STRATEGIC INVESTMENT WITH CONSISTENT CONJECTURES

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It is generally recognised that the flexibility of firms' production will influence the nature of competition in an industry. For example, where production is very inflexible the Cournot outcome seems most appropriate: the Bertrand outcome, however, depends on production being perfectly flexible. The model presented makes both the firms' cost structure (flexibility of production) and the nature of competition endogenous, and thus provides a framework in which both elements of industrial structure and the conduct of firms are explained. We combine two ideas in the recent literature on oligopoly theory: models of strategic investment (Brander and Spencer (1983), Dixon (1985), Eaton and Grossman (1984), Yarrow (1985) inter alia) and notions of the consistency of conjectures (Bresnehan (1981) in particular). The fundamental idea underlying this synthesis is very simple. As we discuss below, there is a precise sense in which Bresnehan's consistency condition relates firm's conjectures about each other's responses—and hence the degree of competition in the product market—to the firms cost functions. Strategic investment models, on the other hand, provide a framework for making the firm's cost functions endogenous: by choosing a level of investment, the firm decides which short-run cost function it will have. By combining the strategic investment framework with Bresnehan's consistency condition, we have a model in which both firm's cost structures and the degree of competition in the product market are endogenously determined. Thus we have a framework in which both a structural characteristic of the market (firms costs), and the conduct of firms are endogenously determined.

For a wide range of industrial processes economists since Marshall have taken the view that it is appropriate to treat the capital stock decision of firms as being taken on a different time scale to output decisions. Insofar as it is appropriate to treat capital as a fixed factor and labour as a variable factor,[1] it follows that when firms compete in the product market they treat their current capital stock as given (output decisions are taken in the long run, output and employment in the short run). The fact that the capital stock is thus committed "before" the firm makes it output decisions implies that the firm can use its investment decision *strategically*: the firm can influence the market outcome through its choice of capital stock.

This paper was awarded the P. W. S. Andrews Memorial Essay Prize for 1985. I would like to thank the Ulph Brothers for useful discussions on this subject, and the referees for their comments. Errors remain inescapably mine.

[1] This can be made a tautology if we treat the capital variable as representing fixed factors in general—including managerial services and skilled labour for example. Alternatively, investment can be interpreted as R&D expenditure as in Brander and Spencer (1983).

Several recent papers have explored the implications of strategic investment. The structure of these models is relatively simple. There are two stages, which capture the distinction between the long[2] and the short-run. In the first "strategic" stage firms choose their capital stock. In the second "market" stage a product market equilibrium occurs given the capital stocks chosen in the strategic stage. The capital stock decisions in the first stage are taken strategically in the sense that firms take into account the effect that investment will have on the outcome in the market stage. In essence, the choice of capital stock determines which short-run cost function the firm will operate on in the market stage. This can also be interpreted as the firm determining the flexibility of production which it will have in the market stage. Following Marshak and Nelson (1972), Production can be characterised as being more "flexible" the less steep the slope of the marginal cost function. For a wide class of cost functions more investment will lead to a decline in the slope of the short run marginal cost function at any given output. An overall equilibrium in the two-stage model is a Nash-equilibrium in capital stocks, since each firm's profits can be given as a function of the capital stocks chosen in the first stage.

What differentiates the models is the assumption made about the market stage, the nature of competition in the product market. Brander and Spencer (1983) explore the model with a Cournot–Nash market stage, Dixon (1985) explores the case with a competitive market stage, whilst Eaton and Grossman (1984) and Yarrow (1985) consider a general conjectural variations model (see also Bulow *et al.* (1985), and Fudenburg and Tirole (1984)). In the case of Cournot–Nash or conjectural variations in the market stage, the firm's investment decision determines the (short run) cost function that the firm will have in the market stage, and hence its reaction function in output space. Thus in essence the firms's choice of capital is ipso facto a choice of reaction function. An equilibrium in this type of strategic investment model is a Nash-equilibrium in a game where firm's strategies are reaction functions. Similarly, in the case of a competitive market stage the firm's investment decision determines its supply function in the market stage and hence we have a Nash-equilibrium in supply functions.[3]

A general (though not universal) property of these models is the phenomena of factor-bias. The strategic use of capital in the first stage means that there is an asymmetry between capital and labour. This asymmetry generally leads to a non-cost minimising capital-labour ratio. In essence, production is inefficient in the sense that firms, whilst on their short-run cost functions, are not on their long-run cost functions. This strategic inefficiency in production gives rise to a welfare loss.

