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Keeping up with the Joneses: competition and the evolution of collusion

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Abstract

An economy consists of many markets, each of which is a duopoly. Firms must earn normal-profits in the long-run if they have to survive. Normal-profits are interpreted as the long-run limit of average profits in the whole economy. We adopt an aspiration based model of firm behaviour, linking it to the economy with the requirement that in the long-run, the profit aspiration must be at least as great as normal-profits. We assume that the joint-profits can be maximized with symmetric payoffs, and with very few other assumptions are able to show that the (almost) global attractor is the cooperative outcome. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

"*The best monopoly profit is a quiet life*" Hicks (1935).

"*This is the criterion by which the economic system selects survivors: those who realize positive profits are the survivors; those who suffer losses disappear*" Alchian (1950, p. 213)

It has long been argued that firms must earn at least normal-profits to survive in the long-run. [1] Failure to achieve this will activate some market mechanism such as bankruptcy, the possible replacement of managers by shareholders, or takeover. In general, we can think of the mechanisms as reflecting the operation of the capital market in its widest sense.

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[1] There are obvious exceptions here, such as non-profit organisations and owner-managed firms. We are considering the "typical" managerial public corporation.

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The capital market reflects the aggregate performance of the economy as represented by average profitability. In this paper the level of normal-profits is taken to be the average level of profits in the economy and explores the implications of this hypothesis in the context of an economy consisting of many oligopolistic markets. *Under fairly general assumptions there are powerful long-run forces pushing the firms in each market towards collusion*. What differentiates the approach here is that the evolution of the economy is inherently social, in that it is the level of average profits in the whole economy over time which drives the behaviour of firms.

We model the behaviour of firms using an aspiration based model of bounded rationality. Firms at any time adopt a pure-strategy. If they are achieving their aspiration level, then they are likely to continue with the same strategy (Hick's "quiet life" alluded to in the opening quote). If however, they are below their aspiration level then they are likely to experiment and try something new. This approach has been put forward both as a good model of *individual* decision making in the mathematical psychology literature Lewin (1936) and Siegel (1957) and as a model of *organizational* decision making with relevance to the firm Simon (1947,1981), Cyert and March (1963) and Kornai (1971). Furthermore, we adopt a formulation which allows the aspiration level to be endogenous (as in Borgers and Sarin, 1997; Karandikar et al., 1998; Palomino and Vega-Redondo, 1999), reflecting the past and the current profitability of the economy.

The key feature of this paper is to link together the aspirations of firms with the level of normal-profit by requiring that in the long-run the aspiration level of all firms is to have at least normal-profits. The structure of the economy envisaged is that of an economy consisting of a large number of identical duopolies. Firms have a finite strategy set, and we need assume very little about the structure of the payoff matrix of the constituent duopoly game, except that the joint-payoff can be maximized by a payoff-symmetric outcome. We need to make some assumptions about aspirations and experiments: if firms are achieving their current aspiration level, then they do not experiment, whilst if they are below aspiration they do. In the case that firms decide to experiment and try out a new strategy, we need assume only that the probability of choosing any particular strategy is bounded away from zero over time.

The main result of the paper is Theorem 1, which states that the collusive (joint-profit maximizing) outcome is the (almost) global attractor for this economic system. *What is novel about this result is that cooperation is not only possible, but almost inevitable*. In the case of the Prisoner's dilemma the dominant strategy of defection will disappear, and all firms will end up cooperating.

The intuition behind this result is that if one or both firms is behaving competitively, then one or both of the firms will earn relatively low profits and experiment. The only state of rest is where both firms in an industry are earning at least average profits: in the long-run, this can only happen if all firms in the economy are collusive. If a firm deviates from the collusive outcome, there is a sense in which it will be "punished" as the other firm searches for an acceptable outcome. Hence the profitability of short-term deviations from collusion is not sufficient to undermine it in the long-run.

The policy implications of the model are quite strong, despite the abstract nature of the model. In particular, collusive outcomes will tend to emerge from the market process rather than competitive outcomes. This does not happen due to explicit or implicit collusion between firms in particular markets. Rather, it is the economy-wide social pressure of the capital markets forcing firms to earn average profits in the long-run that enforces collusion.

In Section 2 of the paper, we outline the basic model in terms of payoffs and strategies. In Section 3, we consider how the economy/population evolves over time, and state the main results. Section 4 explores two examples: the PD and Cournot duopoly. In Section 5, I consider an example to explore the model when payoffs are not quite symmetric: if we allow for aspirations to be a little below average, then collusion still has a large basin of attraction (although other attractors will exist). In Section 6, I discuss the recent related literature, and then conclude.

2. The model

Time is discrete and eternal, with $t = 0, \ldots, \infty$. There are K pure-strategies, $k = \{1, \ldots, K\}$. Π is the $K \times K$ matrix of payoffs π_{ij}, where π_{ij} is the payoff when strategy i plays strategy j. We can define the set of pairs of strategies as $\mathcal{L}$: where $\mathcal{L} \equiv \{(i, j) : (i, j) \in \{1, 2, \ldots, K\}^2$ and $i \leq j\}$, so that $L = \#\mathcal{L} = \frac{1}{2}K(K+1)$. Elements of $\mathcal{L}(r, q \in \mathcal{L})$ may sometimes be referenced by the underlying pair (i, j). $\mathcal{A}$ is the set of subsets A of $\mathcal{L} : \mathcal{A} \equiv \{A : A \subseteq \mathcal{L}\}$. In particular, the set of *payoff-symmetric pairs* is $\mathrm{Sym} \equiv \{q \in \mathcal{L} : \pi_{ij} = \pi_{ji}\}$. The average payoff earned by a pair q is: $\pi(q) \equiv \frac{1}{2}(\pi_{ij} + \pi_{ji})$. Define (a) $\mathrm{Maxav} = \max_{q \in \mathcal{L}} \pi(q)$, (b)[2] $\Pi S = \max_{q \in \mathrm{Sym}} \pi(q)$, (c) $S = \arg\max_{q \in \mathrm{Sym}} \pi(q)$.

