

## UNIONS, OLIGOPOLY AND THE NATURAL RANGE OF EMPLOYMENT\*

*Huw Dixon*

This paper aims to provide a simple but sensible model to explore some of the implications of imperfect competition for the nature of macroeconomic equilibrium and the effectiveness of macroeconomic policy. In microeconomics the study of imperfect competition in labour, product or other markets is well developed, recognising the importance of oligopolies and unions in modern industrial economies, where highly concentrated product markets often coexist with unionised labour markets. In macroeconomics the implications of this state of affairs for policy analysis have hardly begun to be examined (more recent exceptions include d'Aspremont *et al.* (1985), Benassy (1987), Blanchard and Kiyotaki (1987), Dixon (1987*a, b*), Layard and Nickell (1986)). Whilst there are strong differences of opinion as to the implications of imperfect competition for macroeconomic policy, no general framework exists for examining these issues (most standard macroeconomic models assume competitive markets). I have tried to construct a model which whilst simple, includes what I believe to be the most essential ingredients for a sensible macroeconomic model of imperfect competition. The resulting model has some interesting features which are not captured in existing macroeconomic models.

I will first outline the ingredients which I believe to be important in constructing a macro-model of imperfect competition. First, wages are less flexible than prices. In practice, wages are often fixed by long-term contracts (annual in the United Kingdom, often as long as three years in the United States), perhaps because changes are associated with large transactions costs.<sup>1</sup> Firms' pricing and output decisions are commonly variable over a much shorter period of time. This suggests that a two stage model is appropriate: in the first stage, nominal wages are determined in the labour market; in the second stage, outputs and prices are determined in the product market. The first stage represents the long-run wage contracts; the second stage the short run fluctuations of firms' output decisions. Secondly, real wages are relative wages, not own-product wages. Real wages are nominal wages deflated by an appropriate price index of consumption goods. Since output prices will be some mark-up over wages, the real wages in a particular industry will be determined by relative wages. In practice wage relativities play an important part in the

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<sup>1</sup> In British manufacturing, over 90% of labour contracts are for one year or more (Gregory *et al.* 1985).

wage bargaining and some of the macroeconomic implications of this are explored in Oswald (1979) and Gylfason and Lindbeck (1986). We set this phenomenon within the context of a fully specified macromodel. Thirdly, if real wages are seen as being determined by relative wages, then clearly one sector macromodels are inappropriate: we need a multi-sector macromodel. As our results show, a two-sector macromodel is able to capture effects which are just not possible in single-sector or 'representative' sector macromodels: *asymmetries* between the two sectors play a crucial role in determining aggregate employment, the structure of wages and the effect of macroeconomic policy.<sup>2</sup> With more than one sector, fiscal policy inevitably has a 'microeconomic' dimension: the government has to decide how to allocate its total expenditure across sectors. This microeconomic decision can have important macroeconomic consequences.

The model presented in this paper is a simple two-sector general equilibrium macromodel with oligopolistic price determination in the product market, and unionised labour markets. The government controls expenditure and the money supply. There are two industries in the economy, each with a monopoly union that sets the nominal wage, and duopolistic firms that determine prices given wages. Macroeconomic equilibrium is modelled as a Nash equilibrium between the unions, which determines equilibrium wages, prices and employment in each sector for given government policy. This two-stage solution can be interpreted as a subgame perfect equilibrium. The interactions between unions, firms and households are much more complex than those involved in a competitive economy where all such issues are swept under the carpet of the 'price-taking' assumption. In Sections I–III we present a simple log-linear model based on specific assumptions, which possesses an explicit solution, from which we derive clear policy results. Whilst the assumptions are specific, they are also 'standard': households have Cobb–Douglas preferences, firm technology involves constant returns, and unions have Stone–Geary utility. In Section IV we present a much more general framework, which shows that certain key results of the log-linear model are fairly general.

What are the results of this approach? First, the equilibrium will generally involve involuntary (union voluntary) unemployment in the labour market. Secondly, the model possesses a continuum of equilibria. For any given mix of macroeconomic policy, there exists a unique equilibrium level of employment: however, by altering the level and/or sectoral mix of government expenditure, the government can alter the aggregate employment level – there exists a *Natural Range* of employment. It should be emphasised that the Natural Range feature of the model is generated by the unionised wage-determination process: in the corresponding Walrasian model, the assumptions made imply a unique Natural Rate at full employment, unaffected by policy. The model also

<sup>2</sup> Existing multisectoral models (e.g. Blanchard and Kiyotaki, 1987; Nickell and Layard, 1986) do not allow for such asymmetries. Lillian (1982) presents and estimates a model in which asymmetries in demand across sectors lead to multiple equilibria: with search, rapid structural shifts lead to higher search unemployment in equilibrium. The Natural Range in this paper is generated by unionised wage setting rather than search.

contrasts with Macdonald (1987), who derives a range result only by introducing lump-sum costs of price-adjustment, which mean that prices and wages become fixed for small changes in demand.

The basic reason for the Natural Range result is that a change in the relative demand across sectors leads to a change in relative wages, and hence in relative employment. Total employment will be unaffected by this only if an increase in employment in one sector is *exactly* offset by a decrease in the other: this property is not satisfied in the log-linear model (Section III), nor is it compatible with a wide class of functional forms (Theorem 1, Section IV).

In the explicit log-linear model the basic feature which determines where employment lies within the Natural Range is the degree of asymmetry in government fiscal policy in the two sectors. Higher levels of aggregate employment can be obtained by concentrating expenditure in one of the sectors. The reason for this result is that equilibrium employment in each sector is log-linear in the ratio of final demands in the two sectors. An increase in demand in sector 1 relative to sector 2 will bring a proportionate increase in employment in sector 1, and an equiproportionate decrease in employment in sector 2. The size of the *absolute* changes will be larger for the larger sector: thus if sector 1 is larger than sector 2, a 10% increase will outweigh the 10% decrease in employment in sector 2. This implies that total employment is minimised when government expenditure is equal in both sectors. This property is not peculiar to the log-linear model, and though not general it will hold for a range of functional forms (Theorem 2(b), Section IV).

The other results for the log-linear model concern specific policy effects (Section III). A general across the board increase in government expenditure in fixed proportions between sectors will increase aggregate employment unless expenditure is equal in both sectors (Proposition 2). Holding total government expenditure constant, a reallocation of expenditure towards the high expenditure sector will lead to an increase in total employment (Proposition 3). An increase in government expenditure in the high (low) expenditure sector will increase (decrease) employment (Proposition 1). Monetary expansion leads to a fall in employment unless fiscal policy is balanced, when it is neutral. These results are not general, and arise from the specific assumptions made. However, given that the assumptions themselves are fairly standard, and certainly not pathological, they are perhaps an interesting special case.