The existing models of strategic investment discussed above have assumed

[2] This is not the Marshallian long run, since there is no entry.

[3] This contrasts with other models with Nash equilibria in supply functions—see Grossman (1981) and Hart (1982). These models allow for a much wider set of supply functions. The main point is that the set of admissable supply functions in Dixon (1985) is defined technologically.

a given degree of competition in the market stage: it is assumed that whilst the investment decision of firms will alter the degree of flexibility of production, this will have no effect on the nature of competition in the product market. It has been argued, however, that the nature of competition will be influenced by the degree of flexibility of production. One particularly interesting way of capturing this is to impose a consisteny condition (as in Bresnehan (1981)) in the market stage with a conjectural variation model. Loosely speaking, a consistency condition can be interpreted as requiring that firms' conjectures about each others responses equal the actual responses firms would make (given their conjectures). In the context or our model, one implication of this is that the more flexible is production, the more competitive will consistent conjectures be (see Section 1). If firms have totally inflexible production ("vertical" marginal cost) then the Cournot conjecture is consistent; if firms have perfectly flexible production ("horizontal" marginal cost) then the Bertrand conjecture is consistent; for intermediate degrees of flexibility, the conjecture will be between the Cournot and Bertrand values (see, Propositions 1–2, and also Bresnehan (1981 pp. 36–7).

In this paper, we combine the strategic investment framework with consistency of conjectures in the market stage. The firm's investment decision will determine its short run cost function, and hence its flexibility of production in the market stage. The flexibility of production then determines the nature of competition in the market stage. This enables us to capture the idea that the firm's investment decision will alter the nature of competition in the market stage, and that the degree of competition in the product market will hance become endogenous in a strategic investment model.

There are three main results in this paper. First, in equilibrium the degree of competition will lie between the Bertrand and Cournot values (Proposition 4(b)). Firms will choose to have an intermediate degree of flexibility of production. This contrasts with Dixon (1986), where with a given degree of competition firms will prefer to have totally inflexible production. Secondly, the wage-rental ratio will influence the degree of competition: a very small wage-rental ratio will lead to Cournot conjectures (see Proposition 4). Thirdly, we are able to evaluate the result that with consistent conjectures there will be no inefficiency in production (Eaton and Grossman (1984 p. 6), Yarrow (1985, Section 6)). This result is shown to hold only when conjectures are exogenous and happen to be consistent in equilibrium. When conjectures are made endogenously consistent, there will generally be factor-bias. In any symmetric equilibrium this factor-bias will lead to undercapitalisation, a capital-labour ratio below the cost minimising level (Proposition 3).

1. Investment, conjectures, and consistency

In this paper we examine a two stage model of strategic investment, where the second market stage is a conjectural variations equilibrium, as in

Eaton and Grossman (1984) and Yarrow (1985). However, unlike these papers, we make the two firms conjecture about each other's output responses endogenous, by imposing a "consistency" condition. There are several related notions of "rationality" or "consistency", which originated out of Hahn's work on conjectural equilibria (1977, 1978)—for example Bresnehan (1981), Hart (1982), Laitner (1980), Ulph (1983). In the framework adopted in this paper these differences are unimportant, although our results draw most on Bresnehan's (1981) work. We restrict our attention to a simple conjectural variations model where reaction functions are linear: consistency here means that firms know the slope of each other's reaction functions. Thus actual and conjectural responses are consistent.

In this section we explore the way in which investment influences the flexibility of production, and how this influences the nature of product market competition through consistency. Firms choose outputs x_i, and have linear conjectures about how the other firm will respond to changes in their own output. Furthermore, this conjecture does not depend on where firms are in the strategy space: that is, from whatever output firms start at, their conjectures about each other's responses are unaffected. Because of these two features—linearity and independence of initial position—firms' conjectures can be expressed as a scalar ϕ_i, which equals firm i's conjecture about j's proportional response to change in x_i, i's belief about $\mathrm{d}x_j/\mathrm{d}x_i$. If $\phi_i = 0$, $i = 1,2$, we have the Cournot model. If $\phi_i > 0$ we obtain a "collusive" model where firms tend to follow each other. If $\phi_i = -1$ and there is a homogeneous product, then we have the competitive or Bertrand model, since firm i believes any change in its own output will be exactly offset by a change in the other firm's output, so that the price is unaffected. The more negative firm i's conjecture, the more accommodating it believes the other firm to be, in the sense that firm j will reduce its output in response to an increase i's output. Given firms' conjectures about each other, we can derive their reaction functions in output space, which give their actual responses. The consistency condition requires that at the equilibrium vector of outputs, the conjectured response equals the slope of the reaction function.