Assumption 1 (A1). $\Pi S = \mathrm{Maxav}$.

Assumption 1 requires that the maximum joint-payoff can be attained at a payoff-symmetric pair of strategies. For example, consider the standard Prisoner's dilemma (PD) with

$$\Pi = \begin{bmatrix} 2 & 0 \\ a & 1 \end{bmatrix}$$

where 2 is the cooperative payoff, 1 the payoff when both defect, 0 the sucker's payoff and $a > 2$ the double-crosser's payoff. A1 is satisfied if $a \leq 4$: that is the combined defect/sucker payoff is less than the combined cooperative payoff. This is of course the standard assumption in the PD. In fact, if we consider any symmetric payoff function $U(x, y)$, where x and y belong to a compact strategy set $X \subset \Re$ and generate Π by taking a finite subset of X (e.g. constructing a grid), then A1 is satisfied if U is strictly concave.

The economy consists of a continuum of markets $\lambda \in [0, 1]$, each consisting of a duopoly. $J_t(A, \lambda)$ is a characteristic function, such that $J_t = 1$ iff duopoly λ is playing some pair $q \in A$ at t, which defines the Borel measure $P_t : \mathcal{A} \to [0, 1]$

$$P_t(A) \equiv \int_0^1 J_t(A, \lambda)\, \mathrm{d}\lambda$$

$P_t(A)$ gives the proportion of markets that have duopolies of type $q \in A$ at time t.

[2] Note that $S \subseteq \mathcal{L}$ and may contain more than one element.

The average level of profits at t in markets with pair $q \in A$ is [3]

$$\bar{\Pi}_t(A) = \sum_{q \in A} P_t(q) \frac{\pi(q)}{P_t(A)}$$

The average level of profits in the economy at t is $\bar{\Pi}_t = \bar{\Pi}_t(\mathcal{L})$.

2.1. Aspirations and learning

In any one period, a firm plays a pure strategy. As in the Atkinson and Suppes (1958) "finite Markov model" there is a probability that at time $t+1$ the firm will switch from the strategy it plays at t: the key difference with the present paper is that we use an explicit aspiration based model.

Each firm follows the following simple learning rule. It has an aspiration level α_t. If it is earning less than α_t, then it decides to experiment with probability 1; if the firm is earning at least α_t, then it will continue with the existing strategy. In this paper, we assume that all firms share the same aspiration level, with the aspiration level satisfying the condition that in the long-run it has to be no less than average profits. This seems a reasonable assumption reflecting the role of capital markets in industrialized economies.

Assumption 2 (A2). There exists $\{\nu_t\}$ such that $\nu_t \geq 0$, $\nu_t \to 0$ as $t \to \infty$ and

$$\alpha_t \in [\bar{\Pi}_t - \nu_t, \Pi S]$$

One possibility satisfying A2 is to have $\alpha_t = \bar{\Pi}_t$ (Dixon, 1995). The upper bound is imposed because it ensures that aspirations are not overoptimistic: under A1 it is not possible for all firms to earn over ΠS. The results of this paper would not hold if firms had aspirations which were in excess of ΠS in the long-run (e.g. if firms aspired to the highest payoff achieved in the past by an individual firm, this might well exceed ΠS). [4]

Given that a firm decides to experiment, we can define its *conditional switching probabilities*: $s_t(i, g)$ is the period t probability that a firm switches from strategy i to strategy g, conditional upon deciding to experiment. These probabilities will reflect the learning process of the firms: it could be based upon imitation, best response or random noise or some mixture. The only restriction we put on switching behaviour is that switching probabilities are bounded away from 0.

Assumption 3 (A3, Conditional switching probabilities). There exists $\gamma > 0$ such that for all $i, g \in \{1, \dots, K\}$, $t = 0, \dots, \infty$, $s_t(i, g) \geq \gamma > 0$.

The assumption that switching probabilities are bounded away from zero captures the notion that there might be some noise in the switching process. Since this can be arbitrarily

[3] For convenient shorthand, I represent $P_t(\{q\})$ as $P_t(q)$, the proportion of markets adopting actions in the singleton set $\{q\} \in \mathcal{A}$.

[4] In the case of PD, since $a > \Pi S$ this would certainly be the case.

small, we do not consider this to be a demanding assumption. Whilst we assume for simplicity that all firms have the same aspiration level and conditional switching probabilities, the results go through if we allow for firm specific attributes. [5]

Whilst we have interpreted switching behaviour as the same firm in two periods changing behaviour, the formal model would be exactly the same if we think of a different firm in each period. For example, a firm in a particular market might exit (due to bankruptcy or death). In this case the switching probability would pertain to the "place" of the firm: the probability that next period the firm taking the place of the existing firm would play a particular strategy.