In the more general framework in Section IV, the results are of course rather less clear. However, Theorem 2(a) demonstrates that any symmetric model (e.g. representative sector models such as Blanchard and Kiyotaki (1987), Dixon (1987a), Layard and Nickell (1986)) will appear to have a Natural Rate. This is because such models assume that demand is the same in each sector. For small changes in the mix of demand, there will be a local neutrality of policy, although there would be a Natural Range if the models allowed for larger changes in the mix of demand.

## I. THE MODEL

There are two industries  $i = 1, 2$ . In each industry there is a labour market and a product market. In each industry two firms buy labour to produce output, the labour is supplied by  $H$  households who supply labour only to that industry. There is thus no mobility of labour between the two industries, perhaps due to sector specific skills. Whilst nominal wages are treated as exogenous in the next two sections, in Section III we will introduce a monopoly union which sets the nominal wage in the labour market. The government purchases output from the two sectors and determines the money supply. We shall now outline the basic assumptions of the model.

(a) *Households*

Each household is allocated to one of the industries for which it has the appropriate skills, and there are  $H$  households in each industry. Households have one unit of labour which they supply with no disutility, and initial money balances (net of lump-sum taxes or subsidies)  $M^0/H$ . Households can be employed or unemployed. For the  $N_i$  employed households income is  $(M^0/H) + W_i$  where  $W_i$  is the nominal wage: for the  $H - N_i$  unemployed households income is just  $M^0/H$ . Households do not receive any profits: these are distributed capitalists who accumulate money balances.

Households in sector  $i$  have a Cobb–Douglas utility function defined on money balances, and consumption of the output from the other sector ( $X_j$ ) – the ‘Pork-pie’ effect.<sup>3</sup> The Pork-pie effect captures the notion that with many sectors, household consumption from the industry in which they work is not significant. It is a simplification which enables us to obtain an explicit solution for the union’s reaction function. Whilst we deal with a two-industry model here, the framework can be easily generalised to an  $n$ -sector model: the ‘other sector’ stands for the rest of the economy. With many sectors, the importance of the output of industry  $i$  for households in  $i$  would become very small, and hence the Pork-pie effect would become a good approximation (see Dixon (1987*b*) for a formal treatment of a large unionised economy). The presence of money balances in the utility function can be seen as arising either through ‘liquidity services’ provided, or as a proxy for the utility of future consumption as in ‘mixed’ indirect utility functions – see Grandmont (1984). ‘Money’ could also be interpreted as a non-produced good as in Hart (1982). With Cobb–Douglas utility, it makes no difference whether real or nominal money appears in  $U_i$  – so we have not deflated by price. Households in sector  $i$  thus solve the programme:

$$\max_{X_j, M_i} X_j^\alpha (M_i)^{1-\alpha}, \quad (1)$$

$$\text{s.t.} \quad P_j X_j + M_i \leq M^0/H + W_i \quad (\text{employed}), \quad (2)$$

$$\leq M^0/H \quad (\text{unemployed}). \quad (3)$$

<sup>3</sup> The name ‘Pork-pie effect’ derives from the experience of a friend of mine who worked at a pork-pie factory, and as a result has not wished to consume pork-pies since then.

Given identical Cobb–Douglas preferences, we can aggregate over employed and unemployed households who of course face the appropriate budget constraint (2) or (3) respectively yielding:

$$P_j X_j = \alpha(M^0 + W_i N_i) \quad i, j = 1, 2 \quad i \neq j. \quad (4)$$

Of course (4) has unit price elasticity. The capitalists net demand for money is equal to profits.

(b) *Government*

The government chooses its *nominal* expenditure in each sector  $G_i$ ,  $i = 1, 2$ . We have used nominal expenditure rather than real expenditure for two reasons. First, it is more realistic in the United Kingdom context. Since 1982 government expenditure has been planned in nominal terms: since 1978 Cash Limits and nominal plans have been in operation alongside ‘real plans’.<sup>4</sup> The second reason is convenience: with cash-limits the government’s elasticity of demand in any particular sector is unity, the same as the consumer demand generated by Cobb–Douglas households. Whilst our explicit results depend upon this, for analytical purposes we could simply have a government demand function  $G(p)$  which is decreasing in price. This would allow government expenditure to influence the economy through altering the elasticity of industry demand (as in Rankin, 1987).

$M^0$  is the total initial money balances in each sector, net of lump-sum taxes/subsidies. Since money is the only financial asset in this model, there is no monetary policy in any real sense: a ‘Helicopter drop’ of money is equivalent to a fiscal subsidy. Initial money balances are given, and the government ‘chooses’  $M^0$  by applying an appropriate lump-sum tax/subsidy. The government does not finance expenditure by taxation, so that the government’s budget constraint implies that the aggregate ‘end-of-period’ money balances equal ‘initial’ money balances  $2M^0$  plus expenditure  $G_1 + G_2$ .<sup>5</sup>

(c) *Firms*

In each industry, there are two firms which produce a homogenous product under constant returns to scale. Labour productivity is normalised to unity, so that output equals employment in each sector:

$$X_i = N_i. \quad (5)$$

The Duopolists buy labour taking the wage  $W_i$  as given, so that their marginal costs are  $W_i$ . Prices are determined in the industry by a conjectural-variations Cournot model: firms choose outputs, taking into account their conjecture about the other’s response, and the price clears the market given the

<sup>4</sup> How real budget plans should be interpreted is not straightforward. In democratic countries, the legislature usually approves a *nominal* budget proposed by the executive. A real plan is only implementable if revisions to an initial budget can be made within the relevant time period as new information about wages and prices becomes available. A nominal plan is a commitment to make no such revisions.

<sup>5</sup> Taxes could easily be introduced.  $M^0$  could simply be interpreted as net of taxes. No significant distinction between lump-sum and income tax exists here, since labour supply is perfectly inelastic.

outputs chosen. Each firm believes that the other firm's proportional response to changes in output is  $\phi$ . If  $\phi = 0$ , each firm treats the other's output as given, and we will obtain the standard Cournot–Nash equilibrium. If  $\phi = -1$ , then both firms believe that any increase in their own output will be matched exactly by a decrease in the other firms: this is called a Bertrand conjecture, since total output (and hence price) is conjectured to be independent of the quantity chosen by the firm. If  $\phi = +1$ , each firm believes that any changes will be exactly matched by its competitor, so that the actions of the two firms are perfectly coordinated, which will lead to the collusive joint-profit-maximising solution. We adopt the conjectural-variations approach because of its generality; it can embrace many different market solutions – from perfect competition, through Cournot oligopoly, to collusion.

