i, \gamma_i) \in A_i = R_+ \text{x}(1,2) \cup (0,0), \tag{14}$$

k_i is restricted to 0 when $\gamma_i = 0$. The Metagame $[A_i,\ U_i{:}i = 1, \dots n]$ thus encompasses the three cases considered in section II.

Before outlining the formal structure of the strategic metagame we shall briefly discuss how the firm might precommit its inputs. Consider the choice between total and partial precommitment. Since we are dealing with a voluntary fixed precommitment, the firm has to place upon itself a binding upper-bound on the labour employed in the market stage. If we interpret the labour input as men employed, the firm must prevent itself from being able to take people on during the market stage. A firm can clearly manage to 'bind' itself thus in a number of ways. Most importantly, it can determine its own organisation and operating rules which it cannot override except at a large cost to itself. Thus the firm can create an overly bureaucratic personnel department with complex and lengthy hiring procedures. The firm may impose a long (firm-specific) training period on newly hired individuals, or the firm can simply choose a particular labour hiring policy and embody this in the administrative structure and procedure of the firm. Such a policy might take the form of manpower specifications per unit of plant (in our simple model, $L = l_a k$, where l_a is the optimal labour–capital ratio).

When we talk of the 'firm' thus precommitting itself, we can mean a variety of things. For example, we can conceive of top management delegating the firm's manpower policy to lower management, who by reason of incentives or preference will pursue efficiency in production, thus facilitating the strategic behaviour of the top management.[3] Perhaps less plausibly shareholders can choose managers with a preference for such a manpower policy. Yet another possibility is that the 'firm' binds itself to a specific policy via collective bargaining: management and unions can agree on operating procedures, shift lengths, overtime and so on. Alternatively, we can interpret the precommitment of labour as occurring through the choice of a putty–clay technology as opposed to a putty–putty technology. This last interpretation does not fit in with the formal model actually presented here (since with total precommitment there are no variable costs in the market stage), but the results would still hold.

When we expand firms' strategy spaces to allow for the type of precommitment there are two types of equilibria in the resultant model: Cournot and Bertrand. However, before this is proven in the Theorem, we establish a crucial Lemma, which tells us that in some sense total precommitment is dominates partial precommitment:

LEMMA Let $(\mathbf{k},\gamma)\varepsilon A$, and $\theta(\mathbf{k},\gamma) > a$. If for some i $\gamma_i = 1, k_i > 0$, then there exist some strategy $(k_i^1, 2)$ such that given other firms' strategies k_{-i}, γ_{-i}: $U_i(k_i^1, k_{-i}, 2, \gamma_{-i}) > U_i(\mathbf{k}, \gamma)$.

Proof. Since $\theta > a$, firm i produces output $k_i s(\theta)$ inefficiently. Hence the firm can increase its profits by totally precommitting itself to produce the same output efficiently: the market price given by (10) is unchanged, revenue is unchanged, costs fall and profits increase. Q.E.D.

For $\theta(\mathbf{k}, \gamma) > a$, any strategy $(k_i, 1)$ is strictly dominated by some strategy $(k_i^1, 2)$. This is very important, since it implies that whenever firms earn positive profits (note $U_i > 0$ only if $\theta > 0$) they will choose to be totally precommitted to an inflexible production plan.

The Theorem demonstrates that there are two types of equilibrium $(\mathbf{k}^*, \boldsymbol{\gamma}^*)$ in the strategic metagame $[A_i, U_i; i = 1, \ldots, n]$: one where all firms choose total precommitment so that $\boldsymbol{\gamma}^* = \mathbf{2}$ and hence $\mathbf{k}^* = \mathbf{k}^c$ which yields the Cournot outcome, and one where at least one firm chooses $\gamma_j = 0$, which yields the Bertrand outcome.

THEOREM. Let $(\mathbf{k}^*, \boldsymbol{\gamma}^*)$ *be an equilibrium of the game* $[A_i, U_i{:}i = 1 \ldots n]$
(i) there exists an equilibrium $(\mathbf{k}^c, \mathbf{2})$, *where* $\theta^* = \theta^c$ *(Cournot)*
(ii) if $n \geqslant 2$ *there exist equilibria where* $\gamma_j = 0$ *for at least one* j, *where* $\theta^* = a$ *(Bertrand).*

Proof. Let $(\mathbf{k}^*, \boldsymbol{\gamma}^*)$ be an equilibrium. Either $\theta^* = a$, $\theta^* > a$ or $\theta^* < a$. Clearly $\theta^* < a$ cannot be an equilibrium, since firms will lose money.

If $\theta^* > a$, then no firm is uncommitted. Furthermore, each firm has

[3] For a general treatment of delegation, see Vickers (1985).

positive capital, $k_i > 0$, and $U_i > 0$ (if $k_i = 0$ so that $U_i = 0$, then the firm could choose small k_i so that $\theta > a$, and hence $U_i > 0$). Therefore $\boldsymbol{\gamma}^* = \mathbf{2}$ (from the Lemma, any strategy with $\gamma_i = 1$ will be dominated). So $\mathbf{k}^* = \mathbf{k}^c$, the Cournot vector of capital stocks. To see that $(\mathbf{k}^c, \mathbf{2})$ is actually an equilibrium, note that no firm will want to defect by choosing $\gamma_i = 0$, since then profits will fall to $U_i = 0$. Furthermore, no firm will defect by choosing $\gamma_i = 1$, since production is ineffecient. Therefore $(\mathbf{k}^c, \mathbf{2})$ is the only equilibrium where $\theta^* > a$.