3. The evolution of the population

At any time t, we can divide the set of markets into two groups: (i) AA_t the "above aspiration" markets and (ii) BA_t the "below aspiration" markets in which one or both firms have profits strictly below aspiration. Since the payoffs of each firm are determined by the pair of strategies played at its market, we can classify markets according to the pair of strategies played at that market

$$\mathrm{AA}_t = \{(i, j) \in \mathcal{L} : \pi_{ij} \geq \alpha_t \text{ and } \pi_{ji} \geq \alpha_t\}, \qquad \mathrm{BA}_t = \mathcal{L} - \mathrm{AA}_t$$

Since α_t may vary with time, the set of pairs $\mathrm{AA}_t \subseteq \mathcal{L}$ will in general vary over time. The two subsets S and Sym are time invariant. Clearly, under A1 and A2, $S \subseteq \mathrm{AA}_t$ for all t. Furthermore, define $\mathrm{BSym}_t \equiv \mathrm{BA}_t \cap \mathrm{Sym}$.

The evolution of the population is captured by the proportion of markets that belong to strategy pairs in these sets: in particular, $P_t(S)$ is the proportion of industries which are collusive at time t. Under A1 and A2 the sequence $P_t(S)$ satisfies some important properties. First note that (a) there exists $P^* \in [0, 1]$ such that $P_t(S) \to P^*$ and (b) if $P_t(S) > 0$ for some t, then $P_\tau(S) > 0$ for $\tau \geq t$ and $P^* > 0$. These both stem from the fact that $P_t(S)$ is monotonic and bounded. Monotonicity follows since once an industry is colluding, its profits must be at least equal to the economy average (A1) and hence they do not switch strategy (A2).

We first prove that the proportion of industries with one or both firms below aspiration $P_t(\mathrm{BA}_t)$ tends to zero fast enough.

Lemma 1 (based on A1–A3). *$\sum_{t=0}^{\infty} P_t(\mathrm{BA}_t)$ is bounded.*

Proof. First, we establish that $P_t(\mathrm{BSym}_t) \to 0$. Consider the change in the proportion of firms in duopolies with strategy pairs in S, $P_t(S) - P_{t-1}(S)$. This change is the result of

[5] In a longer and more general version of the paper (CEPR Discussion Paper 1810, same title) I allowed for firm-specific switching probabilities: in particular they are able to depend upon the history of the individual firm, the current and past value of average profits and so on so long as A3 is satisfied. Similarly, aspirations can be firm specific so long as they satisfy A2. I also allowed for the probability of experimenting if above aspiration to be non-zero but to tend to zero with the infinite sum being bounded. Furthermore, the probability of experimenting if below aspiration need not be 1, so long as it is bounded away from zero. All of these generalizations taken together are consistent with Theorem 1.

inflows: a lower bound on inflows is $\gamma^2 P_{t-1}(\mathrm{BSym}_{t-1})$ (from A2 and A3 and the definition of BSym_t). Hence

$$P_t(S) - P_{t-1}(S) \geq \gamma^2 P_{t-1}(\mathrm{BSym}_{t-1}), \quad 1 \geq P^* \geq P_0(S) + \gamma^2 \sum_{t=1}^{\infty} P_{t-1}(\mathrm{Bsym}_{t-1})$$

Hence $\sum P_t(\mathrm{BSym}_t)$ is bounded and so $P_t(\mathrm{BSym}_t) \to 0$.

An analogous argument shows that if $P_t(\mathrm{BSym}_t)$ tends to zero, so must $P_t(\mathrm{BA}_t)$. Again, finding a lower bound for inflows, and an upper bound for outflows into Sym

$$P_t(\mathrm{Sym}) - P_{t-1}(\mathrm{Sym}) \geq \gamma^2 P_{t-1}(\mathrm{BA}_{t-1}) - P_{t-1}(\mathrm{BSym}_{t-1})$$

The lower bound for inflows comes from the fact that if at least one firm is below α_t, it may experiment and choose the same strategy as its competitor (A3): if both experiments they may choose a payoff symmetric pair with a probability of at least γ^2. The upper bound on outflows $P_{t-1}(\mathrm{BSym}_{t-1})$ is based on the assumption that *all* industries in the subset BSym_{t-1} leave Sym. Hence:

$$1 \geq P_T(\mathrm{Sym}) = P_0(\mathrm{Sym}) + \sum_{t=1}^{T} [P_t(\mathrm{Sym}) - P_{t-1}(\mathrm{Sym})]$$

$$1 - P_0(\mathrm{Sym}) \geq \gamma^2 \sum_{t=0}^{\infty} P_t(\mathrm{BA}_t) - \sum_{t=0}^{\infty} P_t(\mathrm{BSym}_t)$$

Since $\sum P_t(\mathrm{BSym}_t)$ is bounded and $P_t(\mathrm{BA}_t) \geq 0$, it follows that

$$\left[1 - P_0(S) + \sum_{t=0}^{\infty} P_t(\mathrm{BSym}_t)\right] \gamma^{-2} \geq \sum_{t=0}^{\infty} P_t(\mathrm{BA}_t) \geq 0$$

and hence $P_t(\mathrm{BA}_t) \to 0$. □

Lemma 1 states that the proportion of markets where one or both firms are below least aspiration tends to zero. The reasoning here is in two stages. First, consider payoff-symmetric markets with both firms below aspiration: some of these will become collusive, and since $P_t(S)$ is bounded it follows that the proportion of payoff-symmetric pairs must go to zero. The second step is to show that for the proportion of payoff-symmetric pairs to go to zero, so must $P_t(\mathrm{BA}_t)$.