Since the industry demand is unit elastic, the equilibrium price-cost margin  $\mu$  is given by<sup>6</sup>

$$\mu_i \equiv \frac{P_i - W_i}{P_i} = \frac{1 + \phi_i}{2}. \quad (6)$$

Following Lerner (1934) we call  $\mu_i$  the degree of monopoly. For simplicity, we assume for most of the paper that  $\phi$  and hence  $\mu_i$  are the same in both sectors. For Bertrand conjectures,  $\phi = -1$ , we have the competitive outcome  $\mu = 0$ ; with Cournot conjectures ( $\phi = 0$ ) we have  $\mu = \frac{1}{2}$ ; as  $\phi$  tends to one from below  $\mu \rightarrow 1$  (note that since demand is unit elastic, no equilibrium exists when  $\phi \geq +1$ : the collusive case is a limiting result).

Our assumption of Cobb–Douglas preferences for households combined with Cash-Limits for government expenditure means that industry demand is unit elastic. This, combined with constant returns to scale, gives the model a very simple solution for prices which are a constant mark-up over wages, reflecting the degree of competition in the industry. From the partial equilibrium perspective, an increase in demand will lead to a pure output response, there being no increase in price for the given wage. The own-product real wage is determined solely by the nature of competition in this product market, since this determines the mark-up of price over nominal wages:

$$W_i/P_i = 1 - \mu_i \quad (i = 1, 2). \quad (7)$$

The fact that the mark-up  $\mu$  is independent of the level of demand is a convenient result which arises from the fact that households and government have constant elasticity of demand. In a more general model an increase in demand could lead to a change either way in  $\mu$ . There seems no obvious way to relate clear comparative statics for  $\mu$  to assumptions about household preferences.

#### (d) *The Demand for Labour*

Wage and price determination is a two-stage process: in the first stage wages are determined (by monopoly unions), and in the second stage prices are

<sup>6</sup> An alternative approach is to assume Cournot–Nash behaviour, and vary the number of firms  $n$ : in this case  $\mu = 1/n$ . The market will be more competitive as the number of firms increases, with the perfect competition as the limiting case.

determined *given* wages. This time structure of the model corresponds to the plausible notion that wages adjust more slowly than prices. We will now examine what happens in the second stage with exogenous 'fixed' wages: this determines each union's demand curve, which gives the relation between the nominal wage set and labour demanded by the firms.

Suppose that there is a given level of wages,  $\mathbf{W}$ , government expenditures  $\mathbf{G}$ , and initial money  $M^0$  ( $\mathbf{W}$ ,  $\mathbf{G}$  are 2-vectors of  $W_i$ ,  $G_i$  respectively). What will the corresponding levels of employment  $N_i$  be in both sectors? If we look at market  $i$ , the demand for output is

$$P_i X_i = G_i + \alpha(M^0 + W_j N_j) \quad i, j = 1, 2 \quad i \neq j. \quad (8)$$

From (7) and (5) we can replace  $P_i$  and  $X_i$  by  $W_i$  and  $N_i$

$$N_i = \frac{G_i}{W_i}(1 - \mu) + \alpha(1 - \mu) \left( \frac{M_0}{W_i} + \frac{W_j}{W_i} N_j \right). \quad (9)$$

This gives us two linear equations (9) in two unknowns  $N_i$ .<sup>7</sup> Solving we have

$$N_i = K_i / W_i \quad (10)$$

where 
$$K_i \equiv \frac{1 - \mu}{1 - \alpha^2(1 - \mu)^2} \{G_i + \alpha(1 - \mu) G_j + M^0[\alpha + \alpha^2(1 - \mu)]\}.$$

$K_i$  is the 'reduced-form' measure of demand for output/employment in industry  $i$ , once the demand spillovers and feedbacks between the two sectors have been worked through. Thus an increase in  $G_1$  will first increase demand and employment in industry 1. As a result, the income of households in sector 1 will rise, which will raise expenditure in sector 2, and so on. The familiar income-expenditure feedback will converge at the levels of employment given by (10). If we consider the level of employment in sector  $i$  as we vary  $W_i$ , we have a rectangular hyperbole, since  $W_i N_i = K_i$ . The wage bill is thus constant, which is due to the assumption of Cobb–Douglas preferences, Cash Limits and constant returns. In the next section, this demand for labour curve will define the trade-off between wages and employment for the union for a given macroeconomic policy chosen by the government. From (10) if we treat wages as fixed, there will be standard multiplier effects for fiscal and monetary policy.

(e) *The Walrasian equilibrium*

Setting  $\mu = 0$ , with no disutility of labour both labour markets clear with  $N_i = H$ , and nominal wages  $W_i = K_i/H$ . Fiscal and monetary multipliers are both zero. Macroeconomic policy affects only nominal and relative wages and prices, not output or employment. It is also very important to note that there is a unique Natural Rate of employment in the Walrasian economy which is unaffected by  $(\mathbf{G}, M^0)$ . Of course, it is possible to get natural range results in one-sector Walrasian models where leisure is normal, and in two-sector models with a different marginal product of labour (diminishing returns, or a different

<sup>7</sup> There are also the two inequality constraints  $N_i \leq H$ . To avoid unrewarding details, we ignore these.

labour input-output ratio in each sector). In the present model, none of these effects is present: hence the Natural Range result in the unionised economy is due purely to the process of wage determination underlying the equilibrium.

## II. EQUILIBRIUM IN A UNIONISED ECONOMY

We now turn to the first stage of our macroeconomic equilibrium, the wage-determination process. In each industry, there is one monopoly union which has the power to set the nominal wage. When each union sets the nominal wage in the industry, it treats the nominal wage in the other industry as given. However, the union takes into account the effect of the wages set on prices and employment chosen by firms in the second stage (this is given by the demand for labour). We will therefore be able to represent the overall macroeconomic equilibrium as a Nash-equilibrium between the two unions in the first stage, where the second-stage outcome is summarised by the labour demand function (10).

There are many possible ways of modelling trade union behaviour. We assume that there is a monopoly union in each industry, with the objectives of the union leadership being represented by a utility function defined on real wages and employment. Fortunately, the standard Stone–Geary specification of union utility enables us to solve explicitly for macroeconomic equilibrium. This functional form is standard in the monopoly union literature (see Dertouzos and Pencavel, 1981; Oswald, 1985; Pencavel, 1984).

The real wage in this model is the nominal wage deflated by the appropriate cost-of-living index. Since households in industry  $i$  only consume output from the other sector  $j$ , real wages are here:

$$\frac{W_i}{P_j} = \frac{W_i}{W_j} (1 - \mu) \quad i, j = 1, 2 \quad i \neq j. \quad (11)$$

Thus *relative* wages determine *real* wages: unions care about relative wages because this determines the purchasing power of wages in their own sector. Each union's own-product real wage is a matter of indifference because of the Pork-pie effect. With a symmetric Stone–Geary specification, the union's objective is to maximise

$$U_i = (N_i - F) \left[ \frac{W_i}{W_j} (1 - \mu) - \theta \right], \quad (12)$$

where  $F$  and  $\theta$  are 'reservation' levels of real wages and employment, and are open to various interpretations.  $F$  can be taken to represent the 'core' employment of union 'insiders' in the sector:  $\theta$  can be interpreted as a wage-push parameter.<sup>8</sup> From (10) for sector 1 (12) becomes

$$U_1(\mathbf{W}) = \frac{K_1}{W_2} (1 - \mu) + F\theta - F \frac{W_1}{W_2} (1 - \mu) - \theta \frac{K_1}{W_1}. \quad (13)$$

<sup>8</sup>  $\theta$  could be interpreted as the disutility of labour: although we have assumed there is zero disutility, it would be simple enough to allow for a fixed disutility in the present model. Dertouzos and Pencavel (1981) interpret  $\theta$  as capturing an envy effect from wage relativities.