If $\theta^* = a$, then $\gamma_i = 0$ for at least one firm. If $\gamma_i = 0$ for at least two firms, then whatever any one firm does $\theta = a$ from (10), and $U_i = 0$. Hence $(\mathbf{k}, \boldsymbol{\gamma})$ is a metagame equilibrium whenever $\gamma_i = 0$ for at least two firms. If $\gamma_i = 0$ for one firm, then there is an equilibrium iff for $j \neq i$ $v\sum k_j = F(a)$, and $\gamma_j = 1, 2$ (this presumes that committed firms have priority in meeting demand over uncommitted firms[4]). If $v\sum k_j = F(a)$, then whatever any committed firm j does, $\theta = a$ since $\gamma_i = 0$. The uncommitted firm i will make losses if it produces any output at all. If $v\sum k_j < F(a)$, however, the uncommitted firm can gain by precommitting a little capital and earning positive profits. Q.E.D.

The intuition behind the result is simple enough. The Cournot outcome occurs because firms will want to precommit both factors of production, since to precommit only capital will involve productive inefficiency, and no precommitment will condemn firms to zero profits. The second type of equilibrium yields the Bertrand outcome, and occurs because no firm can gain from precommitment, since production is efficient and any reduction in a firm's own output will be matched by a rise in the outputs of other firms which are uncommitted. Hence no one firm can prevent the zero profit Bertrand outcome.

That the Bertrand outcome is a metagame equilibrium depends crucially on two features of the model presented: the competitive market stage, and the imposition of constant returns to scale in A1. Even with a competitive market stage, the presence of diminishing returns will lead to the Cournot outcome as the unique metagame equilibrium. The Bertrand equilibrium depends on the fact that firms are willing to expand their own output as much as is necessary at the relevant price. However, with diminishing returns there is a well defined, upward sloping 'long-run' supply function. Thus if one firm totally precommits itself, it will be in the position of a monopolist with a competitive fringe, being able to increase profits by restricting output. The result that the Cournot outcome is a metagame equilibrium is more robust, and does not depend on a competitive market stage.

IV. UNCERTAINTY, IMPERFECT COMPETITION, AND ENTRY

If there is uncertainty, e.g. in factor prices or demand, there may be an additional cost to precommitment, since there is a reduction in the flexibility of production during the market subgame, which can impose a cost on the

[4] I would like to thank Paul Klemperer and Meg Meyer for pointing this out to me.

firm. With sufficient uncertainty it is possible to construct examples in which partial precommitment is not dominated by total precommitment.[5]

In the previous sections it has been assumed that the market stage is competitive, at least in the sense that the price is determined so as to clear the output market given firms' supply functions. This section briefly discusses the alternative assumption that the market stage is played according to Cournot rules. In the market stage, the equilibrium will be determined by the firm's reaction functions, which in turn are determined by the firms cost function in the market stage. Thus with a Cournot market stage firms choose reaction functions rather than supply functions. The nature and level of precommitment will determine the firm's reaction function in the market stage. The analysis in this section is very brief indeed, and draws upon Brander and Spencer (1983), who analyse the strategic investment case with a Cournot market stage.

As in Section II, consider first the model with imposed precommitment. In the case of total precommitment ($\gamma = 2$), the fact that the market stage is Cournot rather than competitive will make no difference to the overall equilibrium, which will be the standard one-shot Cournot outcome discussed in Section II (*a*). Since capital and labour are precommitted together, production will be efficient. Turning next to the case of no precommitment, the equilibrium is by definition the standard Cournot equilibrium. Unlike the model with a competitive market stage, there is no difference between the equilibrium outcome in the cases of total precommitment and no precommitment.

The strategic investment case with a Cournot market stage has already been analysed in great detail by Brander and Spencer (1983).[6] Precommitting capital has the effect of reducing costs in the market stage, and hence shifting the firms reaction function outwards. One implication of the market stage being Cournot instead of competitive is that in equilibrium productive inefficiency is due to over-capitalisation rather than under-capitalisation: there is more capital than required to minimise the costs of the outputs produced in equilibrium (this depends on the assumptions made: see Brander and Spencer (1983) p. 227, fn. 4, and Bulow *et al.* (1985)).

What of the metagame equilibrium in reaction functions? Brander and Spencer consider the case where firms choose whether or not to precommit capital given that labour is uncommitted. They show that if firms choice of strategic precommitment is limited to the capital variable, then firms will choose to precommit capital.[7] If we extend the firms' decision to allow precommitment of labour as well, then this becomes the dominant strategy. The structure of the argument is the same as in our Theorem: if the firm precommits only capital, then there will be a strategic inefficiency in production

[5] An example of demand uncertainty is laid out in detail in Dixon (1984, chapter 6).