There are several conceptual and technical problems with the concept of consistency. On the technical level, there are no general results about either existence or uniqueness of consistent conjectures. Furthermore, the equilibrium may not be easy to characterise (see Ulph (1983)). On the conceptual level, the conjectural variations model only allows firms to have a very specific type of conjecture: surely we would like to allow for firms to have much more general conjecture (non-linear conjectures, conjectures which allow for initial position). However, if we allow for a more general class of conjectures, the power of the consistency condition is greatly weakened: as Laitner (1980) shows, a continuum of consistent conjectural equilibria will exist (see also Boyer and Moreaux (1983)). Furthermore, it can be very reasonably argued that the conjectural variations model tries to capture in a static model what is really a dynamic problem of firms responses to each other over time. The best way to concieve of a consitent conjectural equilibrium is within a framework of instantaneous responses, where firms can respond immediately to changes in each other's output. This paper does not aim to answer or solve these issues. Rather, we make assumptions about industry demand and firms cost that overcome the technical problems, and partly alleviate the conceptual issues.

There are two firms (the results obviously generalise) $i = 1,2$, which choose outputs $x_i > 0$. They produce a homogeneous product, and there is linear demand.

A1: Industry Demand:

$$p = p^0 - \sum_{i=1}^{2} x_i$$

There are two factors of production, capital and labour. Capital is treated as fixed when output is chosen, but labour is freely varied.

A2: Technology

$$x_i = k_i^{0.5} \cdot L_i^{0.5}$$

Letting wages be the numeraire, r the rental-wage ratio, the firm's short run cost function under A1 is given by:

$$c(x_i, k_i) = r \cdot k_i + x_i^2/k_i \tag{1.1}$$

$$\frac{\partial c}{\partial x_i} = \frac{2}{k_i} \cdot x_i \tag{1.1a}$$

Thus firms have quadratic costs functions, and linear marginal cost, the slope of which is inversely related to investment. In this way the flexibility of the firms production is determined by the level of its investment. We use the term "flexibility" in the technical sense employed in the literature on competitive markets under uncertainty (Stigler (1939), Marshak and Nelson (1972), Mills (1984)). With quadratic costs, the output response of a firm to a change in price will be greater the less steep its marginal cost function is. This sensitivity of output to price is interpreted as flexibility, and defined as $\gamma = (\partial^2 c/\partial x_i^2)^{-1}$. In terms of the Marchak–Nelson definition from (1.1) more capital leads to greater flexibility of production ($\gamma = k_i/2$), since the marginal costs function becomes flatter. In essence, higher investment leads to a shift from variable to fixed costs in the market stage, and low variable costs lead to flexible production.

Given firms investment $\underline{k}$ and conjectures about each other's responses $\underline{\phi} = (\phi_1, \phi_2)$ we can derive the firm's reaction function in output space. Under A1–2 the firm's profits are:

$$\Pi_i = x_i \cdot (p^0 - x_i - x_j) - rk_i - x_i^2/k_i$$

To derive the firm's reaction function in output space, treating capital as fixed, we set $\partial \Pi_i/\partial x_i = 0$, yielding:

$$x_i = r_i(x_j, k_i, \phi_i) = \frac{p^0 - x_j}{2 + \phi_i + 2/k_i} \tag{1.2}$$

where $i, j = 1,2$, $i \neq j$. The reaction function is linear in x_j, with slope:

$$\frac{\mathrm{d}r_i}{\mathrm{d}x_j} = \frac{-1}{2 + \phi_i + 2/k_i} \tag{1.3}$$

The consistency condition requires that the actual responses equal the conjectural responses, $\phi_i = \mathrm{d}r_j/\mathrm{d}x_i$, which under A1–2 yields the two equations:

$$\phi_i(2 + \phi_j + 2/k_j) + 1 = 0 \qquad i,j = 1,2,\ i \neq j \tag{1.4}$$

Under A1–2, firms have linear reaction functions, and the consistency condition seems particularly appropriate. Although consistency only requires the conjecture to equal the slope of the reaction function at a point, with linear reaction functions this is equivalent to each firm knowing the whole of the other firm's reaction function. Also, since the reaction functions are linear, we don't need to solve for the equilibrium outputs to solve for consistent conjectures, as is clear from (1.4). By convention, we let $\phi_i = 0$ whenever $k_j = 0$.