The nominal wage  $W_1$  is set to maximise (13) treating  $W_2$  as given

$$\frac{\partial U_1}{\partial W_1} = -\frac{F}{W_2}(1-\mu) + \frac{\theta K_1}{W_1^2} = 0.$$

Hence

$$W_1 = \left[ \frac{\theta}{F(1-\mu)} K_1 W_2 \right]^{\frac{1}{2}}. \quad (14)$$

Equation (14) gives the best wage for the union in sector 1 to set given the level of demand it faces ( $K_1$ ) and the nominal wage set by union 2: the *reaction-function* for union 1.

For obvious reasons, it is convenient to express (14) in logarithms, so that we have the two reaction functions

$$\log W_i = r_i(W_j) = \frac{1}{2} \log \left[ \frac{\theta}{F(1-\mu)} \right] + \frac{1}{2} \log K_i + \frac{1}{2} \log W_j. \quad (15)$$

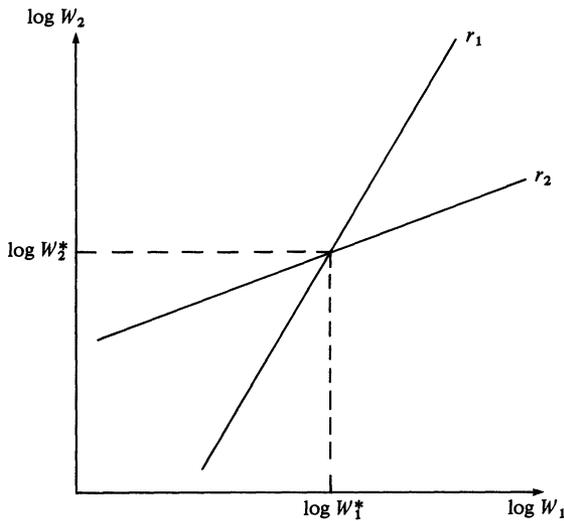


Fig. 1. Nash equilibrium in wages.

These are depicted in Fig. 1. For an equilibrium to exist, we require the unions to achieve positive utility, which implies<sup>9</sup>  $\theta < (1-\mu)$ ,  $F < H$ .

The Nash-equilibrium between the two unions is represented by the crossing of the two reaction functions, at  $\mathbf{W}^*$ . We can solve (15) explicitly for the equilibrium wages, and resultant employment levels

$$\log W_i^* = \log \left[ \frac{\theta}{(1-\mu)F} \right] + \frac{2}{3} \log K_i + \frac{1}{3} \log K_j, \quad (16)$$

$$N_i^* = (1-\mu) \frac{F}{\theta} \left( \frac{K_i}{K_j} \right)^{\frac{1}{3}}. \quad (17)$$

<sup>9</sup> If  $\theta > 1-\mu$ , then it is not possible for both unions to attain positive utility. For obvious reasons, this might result in inflationary pressure and a wage-wage spiral.

Where from (10)

$$\frac{K_1}{K_2} = \frac{G_1 + \alpha(1-\mu)G_2 + M^0\alpha[1 + \alpha(1-\mu)]}{G_2 + \alpha(1-\mu)G_1 + M^0\alpha[1 + \alpha(1-\mu)]}. \quad (18)$$

Equations (16) and (17) will only characterise the equilibrium when the full-employment constraint is non-binding in both sectors, i.e.  $N_i^* \leq H$  ( $i = 1, 2$ ). For  $F/\theta$  large enough, the equilibrium will of course be one with full-employment. In this paper, we concentrate only on those cases where the full employment constraint is non-binding, and there is unemployment in both sectors.

The introduction of monopoly unions in the two industries has determined the equilibrium level of wages and employment in both sectors, which are unique for any given government policy. In the case of a 'balanced' fiscal policy,  $G_1 = G_2$ , employment is given in each sector by  $(1-\mu)F/\theta$ , which we will call the Balanced Rate of employment ( $N_B$ ).  $N_B$  is thus determined by the degree of monopoly and union preferences – less competition in product markets leads to a lower  $N_B$ , as does higher wage push  $\theta$ ; higher reservation employment will of course increase  $N_B$ . The actual level of employment in either sector may be above or below  $N_B$ , depending on the ratio  $K_i/K_j$ . The deviation of  $K_i/K_j$  from unity will occur if fiscal policy favours one sector more than another. As government policy favours sector 1,  $K_1/K_2$  rises: as fiscal policy favours sector 2,  $K_1/K_2$  falls.

From (18) it can be seen that money enters symmetrically into  $K_1$  and  $K_2$ . Thus an increase in  $M^0$  moves  $K_1/K_2$  nearer to unity, and hence both levels of equilibrium employment nearer to the balanced rate  $N_B$ . Thus if  $K_1 > K_2$  so that  $N_1 > N_B > N_2$ , an increase in  $M^0$  will reduce  $N_1$  and raise  $N_2$ , and vice-versa. If  $K_1 = K_2$ , changes in  $M^0$  will have no effect on employment. Money is not neutral in this model, which is unsurprising given that government expenditure is fixed in *nominal* terms.

From (16) it is clear that what determines real and relative wages is the relative level of demand across sectors. A higher level of government expenditure in sector 1 will shift the two unions' reaction functions out ( $K_i$  both increase), but the effect will be stronger in sector 1 (the increase in  $K_1$  is larger). Thus  $W_1$  will rise by more than  $W_2$ : and since  $P_2$  is a markup on  $W_2$ ,  $W_1/P_2$  will also rise. Solving (16) for the equilibrium real wage for union 1 we have

$$\frac{W_1}{P_2} = \frac{W_1}{W_2}(1-\mu) = (1-\mu)\left(\frac{K_1}{K_2}\right)^{\frac{1}{3}}. \quad (19)$$

In the case of balanced government fiscal policy, the real wages in both sectors are equal to the own-product real wage  $1-\mu$ . If policy favours sector 1, the real wages in sector 1 will be larger than the own-product real wage, and real wages in sector 2 smaller. As in the case of employment, an increase in the money supply will increase  $K_1$  and  $K_2$  equally, thus reducing any imbalance between demand in the two sectors. This will lead to an equalisation of real wages, as real wages in both sectors move towards the own-product real wage.