[6] Although they consider the strategic variable as R & D, the model is formally equivalent to a model with capital stocks as the strategic variable (*op. cit.* p. 226).

[7] 'If one firm ignores the possibility of strategic use of R & D while the other firm does not... the first firm loses, whilst the second firm gains relative to the pure Cournot rules' (Brander and Spencer, 1983, p. 230).

hence the firm can gain by precommitting labour and producing the same output.

We have considered two alternative assumptions about the market stage (i.e. competitive and Cournot). The main difference in the equilibrium outcomes between these approaches is that the Bertrand outcome can only be an equilibrium with a competitive market stage. The basic incentives for precommitment are similar in both models: strategic advantage and productive efficiency. Given the wide range of possible assumptions that could be made about the equilibrium in the market stage, can we generalise from these two cases?

Within the context of static oligopoly, the conjectural variations equilibrium is a useful way of embracing many possible solution concepts. The basic Cournot model is extended to include the firm's 'conjectures' about how other firms will react to changes in its output. Whilst this model is quite restrictive (see Ulph (1983)) it has been employed in strategic investment models (Eaton and Grossman (1984), Venables (1984), Yarrow (1985)). From the point of view of the strategic metagame, the most important point is that inefficiency in production is endemic with partial precommitment: all the exact nature of product market competition influences is the direction and level of the factor bias (see also Bulow *et al.* (1985) and Fudenberg and Tirole (1984)). Eaton and Grossman (1984) do find, however, that if the market stage is a 'consistent' conjectural equilibrium then there will be no factor bias in the strategic investment regime, and production will be efficient ('consistency' here being the condition suggested by Bresnahan (1981), namely that the conjectures and the actual reaction functions coincide locally at equilibrium). However, in Dixon (1985*b*) I show that Eaton and Grossman's result holds only for conjectures that are exogenous and happen to be consistent. With endogenous consistent conjectures strategic inefficiency will still occur in general. The result that the Cournot outcome is a metagame equilibrium is therefore very robust with respect to the exact nature of the market stage.

Allowing entry can have different effects on the model depending on when entry is allowed, and whether there are sunk costs. With a competitive market stage and constant returns, the presence of strictly positive sunk costs rules out the Bertrand equilibria. If there are no sunk entry costs, then the equilibria will all be Bertrand. One way of conceptualising free entry is that whilst incumbents might have a choice of precommitment type, potential entrants are always uncommitted, and hence any reduction in output by incumbents can be matched by output from new entrants (this obviously relates to Grossman (1981), and the metagame equilibrium in our Theorem when one firm is uncommitted).

V. CONCLUSION

This paper has provided a framework for analysing voluntary precommitment in the context of a simple oligopoly model. Two themes have emerged from the results. The first theme concerns the incentives for the firm taken on its own to precommit. The second theme concerns the nature of the equilibrium which results from the firms' behaviour. Precommitment determines the flexibility of production, the firm's cost structure. The flexibility of production determines the market outcome – via firms' supply functions (or Cournot reaction functions).

In the absence of uncertainty two considerations enter into the firm's precommitment decision productive efficiency, and strategic advantage. These considerations may work in different directions,[8] but in the case of total precommitment as opposed to partial precommitment, both considerations lead the firm to favour a totally inflexible production plan. This leads to the Cournot outcome being a metagame equilibrium. With uncertainty, however, the incentive to maintain flexibility of production is present and may predominate.

There were found to be two types of equilibria in the strategic metagame when the market stage is competitive: Cournot and Bertrand. This is a pleasing result, since the metagame framework embraced both concepts not only as possibilities, but also as equilibria. In the case of the Cournot equilibrium, we can see firms attempting to achieve an inflexible cost structure in order to reap the benefits of efficiency and strategic advantage. In the Bertrand case, where firms are uncommitted, any attempt to restrict output through precommitment fails due to uncommitted firms raising output (recall that this depends on the assumption of constant returns). The metagame framework gives a plausible account of why these equilibria might come about by making the cost structure in the industry endogenous.

The framework and results presented here are mainly of theoretical interest, in that they provide another perspective for interpreting some of the factors determining commonly used oligopoly solutions. If we wanted to apply the model, there are two key issues that need consideration. First, the costs of precommitment. These will be determined partly by technological factors (the duration of the period of production, and scope for its variation), and partly by institutional factors which determine how the factor markets function (are there spot markets, is training firm specific?). In many instances these considerations would point fairly unambiguously to one type of precommitment being much cheaper than the others. Secondly, there is the presence of uncertainty, and its exact nature. In Section IV we only examined the possibility of demand uncertainty. In practice other forms of uncertainty may be important – relative factor price uncertainty for example. Lastly the model is essentially a static model. The precommitment decision could be put in an explicit temporal framework, as a dynamic game. However, it is hoped that the framework presented provides an interesting perspective on the issue, and starting point for subsequent analysis.

Birkbeck College
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[8] In Brander and Spencer's model of (1983), where we start from the case of no precommitment. If the firm precommits capital, then its production will become inefficient. Brander and Spencer show that the strategic gains outweigh this effect.

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