What is the relationship between consistent conjectures and the capital stocks $\underline{k}$? From (1.4) it is clear that consistency implies that $\phi_i < 0$ whenever $\underline{k} \gg 0$. In addition to (1.4) we have the second order conditions for the reaction function:

$$\phi_i > -(1 + 1/k_i) \qquad i = 1, 2 \tag{1.5}$$

Equations (1.4) and (1.5) define uniquely the consistent conjectures:

Proposition 1: Let $\underline{k} \gg 0$. There exist unique consistent conjectures

$$\phi_i \in (-1, 0) \qquad i = 1, 2$$

(all proofs are in the appendix.)

Thus, for $\underline{k} \gg 0$, the consistent conjectures are between the Bertrand value -1 and the Cournot value 0.[4] Let the implicit function defined by (1.4) be $\phi : [0, \infty)^2 \to (-1, 0]^2$ where:

$$\phi_i = \phi_i(\underline{k}) \tag{1.6}$$

Total differentiation of (1.4) yields the response of conjectures to changes in

[4] Consistency implies non-positive conjectures within the context of perfect information. Hviid and Ireland (1986) show that allowing for imperfect information may yield positive consistent conjectures.

$\underline{k}$. Differentiating with respect to k_i, using (1.3):

$$\begin{bmatrix} \phi_2 & \dfrac{-1}{\phi_2} \\ \dfrac{-1}{\phi_1} & \phi_1 \end{bmatrix} \begin{bmatrix} \mathrm{d}\phi_1/\mathrm{d}k_1 \\ \mathrm{d}\phi_2/\mathrm{d}k_2 \end{bmatrix} = \begin{bmatrix} \dfrac{2}{k_1^2} & \phi_2 \\ 0 & \end{bmatrix} \tag{1.7}$$

The determinant is $\Delta_\phi = \phi_1\phi_2 - (\phi_1\phi_2)^{-1} < 0$, since $\phi_i \in (-1, 0)$. Hence:

$$\frac{\mathrm{d}\phi_1}{\mathrm{d}k_1} = \frac{1}{\Delta_\phi} \cdot \phi_1\phi_2 \frac{2}{k_1^2} < 0 \tag{1.8a}$$

$$\frac{\mathrm{d}\phi_2}{\mathrm{d}k_1} = \frac{1}{\Delta_\phi} \cdot \frac{\phi_2}{\phi_1} \frac{2}{k_1^2} < 0 \tag{1.8b}$$

and similarly for $\mathrm{d}\phi_i/\mathrm{d}k_2$.

Hence, an increase in k_1 leads to both firms' consistent conjectures becoming more negative, more accommodating. Under consistency $\phi_1 = \mathrm{d}r_2/\mathrm{d}x_1$: so that (1.8a) implies that the increase in k_1 leads to the slope of firm 2's reaction function becoming more negative, and (1.8b) implies that its *own* reaction function becomes more negatively sloped. This will subsequently prove very important: with consistent conjectures, *each* firm's reaction function is determined by *both* firms capital stocks. As either firm invests more, the market will become more competitive since firms will be encouraged to expand their own output as the other firm's reaction function become more accommodating. Consider sequences $\{\underline{k}_n\}$ where $\underline{k}_n \gg 0$ and the corresponding sequences $\{\underline{\phi}_n\}$ where $\underline{\phi}_n =_{\mathrm{def}} \phi(\underline{k}_n)$:

Proposition 2: (a) If $\underline{k}_n \to \underline{\infty}$, then $\phi_{in} \to -1$

(b) If $\underline{k}_n \to \underline{0}$, then $\phi_{in} \to 0 \qquad i = 1, 2.$

Proposition 2 tells us that as *both* firms' production become perfectly flexible ($\underline{k}_n \to \underline{\infty}$), then consistent conjectures tend to the Bertrand value: as both firms' production become perfectly inflexible, the consistent conjectures become Cournot.