As is clear from (19) the structure of relative wages in the unionised economy is determined by the relative intensity of demand in the two sectors: a change in the relative levels of demand will alter relative wages and employment, and as we shall see in the next section total employment will also vary. The exact values of  $\theta$  and  $F$  in the unions' utility function does not influence the structure of real wages, although they will of course influence the equilibrium levels of nominal wages and employment. This is because we have assumed that unions have the same utility function: if we allowed for  $(\theta, F)$  to be union specific then the story would be different. If (say) both unions push harder for real-wages ( $\theta$  rises), the outcome will merely be lower employment and the same real/relative wages. However, if only one union adopts a stronger preference for real wages, then it will push up the equilibrium wage of its members relative to those of the other unions.

### III. MACROECONOMIC POLICY IN A UNIONISED ECONOMY

We have seen how the introduction of wage setting unions and price setting firms provides a framework which determines the level of sectoral employment and the structure of real wages in the economy. Macroeconomic policy is not usually conceived of as being concerned with particular industries, but rather with aggregates such as employment. How can we use the microeconomic analysis of the previous sections to characterise the macroeconomic policy options in the imperfectly competitive economy? This section provides a simple diagrammatic exposition of the policy options open to the government. First, I outline the Natural Range theory in the context of the log-linear model. Secondly, I evaluate the effects of particular policies on aggregate employment in the log-linear model: the policies being an increase in expenditure in one sector; an across the board increase in expenditure; a reallocation of expenditure across sectors; and a helicopter drop of money.

Aggregate employment is simply the sum of employment in the two sectors

$$N = N_1 + N_2 \quad (20)$$

$$= (1 - \mu) \frac{F}{\theta} \left[ \left( \frac{K_1}{K_2} \right)^{\frac{1}{3}} + \left( \frac{K_2}{K_1} \right)^{\frac{1}{3}} \right]. \quad (21)$$

It turns out that it is very easy to represent the feasible employment levels. The *product* of employment in both sectors is constant, since from (17)

$$N_1 N_2 = (1 - \mu)^2 \left( \frac{F}{\theta} \right)^2. \quad (22)$$

In the left-hand quadrant of Fig. 2 we plot employment in the two sectors on the axis, and (22) is represented by the rectangular hyperbola. In the right-hand quadrant we depict union 2's employment as a function of  $K_2/K_1$ . We can also determine upper and lower bounds on employment in each sector using (18) to determine upper and lower bounds for  $K_1/K_2$  as  $(\mathbf{G}, M^0)$  varies. For an

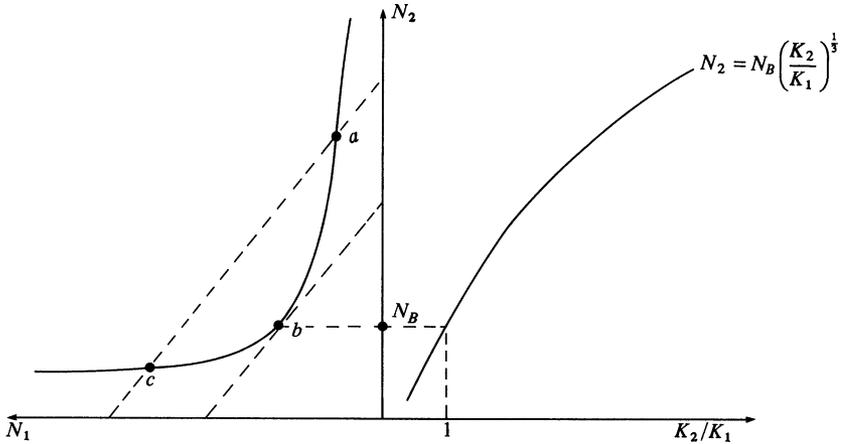


Fig. 2. The Natural Range of employment.

equilibrium to exist,  $M^0 > 0$ , so  $K_1/K_2$  must lie in the open interval  $(\alpha(1-\mu), 1/\alpha(1-\mu))$ . Hence we have upper and lower bounds for  $N_i$ , denoted  $\bar{n}$  and  $\underline{n}$  respectively

$$\bar{n} \equiv (\alpha - \alpha\mu)^{1/3} N_B > N_i > (\alpha - \alpha\mu)^{-1/3} N_B = \underline{n}. \tag{23}$$

When we combine (23) with (24) we can see from Fig. 2 that only the portion of the rectangular hyperbola between points  $a$  and  $c$  is attainable through varying  $(\mathbf{G}, M^0)$ .

What are the implications for aggregate employment  $N$ ? The aggregate iso-employment loci are simply the class of negatively sloped  $45^\circ$  lines. The lowest attainable level of employment occurs at point  $b$ , where the employment levels are equal. This corresponds to the case of balanced fiscal policy, where employment in each sector is  $N_B$ . The upper limit of employment is  $\bar{N}$ , the isoemployment locus passing through points  $a$  and  $c$

$$\bar{N} = \bar{n} + \underline{n}. \tag{24}$$

The *Natural Range* of attainable aggregate employment levels is thus

$$N \in [2N_B, \bar{N}]. \tag{26}$$

The ‘Natural Range’ property of the imperfectly competitive economy is in complete contrast to the underlying Walrasian economy, in which full employment  $N = 2H$  occurs for any  $(\mathbf{G}, M^0)$ . Imperfect competition provides the government with freedom of choice over employment, albeit a limited one. The values of  $\alpha$  and  $\mu$  determine the size of the Natural Range:  $\alpha$  represents the proportion of household income spent (the rest is ‘saved’ to accumulate money);  $1 - \mu$  is the share of wages in national income. A back of the envelope calculation using the relevant United Kingdom magnitudes would indicate that  $\alpha(1-\mu)$  might be around 0.64 ( $\alpha = 0.8, 1 - \mu = 0.8$ ). In this case the maximum level of employment would be some 5% higher than the balanced rate. This magnitude is certainly not small in the context of United Kingdom policy debates, representing a variation of around one million in employment.