Theorem 1 (based on A1–A3). *If $P_t(S) > 0$ for some t, then as $t \to \infty$:*
1. *$P_t(S)$ tends to 1,*
2. *$\bar{\Pi}_t$ tends to ΠS.*

Proof. [6] From the definition for average profits, for all t:

$$\bar{\Pi}_t = \bar{\Pi}_t(\mathcal{L}) = P_t(S)\bar{\Pi}_t(S) + P_t(\mathrm{AA}_t - S)\bar{\Pi}_t(\mathrm{AA}_t - S) + P_t(\mathrm{BA}_t)\bar{\Pi}_t(\mathrm{BA}_t) \qquad (1)$$

[6] Theorem 1 can also be proven using standard results in the Theory of Markov Processes (see, e.g. Futia, 1982, p. 385).

By definition, $\bar{\Pi}_t(\mathrm{AA}_t - S) \geq \alpha_t$, and from A2 $\alpha_t \geq \bar{\Pi}_t - \eta_t$, so that when $P_t(S) > 0$ (1) becomes

$$\bar{\Pi}_t \geq \frac{P_t(S)\Pi S + P_t(\mathrm{AA}_t - S)\eta_t + P_t(\mathrm{BA}_t)\bar{\Pi}_t(\mathrm{BA}_t)}{1 - P_t(\mathrm{AA}_t - S)} \tag{2}$$

Since $P_t(S) + P_t(\mathrm{AA}_t - S) + P_t(\mathrm{BA}_t) = 1$, and the limit of $P_t(S)$ is $P^* > 0$, and of $P_t(\mathrm{BA}_t)$ is 0 (Lemma 1), the limit of $P_t(\mathrm{AA}_t - S) = 1 - P^*$. Hence (2) implies

$$\lim_{t\to\infty}\inf \bar{\Pi}_t \geq \Pi S$$

Since $\lim\sup \bar{\Pi}_t \leq \Pi S$ it follows that $\lim \bar{\Pi}_t$ exists and equals ΠS, with $P^* = 1$. □

The intuition behind Theorem 1 is fairly clear. The pair(s) S is an *absorbing* state in the Markov process. From Lemma 1, the proportion of firms with one or more firms below aspiration will tend to zero, so that all firms will be at or above average profits. The only way that this is possible is to have all firms earning ΠS. We require $P_t(S) > 0$ for some t in order to avoid the process getting stuck at a position where all markets earn exactly the average at a level below ΠS. In Section 4, we examine the Prisoner's dilemma and Cournot duopoly to illustrate each of these points in a concrete way.

Whilst the intuition is fairly clear, the exact evolution of P_t and $\bar{\Pi}_t$ is open to a wide variety of possibilities under A1–A3. In particular, the path of both can be highly non-monotonic, and Theorem 1 does little to tie down the nature of the path towards the long-run stationary state. This would require the examination of specific models for the evolution of aspirations and the switching probabilities.[7] However, Theorem 1 does establish the long-run properties of a very wide class of learning processes.

One interpretation of the result is a model of *group selection*. However, it should be noted that individual firms cannot choose whom they play against: they can only choose their own behaviour. Groups are selected, but only indirectly by the market mechanism: in duopolies that are too competitive, profits of one or both firms are eventually below aspiration, becoming unsustainable. Thus the process outlined in this paper can be interpreted as one where nature (the economy) selects the optimum degree of competitiveness (the cooperative solution). Note that Alchian's original argument (1950) (quoted at the opening of this paper) was conducted at the level of the *individual* firm: either the atomistic competitor or a monopoly. However, in duopoly the individual firm's profits depends upon the *joint-strategy* of the firms: hence it is the joint-strategy that is chosen. Whilst the motivation of our arguments is similar in spirit, the conclusions reached in a strategic environment are very different.

4. Examples: the PD and Cournot duopoly

In this section we will illustrate how the process described in this model operates in two concrete examples. First, the abstract but popular PD model. This is used in particular to

[7] In Dixon and Lupi (1996), we simulate in detail several different switching rules (imitation, best response, random switching) in the context of a homogeneous Cournot model.

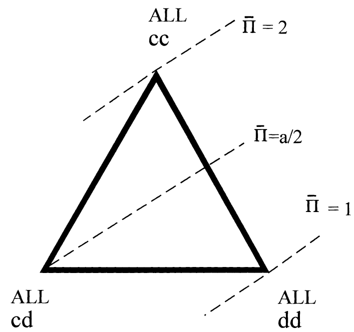

Fig. 1. The PD with $\Pi_{\rm PD}$ and $2 < a < 4$.

show how the relaxation of any of A1–A3 means that the convergence to collusion might be overturned. Second, we use the more concrete example of Cournot duopoly to illustrate how the model works when there is random switching behaviour.

4.1. Prisoner's dilemma

We consider two cases of the PD:

$$\Pi_{\rm PD} = \begin{bmatrix} 2 & 0 \\ a & 1 \end{bmatrix}, \qquad \Pi^*_{\rm PD} = \begin{bmatrix} 2 & -a \\ a & 1 \end{bmatrix}$$

where $a > 2$, $k = \{c, d\}$, $\mathcal{L} = \{cc, cd, dd\}$, $S = \{cc\}$, $\Pi S = 2$, $K = 2$, $L = 3$. With $\Pi_{\rm PD}$, A1 is satisfied if $a \leq 4$: with $\Pi^*_{\rm PD}$ for any $a > 2$.