In this section we have examined the relationship between investment and the flexibility of production, and the relationship between the flexibility of production and consistent conjectures. The more each firm invests in capital, the greater is its flexibility of production, and the more competitive the consistent conjecture of *both* firms become. This relationship implies that firms can manipulate the degree of competition in the product market through their investment decisions.

2. Strategic investment with consistent conjectures

In this section we explore the full two-stage strategic investment model when firms take into account the effect of their investment decisions on the degree of competition in the market stage. As a first step, we will consider the firm's output decision in the market stage *given* conjectures $\underline{\phi} \in (-1, 0]^2$ and $\underline{k} \geqslant 0$, *without* imposing consistency $\underline{\phi} = \underline{\phi}(\underline{k})$. Firms choose outputs to maximise profits given $\underline{\phi}$ and $\underline{k}$. This yields the firms' reaction functions in output space, 1.2). Given the two firms' reaction functions, we can solve for the equilibrium outputs. Whilst 1.2) can be solved explicitly, we shall write the solution outputs as general functions of $\underline{\phi}$ and $\underline{k}$:

$$x_i = R_i(\underline{\phi}, \underline{k}) \qquad i = 1, 2 \tag{2.1}$$

We can totally differentiate 2.1) to discover the response of the equilibrium outputs in the market stage to changes in $\underline{k}$ and $\underline{\phi}$. Defining the determinant $\Delta = (2 + \phi_1 + 2/k_1)(2 + \phi_2 + 2/k_2) - 1 > 0$, this yields:

$$\frac{\partial R_1}{\partial k_1} = \frac{1}{\Delta}\frac{2x_1}{k_1^2}\left(2 + \phi_2 + \frac{2}{k_2}\right) > 0 \tag{2.2}$$

$$\frac{\partial R_2}{\partial k_1} = \frac{-1}{\Delta}\frac{2x_1}{k_1^2} < 0 \tag{2.3}$$

and similarly $\partial R_2/\partial k_2 > 0 > \partial R_1/\partial k_2$. Note that:

$$\frac{\partial R_2/\partial k_1}{\partial R_1/\partial k_1} = \frac{-1}{2 + \phi_2 + 2w/k_2} \tag{2.4}$$

The RHS of (2.4) is the slope of firm 2's reaction function: the LHS the ratio of the change in x_1 to the change in x_2 caused by the shift in firm 1's reaction function resulting from the increase in k^1. Intuitively equality 2.4 must hold since varying k_1 merely shifts firm 1's) reaction function, whilst firm 2's reaction function is unaffected. Hence the change in both firms' output in the market stage is simply a move along firm 2's reaction function as in Fig. 1. Note also that the increase in firm 1's output as it increases investment exceeds the reduction in firm 2's output, so that total industry output increases.

How does the equilibrium output vary with $\underline{\phi}$ given $\underline{k}$? Again, total differentiation of equations (1.2) with respect to $\underline{\phi}$ yields:

$$\frac{\partial R_1}{\partial \phi_1} = \frac{-1}{\Delta} x_1 \cdot \left(2 + \phi_2 + \frac{2}{k_2}\right) < 0 \tag{2.5}$$

$$\frac{\partial R_2}{\partial \phi_1} = \frac{x_1}{\Delta} > 0 \tag{2.6}$$

and similarly $\partial R_1/\partial \phi_2 > 0 > \partial R_2/\partial \phi_2$.

Thus as firm 1 conjectures that firm 2's output response becomes less accommodating (ϕ_1 increases), its own reaction function shifts to the left, again as in Fig. 1, so that:

$$\frac{\partial R_2/\partial \phi_1}{\partial R_1/\partial \phi_1} = \frac{-1}{2 + \phi_2 + 2/k_2} \tag{2.8}$$

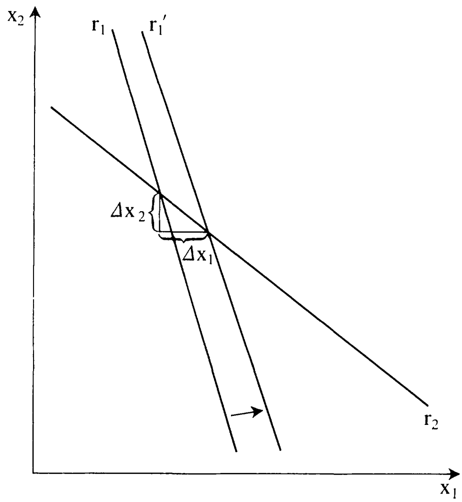

FIG. 1. Output responses to investment by firm 1 with exogenous conjectures

since firm 2's reaction function is unaffected by changes in ϕ_1. Note that as ϕ_1 increases, and firm 2 is believed to be less competitive, total industry output *falls*.