Essentially, the government can increase aggregate employment by adopting a policy leading to a greater imbalance in  $K_i$ . Turning to fiscal policy, suppose that the government starts with a policy where  $G_1 > G_2$ . In this case  $K_1/K_2 > 1$ , and employment in the two sectors is represented by a point on the rectangular hyperbola between  $b$  and  $c$ . In this circumstance an increase in  $G_1$  will make fiscal policy less balanced, pushing  $K_1/K_2$  further away from 1. In terms of Fig. 2 we will move along the rectangular hyperbola further away from point  $b$  towards point  $c$ : the rise in  $N_1$  is larger than the fall in  $N_2$ , so that total employment rises. Suppose that we start from a position between  $a$  and  $b$  on the rectangular hyperbola: in this case  $G_2 > G_1$ ,  $N_2 > N_B > N_1$ , and  $K_1/K_2 < 1$  – fiscal policy favours employment in industry 2. An increase in  $G_1$  will make fiscal policy more balanced, pushing  $K_1/K_2$  nearer to unity and causing a move along the rectangular hyperbola towards  $b$ , resulting in a fall in aggregate employment. Thus an increase in government expenditure can lead either to an increase or a decrease in total employment. What matters is the effect on the *balance* of final expenditures  $K_1/K_2$ . The intuition behind this result is that the equilibrium employment equations (17) are log-linear in  $K_1/K_2$ : a change in  $K_1/K_2$  will have equiproportionate but opposite effects on sectoral employment. The absolute effects are proportional to size, so that the rise (fall) in the larger sector will dominate the fall (rise) in the smaller sector leading to a change in aggregate employment.

We will now explore more formally the effects of policy on employment. First, consider the effect of an increase in nominal expenditure  $G_i$  in one sector (a selective expenditure increase).

**PROPOSITION 1.** *An increase in government expenditure sector  $i$  will increase (decrease) total employment if expenditure is larger (smaller) in that sector, i.e.  $G_i > G_j$  ( $G_i < G_j$ )*

(all proofs are in Appendix). Next we consider the effect of an increase in aggregate government expenditure  $G$ , holding sectoral expenditure shares constant: for some  $0 \leq \delta \leq 1$ ,  $G_1 = \delta G$  and  $G_2 = (1 - \delta)G$ .

**PROPOSITION 2.** *An increase in total expenditure  $G$  in fixed proportions across sectors leads to an increase in aggregate employment unless expenditure is equally divided between sectors, in which case there is no effect on employment.*

The reason behind Proposition 2 is that when  $\delta \neq \frac{1}{2}$  (an unbalanced fiscal policy), an increase in  $G$  will lead to a move in  $K_1/K_2$  away from unity. There are ultimately two autonomous sources of final demands  $K_i$  in the economy: government expenditures  $G_i$ , and initial money balances  $M^0$  which effect  $K_i$  symmetrically. If  $\delta \neq \frac{1}{2}$ , then an increase in  $G$  leads to a rise in the imbalance in sectoral demands. If  $\delta = \frac{1}{2}$ , then  $K_1 = K_2$  whatever  $G$ , and employment is at the balanced rate  $N_B$ . Next, we consider the effect of a reallocation of expenditure *between* sectors: varying  $\delta$  with  $G$  constant.

**PROPOSITION 3.** *If  $G$  is held constant, a reallocation from sector 2 to sector 1 (an increase in  $\delta$ ) will increase (decrease) employment when  $\delta > \frac{1}{2}$  ( $\delta < \frac{1}{2}$ ).*

The reason for this result is that a reallocation of expenditure towards the high expenditure sector will increase the asymmetry of final demand across sectors.

An increase in the money supply is never expansionary, since money enters symmetrically into both  $K_i$ . Thus an increase in  $M^0$  moves  $K_1/K_2$  nearer to unity, increasing the balance between expenditure in the two sectors. If there is an imbalance in fiscal policy, then an increase in the money supply will reduce aggregate employment. In effect an increase in the money supply moves the equilibrium along the rectangular hyperbola towards  $b$  in Fig. 2. Only if fiscal policy is balanced does money have no effect on employment, which will be at its balanced level in both sectors. Note that in this model, 'monetary' policy is not 'pure': a change in  $M^0$  will alter equilibrium wages and prices, and hence *real* government expenditures will also change. An increase in  $M^0$  raises wages and prices, causing a reduction in real government expenditure.

It should be noted that in this model unemployment plays no role: (21) determines equilibrium employment, union behaviour is not influenced by the unemployed 'outsiders'. This is perhaps unrealistic (though standard). A simple *ad hoc* way to allow unemployment to matter is to make union preferences ( $\theta$  and  $F$ ) depend on unemployment: maybe unions care more about employment and less about wages when unemployment is high. This effect would tend to reduce the Natural Range, since union preferences would change with employment, shifting the rectangular hyperbola in Fig. 2 inwards at higher levels of employment. Clearly, however, a full treatment of this would require a more explicit and complicated model of union behaviour.

#### IV. THE NATURAL RANGE – TOWARDS A GENERALISATION

The model presented in the previous sections is specific: this naturally raises the question of how far its results are of more general interest. Is the model typical, or a special case? Whilst the assumptions employed are not pathological, it still remains to be demonstrated what general inferences can be made. In this section we focus on perhaps the most important feature of the model from the policy point of view: the Natural Range property. We present a more general framework which is compatible with the specific model, and within which it is easy to explore the influence of functional forms. Theorem 1 demonstrates that under fairly mild assumptions the Natural Rate Hypothesis is incompatible with an important and general type of functional form – polynomials. The Natural Range result is not a special case.

In order to explore the issue of generality, we have found it necessary to alter the approach to the problem. Rather than deriving the macroeconomic equilibrium explicitly from assumptions about microeconomic agents (firms, unions, and households), we will start from a more aggregated level. Given the complexity of the underlying relationships in an imperfectly competitive economy, this is perhaps as near to a tractable general analysis as we might hope to get. The Natural Rate Hypothesis (NRH) is open to many

interpretations. For our present purposes we shall adopt the following rather strong definition: there exists a unique equilibrium level of employment in the economy which is unaffected by the government's macroeconomic policy. The NRH is certainly satisfied in the Walrasian version of the model in this paper: there is full employment in each sector whatever the government does. As we shall demonstrate, in unionised economies it is very unlikely, and perhaps impossible for the NRH to be satisfied. Thus the presence of imperfect competition yields different policy implications from Walrasian models.

The basic conceptual framework in this section is the same as in the log-linear model: there are two sectors, labour is sector specific, monopoly unions set nominal wages, and so on. We will start the generalisation of the model from the demand for labour function, which defines the wage/employment trade-off faced by the unions. The demand for labour is a derived demand, depending both on demand for output, and the firms' input decision. In its most general form, the demand for labour in sector  $i$  can be written as function of nominal wages and nominal demand for output:

$$N_i = N_i(W_i, W_j, K_i) \quad (i = 1, 2). \quad (26)$$

This corresponds to equation (10).  $K_i$  is a measure of *nominal* demand for sector  $i$ : it could be given the specific interpretation in the paper, or be a more general shift parameter for a sectoral demand. The government is assumed to be able to influence  $K_i$  through its fiscal-monetary policy. What properties can be expected to hold for  $N_i$  in general? Homogeneity to degree zero (HODO) in  $(\mathbf{W}, K_i)$  seems a natural property: if nominal wages and nominal demand for both outputs double, we would expect the labour demanded by firms to remain constant

$$A1 \text{ Homogeneity: } N_i(\mathbf{W}, K_i) \text{ are HODO in } (\mathbf{W}, K_i).$$