The evolution of the population in $\Pi_{\rm PD}$ can be represented on the unit simplex in Fig. 1, where each point represents a three-vector of proportions of markets playing each strategy pair. We represent the iso-average payoff loci on the simplex: these are linear and parallel since the average payoff is a linear combination of the payoffs for each strategy pair, with slope $\frac{1}{2}(a-2)$. $\bar{\Pi} = \frac{1}{2}a$ is the dotted straight line passing through corner all-*cd*; $\bar{\Pi} = 2$ passes through all-*cc*, $\bar{\Pi} = 1$ passes through all-*dd*.

The dynamics of this system are straightforward. Average profits must lie in the interval $[1, 2]$. Except in the case where $P(dd) = 1$ and $\bar{\Pi}_t = 1$, we thus have: $\mathrm{AA}_t = \{cc\}$; $\mathrm{BSym}_t = \{dd\}$; $\mathrm{BA}_t - \mathrm{BSym}_t = \{cd\}$. From any point except where $P(dd) = 1$, all trajectories will lead to the apex where $P(cc) = 1$.

With Π^*_{PD} the sucker-payoff is $-a$, so that $\pi(cd) = 0$. This extends the possible range of average profits to $[0, 2]$. The iso-profit loci are downward sloping as in Fig. 2: the minimum is represented by the dotted line through the cd vertex, and the maximum by the dotted line through *cc*. The line passing through *dd* is the $\bar{\Pi} = 1$ line.

The dynamics here are different depending on whether the economy is in region A or B. In region A, $\bar{\Pi} > 1$, so that $\mathrm{AA}_t = \{cc\}$, $\mathrm{BSym}_t = \{dd\}$, $\mathrm{BA}_t - \mathrm{BSym}_t = \{cd\}$. In region B, $\bar{\Pi} \leq 1$, so that $\mathrm{AA}_t = \{cc, dd\}$, $\mathrm{BA}_t = \{cd\}$. With the simple learning model, only firms playing *c* at *cd* markets will experiment, so that the trajectories must be horizontal lines in B. Hence we can see why we need to assume that $P_t(cc) > 0$ for some t in Theorem 1: all-*dd* has a basin of attraction along the southern edge of the simplex.

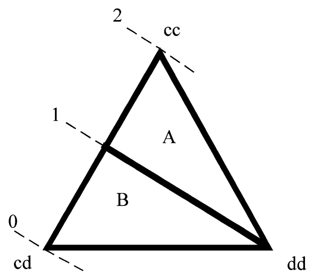

Fig. 2. The PD with Π^*_{PD}.

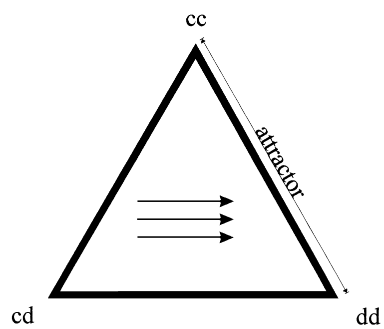

Fig. 3. Best response experimentation with no noise.

Theorem 1 gives *sufficient* conditions for all-*cc* to be the attractor. We will now illustrate how dropping some of the key assumptions will open the possibility that all-*cc* ceases to be the attractor. We have already seen why $P(cc) > 0$ is necessary to rule out all-*dd* in Π^*_{PD}. Next consider a violation of A3: noiseless best-response experimentation, where firms below aspiration switch to the best-response to their competitor. In both Π^*_{PD} and Π_{PD} , the best response is *d* (the strictly dominant strategy). In this case the pairs divide into two disconnected sets: there can be no flow into or out of {*cc*}, and {*dd*} is an absorbing state. The only flows in this system are from {*cd*} to {*dd*}. The resultant dynamics are represented in Fig. 3, where the attractor is the northeastern edge of the simplex, where there are no {*cd*} markets. The paths to this are simply the horizontal lines: the economy starts off with an initial proportion of {*cc*} markets, and eventually all the rest will become {*dd*}. However, whilst the example of noiseless best-response experimentation is an interesting illustration of what can happen when A3 is violated, it is not at all robust. Any level of switching noise[8] $\gamma > 0$, no matter how small, will lead to Theorem 1 becoming valid again, and {*cc*} absorbing all markets, so long as $P_t(S) > 0$ for some t.

Lastly, what happens when A1 is violated? For simplicity, let us consider the Π_{PD} payoff matrix, with $a = 6$ so that Maxav $= 3 > \Pi S = 2$. The iso-payoff loci in Fig. 4 are vertical, passing through all-*cd* ($\bar{\Pi} = 3$), all-*cc* ($\bar{\Pi} = 2$) and all-*dd* ($\bar{\Pi} = 1$). There are two different

[8] By this I mean that there is a probablity γ that the non-best response c is chosen (possibly by mistake), so that A3 is now satisfied.

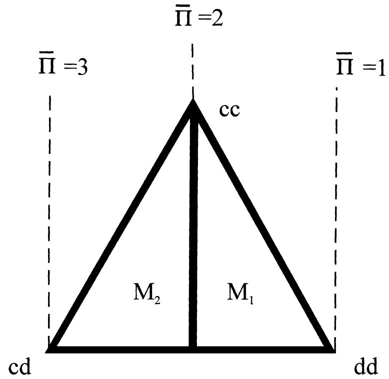

Fig. 4. Violation of Al PD with Π_{PD} and $a = 6$.

regions: in M_1, $\bar{\Pi} < 2$, so that $AA_t = \{cc\}$, $BA = \{dd, cd\}$; whilst in M_2, $AA_t = \emptyset$, whilst $BA_t = \{cc, dd, cd\}$. What exactly happens depends upon the exact switching technology. With random switching $s_t(c, d) = s_t(d, c) = \frac{1}{2}$, the equilibrium is a point in M_2 where $P^*(cc) = 0.125$, $P^*(dd) = 0.5$, $P^*(cd) = 0.375$ with $\bar{\Pi} = 2.125$. There is a perpetual flow of markets between the three pairs of strategies.

4.2. Cournot duopoly

Perhaps the simplest economic application of our model is to Cournot duopoly without costs, so that the two firms in any duopoly produce output x and y, and the price is $P = \max[0, 1-x-y]$, and the profits of the firms are xP and yP, respectively. In this case we have the set S with a unique element: it is the joint profit maximizing (JPM) pair where each firm produces 0.25 (half of the monopoly output 0.5). Furthermore, $\Pi S = \text{Maxav} = 0.125$, so that A1 is satisfied. With the random switching rule $s_t(i, j) = 1/K$ for all i, j, which satisfies A3.

We[9] allowed for 21 firm types. Choosing a grid of granularity 0.025 over the range[10] 0.1–0.6, perturbing it slightly by moving 0.325 to 0.333 ($\frac{1}{3}$), so that the Cournot–Nash output was included. Hence $K = 21$ and $L = 231$. The simulations were initiated from the initial position with a uniform distribution over all pairs. The results of the simulation are depicted in Figs. 5a and b. In Fig. 5, we see the path of average profits over time: in Fig. 5a the evolution of population proportions of the JPM market (0.125, 0.125) and the symmetric Cournot market are depicted (note that the proportions are measured on a logarithmic scale).

From Fig. 5b, we see that the average profits converge to the symmetric joint profit maximum of 0.125. However, the time path of profits is non-monotonic: at particular times there appear large drops in profit. The reason for this is quite intuitive. As the average profit level increases, it surpasses that of one or both firms, which start to experiment. The profits of firms at those markets will then on average fall below the population average as

[9] I would like to thank Paolo Lupi for implementing these simulations for me in Gauss.
[10] We did not allow for a wider grid range (e.g. [0,1]), because the additional strategies are often ones with very low or zero profits: they slow down the simulation without adding any extra insight.

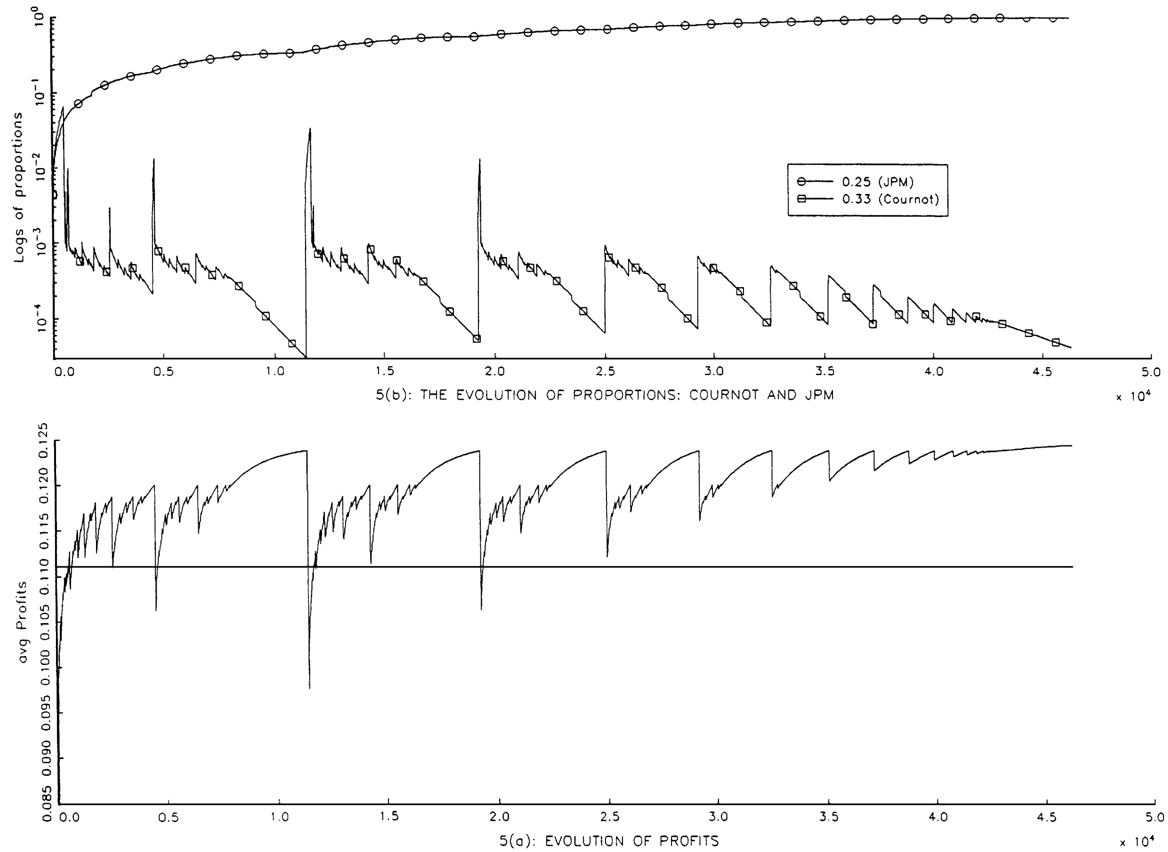

Fig. 5. The evolution of competition.

the firms disperse over some or all output pairs. The effect of this can be quite dramatic: the discontinuity is particularly large when a symmetric market goes critical, since both firms at each such market begin to experiment and spread across all possible output pairs. However, whilst the time-series of profits is non-monotonic and "discontinuous", there is a clear upward trend and convergence to 0.125.