Given the sectoral labour demand functions faced by the monopoly unions, we can define the reaction functions  $r_i$ , which give the optimal wage  $W_i$  as a function of the wage set in the other sector and demand  $K_i$ . In the most general form, union utility can be seen as depending on real wages and employment. As in the model, we will assume a 'Pork-pie' effect, or simply a direct 'envy' effect (as in Oswald, 1979; Gylfason and Lindbeck, 1986), so that relative wages enter the utility function. The union in sector  $i$  chooses the nominal wage  $W_i$  to maximise utility subject to labour demand (26)

$$\max_{w_i} U_i(W_i/W_j, N_i) \quad (27a)$$

$$\text{s.t.} \quad N_i = N_i(\mathbf{W}, K_i). \quad (27b)$$

We will assume that the above programme is well defined, possessing a unique solution continuous in  $W_j$  and  $K_i$ :

$$W_i = r_i(W_j, K_i) \quad (i, j = 1, 2). \quad (28)$$

This corresponds to (14) in the model. The Nash Equilibrium for this economy is obtained by solving (28) for  $W_1$  and  $W_2$ . We assume that there is a unique equilibrium, which is continuous in  $\mathbf{K}$ :

$$W_i = W_i(\mathbf{K}) \quad (i = 1, 2). \quad (29)$$

Equilibrium employment in each sector is obtained by substituting (29) into  $N_i(\cdot)$ :

$$N_i[W_1(\mathbf{K}), W_2(\mathbf{K}), K_i]. \quad (30)$$

Since labour demand is HODO in  $(\mathbf{W}, \mathbf{K})$ , it follows from (27) that reaction functions  $r_i$  are Homogeneous to degree 1 (HOD1) in  $(W_j, K_i)$ , and the equilibrium equations (29) are HOD1 in  $\mathbf{K}$ . If the levels of demand in each sector are increased by the same proportion, then all that happens is that the equilibrium nominal wages double, leaving sectoral and aggregate employment unchanged. This homogeneity allows us to write equilibrium employment solely as a (continuous) function of the *relative* strength of sectoral demand  $k \equiv K_1/K_2$ :

$$N_i = n_i(k). \quad (31)$$

This corresponds to (17). The government may face limitations on the extent to which it can influence  $k$ : we will simply make the general assumption that  $k$  is restricted to a bounded convex set  $A$ , which is not a singleton, and includes unity and some values above and below unity (set  $A$  is  $[\alpha(1-\mu), 1/\alpha(1-\mu)]$  in the log-linear model).

The framework we have presented in this section is very general: for example, there are no monotonicity restrictions on  $n_i$  (although we might have strong priors that  $n_1$  is increasing and  $n_2$  decreasing). We can now define the NRH formally as a linear cross-equation restriction on  $n_i$ . Denoting the Natural Rate as  $NR$ , we have for all  $k$  in  $A$ :

$$\text{Natural Rate Hypothesis: } n_1(k) + n_2(k) = NR.$$

Clearly, such a restriction on  $n_i$  is very strong relative to the set of possible continuous functions, which in itself perhaps indicates that the NRH is a special case.

If both unions have the same utility function (27a), and face the same labour demand (27b), then we have a *symmetric* model. Such symmetry implies that for any  $k$ , employment in sector  $i$  will be the same as it would be in sector  $j$  if the situation were reversed, and demand were at  $1/k$  (which is satisfied in the specific model)

$$\text{Symmetry: } n_i(k) = n_j(1/k) \quad (i \neq j).$$

We can now analyse the generality of the Natural Range result. In the specific model of the previous sections, the Natural Range result was due to the log-linearity of  $n_i(\cdot)$ . How far does the Natural Range property extend? The only restriction that we have placed on  $n_i$  is continuity. Given the general framework, it is easy to test whether or not the Natural Range property holds for specific functional forms of  $n_i$ . Theorem 1 below demonstrates that if  $n_i$  are

polynomials, then under symmetry the NRH cannot hold. This is an important result, since the polynomials are a general class of functions that can be used to approximate any continuous function (see Stone–Weierstrass Theorem, Rudin, 1976, p. 159), and of course differentiable functions by Taylor's Theorem. The only exception to this is the trivial case where  $n_i$  are constant, which we rule out by a non-invariance assumption:

A<sub>2</sub>: *Non-Invariance*: If there exist  $k'$  and  $k''$  such that  $n_i(k') = n_i(k'')$  and  $k' < k''$ , then there exists some  $k$  such that  $k' < k < k''$  and  $n_i(k) \neq n_i(k')$ .

**THEOREM 1.** *Under Homogeneity (A 1), Non-Invariance (A 2), and symmetry, the Natural Rate Hypothesis cannot hold if  $n_i$  are polynomials.*

This result shows that the Natural Range result is not a fluke of log-linearity: if  $n_i$  are polynomials, the Natural Range is typical, whilst the NRH is a special case (for example resting on a particular asymmetry between sectors). Of course, we have adopted a strong interpretation of the NRH: it could well be argued that so long as the Natural Range was 'small', the NRH would be a good working approximation.

We have interpreted the NRH as a global restriction: aggregate employment is independent of macroeconomic policy mix. This does not imply that policy may be ineffective at a particular value of  $k$ . In the specific model we noted that if  $k = 1$  (resulting from a symmetric fiscal policy), then policy multipliers are zero – both monetary and fiscal policy have no effect on aggregate employment. As in the case of the Natural Range, the argument here rested on the log-linearity of  $n_i$ . Theorem 2 demonstrates that this result carries over to the general framework under homogeneity and symmetry: macroeconomic policy will be ineffective when  $k = 1$ . The only additional assumption that needs to be made is that  $n_i$  are differentiable. Part (b) of the Theorem also states the condition for employment to be (locally) minimised at  $k = 1$ :

**THEOREM 2:** *Under A 1, if the economy is symmetric, and  $n_i$  are twice differentiable: (a) macroeconomic policy is ineffective if  $k = 1$ . (b) Total employment is at a local minimum at  $k = 1$  if*

$$-\frac{n''}{n'} < \frac{1}{2}.$$

Part (a) demonstrates that deriving policy results from 'representative' sector models with symmetric policy can be very misleading: an inevitable *local* policy ineffectiveness emerges. As Theorem 1 demonstrates, this local result cannot be generalised. It is clearly essential in such models to allow for asymmetries in policy, and hence the full range of possible policies rather than concentrating on a special case. Part (b) shows that employment will be locally minimised when  $k = 1$  so long as  $n(\cdot)$  is not 'too' concave (the left hand side of the inequality is of course the Arrow–Pratt measure of concavity used to measure absolute risk-aversion). Whilst this concavity condition is not particularly restrictive, there will surely be examples for which it is violated. The conditions for employment to be a global minimum are rather more difficult to interpret (see equation (A 3) in Appendix).