From Fig. 5a, the proportion $P(S)$ is monotonic, but not smooth. Corresponding to the discontinuous falls in population average profit, there are jumps in the proportion of firms at the JPM market, corresponding to the jumps in average profit. The proportion of firms at the Cournot pair $(\frac{1}{3}, \frac{1}{2})$ is a highly non-monotonic time series. The first thing to note is that in the initial stages of the simulation, the proportion of Cournot markets exceeds the proportion of JPM markets. This can occur because during this period the Cournot pair is also in the set AA_t: until average profits reach $\frac{1}{9}$, the Cournot pair will "absorb" markets from BA_t. The fact that the Cournot pair attracts more than JPM is due to the fact that early on more markets in BA_t can reach the Cournot pair than JPM. However, after 50 iterations, the Cournot pair has a smaller proportion than the JPM pair, and is in BA most of the time. The time-series of the Cournot market type is not atypical: most pairs except JPM have a similar time-series profile. The convergence of the proportion of markets towards type JPM is steady but slow: this is because the probability of hitting JPM from other locations is small throughout the simulation: from each market in which both firms experiment there is a probability of $\frac{1}{442}$ of moving to JPM. Convergence is in general quicker with fewer strategies and non-random switching rules. We explore more specific rules in the Cournot model using simulations in Dixon and Lupi (1996).

5. Asymmetric payoffs: an example

Whilst Theorem 1 holds for models where there are symmetric payoffs, in the sense that the payoff for an individual depends only on the strategies played and not the identity of a player. What if the payoffs are not symmetric? This has been analyzed for the case of an economy consisting of asymmetric Cournot duopolies in Cabelka (1999). In this paper, I will consider a simple example to illustrate some of the general issues raised. Consider a perturbed PD in which the payoff of player B is ε less than the other player A for any given strategy pairing. For example, B might an inefficient firm. Denote the game as PD(ε)

$$\begin{bmatrix} 2, 2-\varepsilon & 0, 3-\varepsilon \\ 3, -\varepsilon & 1, 1-\varepsilon \end{bmatrix}$$

In this case, $\{cc\}$ will not be an attractor under the dynamics of A1–A3. To see why, note that if all pairs were playing $\{cc\}$, then the average payoff would be $\bar{\Pi} = 2 - \frac{1}{2}\varepsilon$. The inefficient firm would be earning below aspiration, hence under A2 it would experiment. One way to restore the result is to assume that the inefficient firm has lower aspirations: this is not at all unreasonable if one thinks of aspirations coming from within the firm.[11]

[11] In fact, if we adopt the view the aspiration level reflects the external pressure from the capital market then ν could reflect some kind of transaction cost or other imperfection. Whilst the example assumes that this applies only to the inefficient firms, the model could be analyzed with common aspirations $\bar{\Pi} - \nu$.

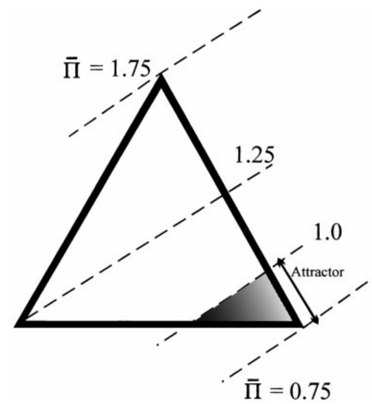

Fig. 6. The PD with an inefficient firm: $\varepsilon = \nu = 0.5$.

Let us amend A2 to allow for inefficient firms to have aspirations bounded away from the average by some $\nu > 0$.

Assumption (A2′). Let $\nu \geq 0$. For inefficient firms the aspiration level is $\hat{\alpha}_t = \bar{\Pi}_t - \nu$; for efficient firms $\alpha_t = \bar{\Pi}_t$.

This assumption reduces to A2 when $\nu = 0$. An exhaustive formal analysis of this model lies beyond the scope of this paper. However, I will consider a special case where $\upsilon = \varepsilon = 0.5$ with random switching. The unit simplex representing the population distribution is shown in Fig. 6.

The iso-average profit lines are the same as in Fig. 1 with $a = 3$, except shifted down by 0.25 ($\frac{1}{2}\varepsilon$) to reflect the lower payoffs of inefficient firms. Let us consider each type of industry pair:

- $\{c, c\}$ This will be an absorbing state. Average profits $\bar{\Pi} \leq 1.75$, so that $\hat{\alpha} \leq 1.25$. Inefficient firms will earn 1.5, which exceeds aspiration (efficient firms also earn above aspiration of course).
- $\{c, d\}$. The cooperator will always be below aspiration and the defector always above aspiration irrespective of whether they are inefficient or not. Hence, with random switching, 50 percent of $\{c, d\}$ industries will become $\{d, d\}$.
- $\{d, d\}$. Both efficient and inefficient firms will be meeting their aspiration if $\bar{\Pi} \leq 1$: neither if $\bar{\Pi} > 1$. The inefficient firms earn 0.5, and $\hat{\alpha} \leq 0.5$ iff $\bar{\Pi} \leq 1$; efficient firms earn 1 and $\alpha = \bar{\Pi}$.