## CONCLUSION

This paper presents a simple two-sector model of imperfect competition with price-setting firms and wage-setting unions, in which the macroeconomic implications of fiscal and monetary policy can be analysed. There are perhaps two general conclusions to be drawn from the exercise.

First, imperfect competition matters. A fundamental issue which is prior to any macroeconomic analysis is how we conceive of macroeconomic equilibrium. The nature of macroeconomic equilibrium will tell us how wages, prices, and employment are determined: it tells us the welfare properties of the equilibrium, it tells us the desirability and possibility for government policy intervention. Given the importance of unions in the economy and the prevalence of industrial concentration, it is clearly important to develop models which reflect these facts. The results of this paper, suggest that the macroeconomic equilibrium in an imperfectly competitive economy is fundamentally different to the competitive equilibrium. The equilibrium may involve unemployment; the process by which nominal, real, and relative wages are determined is wholly different to that in a market-clearing economy. As a consequence, the analysis of macroeconomic policy is also different.

Second, the imperfectly competitive economy displays a Natural Range of Employment: fiscal and monetary policy can be used to obtain a range of equilibrium levels of employment. This is a general result which will apply to a broad range of models. The Natural Range property in this model is due to imperfect competition: the Walrasian case displays a unique Natural Rate at full employment which is unaffected by policy. The size of the Natural Range is an empirical question. However, even if the Natural Range represents a small proportion of employment, it may still be 'large' in policy terms. Calibrating the model using United Kingdom magnitudes suggests that the Natural Range might represent 5% of employment, which represents a potential variation of around one million in unemployment.

The effects of fiscal and monetary policy are determined by the adjustment of wages and prices by unions and firms. Whilst policy can be used to increase employment, there is no simple link between the level of demand and employment: indeed, in an imperfectly competitive economy it is quite possible for wage and price inflation caused by a rise in demand to result in a fall in aggregate employment. By modelling wage and price determination explicitly in an imperfectly competitive economy, the analysis of macroeconomic policy raises a whole set of effects and issues which are not present in a perfectly competitive framework.

Lastly, it should be stressed that the basic framework of the paper – a multisector unionised economy – has great potential. The present paper has addressed policy purely in terms of static equilibrium and its comparative statics. However, the introduction of wage and price setting unions introduces new possibilities for dynamic models: wage adjustment can actually be explained when there are optimising unions setting wages. The model of this paper can easily be adapted to consider alternatives to the static Nash-

equilibrium employed here: an interesting possibility is to have overlapping contracts. We leave the exploration of these possibilities to future research.

*University of Essex*

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#### APPENDIX: PROOFS

**THEOREM 1.** *Define the aggregate employment function  $N$ :*

$$N(k) = n_1(k) + n_2(k).$$

Under symmetry,  $n_2(k) = n_1(1/k)$ , so (dropping sectoral subscripts):

$$N(k) = n(k) + n(1/k). \quad (\text{A } 1)$$

If  $n_i$  are polynomials,  $n(k) = a_0 + \sum_{i=1}^m a_i k^i$

so that

$$N(k) = 2a_0 + \sum a_i (k^i + k^{-i}),$$

$$\frac{dN}{dk} = k^{-m-1} \sum a_i i (k^{m+i} - k^{m-i}). \quad (\text{A } 2)$$

Under Non-invariance, some  $a_i$  are non-zero ( $i = 1, \dots, m$ ), and (A 2) has at most  $2m$  roots. Hence  $dN/dk$  is non-zero almost everywhere, and  $N$  varies with  $k$ : the Natural Rate Hypothesis does not hold, and there is a range of equilibrium employment levels. Q.E.D.

**THEOREM 2.** (a) *From (A 1), differentiating:*

$$\frac{dN}{dk} = \left. \frac{dn}{dk} \right|_k - (1/k^2) \left. \frac{dn}{dk} \right|_{1/k}.$$

When  $k = 1$  this is zero, since the derivatives are evaluated at the same point. (Note that one of the roots of (A 2) is  $k = 1$ .)

(a) The second derivative of employment with respect to  $k$  is

$$N''(k) = n''(k) + n''(1/k)/k^4 + n'(1/k)/k^3. \quad (\text{A } 3)$$

Evaluated at  $k = 1$  we have:

$$N''(1) = 2n''(1) + n'(1)$$

which is positive if  $-(n''/n') < \frac{1}{2}$ .

Q.E.D.

Brief outlines of calculus proofs of Propositions 1-3.

**PROPOSITION 1.** *Define  $k = K_1/K_2$ . Recalling (22)  $N_1 N_2 = N_B^2$ , we have*

$$dN_2/dG_1 = -(N_2/N_1) (dN_1/dG_1). \quad (\text{A } 4)$$

Clearly, since  $k$  is increasing in  $G_1$ , it follows that  $dN_1/dG_1 > 0$ . The change in total employment is

$$dN/dG_1 = dN_1/dG_1 + dN_2/dG_1$$

$$\text{from (A 4)} \quad = dN_1/dG_1(N_1 - N_2)/N_1. \quad (\text{A 5})$$

(A 5) is positive if  $k \geq 1$  ( $N_1 \geq N_2$ ) and negative if  $k \leq 1$  ( $N_1 \leq N_2$ ). Q.E.D.

PROPOSITION 2. Define  $\Gamma = \{\alpha M^0 [1 + \alpha(1 - \mu)]\}/G$ . Hence

$$d\Gamma/dM^0 > 0 > d\Gamma/dG.$$

Since  $G_1 = \delta G$ , and  $G_2 = (1 - \delta)G$ , we have

$$k = \frac{\delta + (1 - \delta)(1 - \mu)\alpha + \Gamma}{(1 - \delta) + \delta(1 - \mu)\alpha + \Gamma}.$$

$$\text{Hence} \quad \text{sign } dk/d\Gamma = \text{sign } \delta - \frac{1}{2}.$$

$$\text{Recall} \quad \text{sign } N_1 - N_2 = \text{sign } \delta - \frac{1}{2}.$$

Analogously to (A 4):

$$dN/dG = (dN_1/dG)(N_1 - N_2)/N_1. \quad (\text{A 6})$$

If  $\delta \geq \frac{1}{2}$  then  $N_1 \geq N_2$  and  $dN_1/dG \geq 0$ , so that (A 6) is positive: if  $\delta \leq \frac{1}{2}$  then  $N_1 \leq N_2$ ,  $dN_1/dG \leq 0$ , and (A 6) is positive. Q.E.D.

PROPOSITION 3. Analogously to (A 4):

$$dN/d\delta = (dN_1/d\delta)(N_1 - N_2)/N_1. \quad (\text{A 7})$$

Since  $dN_1/d\delta > 0$ , if  $N_1 \geq N_2$  ( $\delta \geq \frac{1}{2}$ ) then (A 7) is positive, and vice-versa.

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