Hence for the shaded region in the bottom right corner of the simplex, where $\bar{\Pi} \leq 1$ there is no flow out of $\{d, d\}$ and only from $\{c, d\}$ to $\{d, d\}$. Thus the northeast border of the shaded area is an attractor for the shaded region, with the dynamic being horizontal, since the limiting distribution is $P^*(cc) = P_0(cc); P^*(dd) = 1 - P_0(cc)$. For $\bar{\Pi} > 1$ (the unshaded region), both types of firm wish to experiment in $\{d, d\}$ industries: hence there is a flow from $\{d, d\}$ of 25 percent to $\{c, c\}$ and 50 percent to $\{c, d\}$, and a flow of 50 percent from $\{c, d\}$ to $\{d, d\}$. Hence all points in the unshaded area lead to $P^*(cc) = 1$.

The size of the shaded area is determined by the size of v and ε: in the case where $v = \varepsilon$, as $\varepsilon \to 0$ the attractor and its basin of attraction shrink to zero (as the iso-profit lines $\bar{\Pi} = 1$ and $\bar{\Pi} = 1 - \frac{1}{2}\varepsilon$ will converge). Hence if there is only a little asymmetry in payoffs, most of the simplex will converge to collusion $\{c, c\}$.

This example illustrates the general point that the model can be adapted to allow for asymmetric payoffs and maintain the result that collusion has a large basin of attraction. The key point is that the assumption about aspirations needs to be altered to allow inefficient firms to earn below average, but not too much. Allowing aspirations to remain too far below average may well mean that a range of other attractors is introduced into the model.

6. Related literature

There are several recent papers related to ours.[12] The closest is Palomino and Vega-Redondo (1999). This paper considers a population of players who a randomly matched, and play the Prisoner's dilemma. The mean payoff is known in each period, and this determines the aspiration level.[13] If a player is earning below aspiration with its current strategy, then it switches with a positive probability to the other strategy. They find that for certain parameters, all paths converge to a situation with a strictly positive proportion of cooperators. Our paper differs in that we do not have random matching, and that we consider a very general class of games under A1 (of which the PD is one example).

Bendor et al. (1994) and Karandikar et al. (1998) both consider a two player game, where individual behaviour is driven by a similar aspiration based model. In Bendor et al. (1994) aspiration levels are constant over time, and they impose the condition that the individual aspiration levels are equal to the long-run individual average payoff[14] (*consistent aspirations*); in Karandikar et al. (1998) aspirations can evolve, but are determined by individual payoff histories. In both papers, there are multiple long-run equilibria, which in general include cooperative outcomes. The key difference between our own paper and these papers is the social dimension: here it is the *population* average which ultimately determines the aspiration level.[15]

The local interaction literature ((Ellison, 1993; Oliphant, 1994; Ellison and Fudenburg, 1995) is similar in that here firms only interact with their market competitors. However, the key interactions in our paper are not only local, but also social via the population average payoff. Our results hold even if the individual firms ignore the existence of their competitors, and consider themselves to be solving a non-strategic problem. This feature also differentiates our paper from other learning models (e.g. Blume and Easley, 1992; Marimom and Grattan, 1995). More similar to our approach are papers where there is global interaction through the population average action ((Banerjee, 1992; Canning, 1992).

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[12] Chiappori (1984) also considers the issue of natural selection and optimisation. However, for him survival is based on non-negative profits, not profits at or above average.

[13] In fact, they assume a partial adjustment model, so that the aspiration level changes in accordance with the difference between the current level and average profits.

[14] "A minimal requirement for a model of endogenous aspirations is that in the long-run, aspirations should not be out of line with the average payoffs accumulated from experience" ((Bendor et al., 1994, p. 9).

[15] This also differentiates our paper from Borgers and Sarin (1997).

Lastly, there is a theoretical and experimental literature on learning in oligopoly settings (Kirman, 1995), which again focuses on isolated markets and does not have the social dimension.

One paper which derives a very different result to ours is Vega-Redondo (1997), which considers a single market and obtains the result that imitation of Cournotian competitors leads to the Walrasian outcome. The reason for this result is that it considers only a single market, so that the performance of a single firm is compared only to the performance of its competitors. Since more aggressive behaviour (i.e. producing a lager output) earns higher profits, firms will tend to increase output until the Walrasian equilibrium is achieved. This does not happen here because of the social dimension: the move towards more competition in one market would be prevented when profits fall below the average in the economy.

7. Conclusion

In this paper we have formulated a simple model of social learning which is based on an information structure and matching technology suggested by the economic application of oligopoly, and with a learning model in which aspirations are linked in the long-run to the population average payoff. The results of the paper are very simple and very powerful: the model predicts perfect collusion (cooperation), even in the case where collusion implies the use of a dominated strategy (as in the Prisoner's dilemma). The model does not require strong assumptions on the learning process or payoff matrix, and Theorem 1 will certainly hold in symmetric versions of most economic models.

The results as derived depend on certain assumptions as sufficient conditions. Some of these are more crucial than others. The assumption that there is a continuum of markets and agents is not crucial: analogous result would hold if there were a finite number (see Cabelka, 1999). The assumption that the economy consists of many identical duopolies is more crucial. In Section 5, I considered an example with asymmetric payoffs: the results of Theorem 1 held in a modified form only after changing the assumption about aspirations. Whilst I would conjecture that the result is robust in a similar way to other generalizations, it remains to carry out the formal analysis of the model under the assumption of n-firm markets, non-identical markets, and equilibrium under entry. Lastly, it would be very interesting to model the process of the capital market itself more explicitly.

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