We have up to now examined what happens to the equilibrium outputs in the market stage as $\underline{\phi}$ and $\underline{k}$ vary. We have *not* imposed consistency $\underline{\phi} = \underline{\phi}(\underline{k})$ as in 1.6). Eaton and Grossman (1984) and Yarrow (1985) consider a strategic investment model where firms choose $\underline{k}$, but treat the nature of competition in the product market as exogenous. To briefly outline this type of model, firms' payoffs can be written as a function π_i of capital stocks chosen; given $\underline{\phi}$:

$$\pi_i(\underline{k}) = R_i(\underline{\phi}, \underline{k})\left[P^0 - \sum_{j=1}^{2} R_j(\underline{\phi}, \underline{k})\right] - rk_i - \frac{R_i(\underline{\phi}, \underline{k})^2}{k_i} \tag{2.8}$$

Firms then choose their capital stocks k_i, and we assume a Nash-equilibrium occurs, which seems reasonable given that capital expenditures are irreversible. In essence, this is a model where firms choose their output reaction-functions through teir choice of k_i. The resultant equilibrium can thus be seen as a Nash-equilibrium in reaction functions. A necessary condition for equilibrium is that firms are in their reaction functions. Setting $\partial\pi_i/\partial k_i = 0$ we have:

$$\frac{\partial c}{\partial k_i} = x_1 \cdot \frac{\partial R_1}{\partial k_1}\left[\phi_1 - \frac{\partial R_2/\partial k_1}{\partial R_1/\partial k_1}\right] \tag{2.9}$$

where $\partial c/\partial k_1$ is the partial derivative of the cost function $c(x_1, k_1)$ with respect to k_1, from A2. Turning first to the LHS of (2.9), if $\partial c/\partial k_1 = 0$ then the capital stock minimises the cost of producing x_1. If $\partial c/\partial k_1 < 0$, then there is *undercapitalisation,* too little capital then that which minimises cost, the technology being too labour-intensive. If $\partial c/\partial k_1 > 0$, then we have *overcapitalisation,* with more capital than mimimises the cost of producing x_1. Turning to the RHS of (2.9), if we consider the term in brackets, we have the difference between the firm 1's conjecture about the firm 2's output response ϕ_1, and the actual response of firm 2 (recall that $(\partial R_2/\partial k_1)/(\partial R_1/\partial k_1)$ is the *slope* of firm 2's reaction function, (2.7)). Thus we obtain Eaton and Grossman's result that if the conjecture is greater (less) than the actual response, there will be a factor bias of overcapitalisation (undercapitalisation) (Eaton and Grossman, (1984) Proposition 2.1). If the actual and conjectured responses are equal, however, then there will be no factor bias, and the technology will be efficient.

For example, if firms have Cournot conjectures ($\phi_i = 0$) as in Brander and Spencer (1983), then there will be over-capitalisation under A1–2. To take the other extreme, where firms have Bertrand conjectures as in Dixon (1985), then there will be under-capitalisation. If there is a factor bias of under- or over-capitalisation, then this will lead to a welfare loss relative to the social optimum. If we adopt the consumer surplus approach, then there will be two sources of welfare loss. The first will be the standard "welfare triangle" due to output being restricted below the perfectly competitive level. The second will be due to average costs being above their minimum level, which follows from the factor bias. In Dixon (1985), it is shown that in the case where the market stage is competitive, the lost surplus due to factor bias can exceed the surplus lost due to the restriction of output.

Eaton and Grossman's conclusion (1984 p. 6–7) is that if the product market is a consistent conjectural equilibrium, then there will be no factor bias. This result, however, is derived only for an exogenously given conjecture which happens to be consistent for the values of $\underline{k}$ the firms choose in equilibrium. If $\underline{\phi}$ *happen* to be consistent for a particular (equilibrium) $\underline{k}$, then they will certainly be inconsistent for all other $\underline{k}$, since the slopes of the firm's reaction functions will be different. If we believe that consistency of conjectures is a desirable property, then surely we ought to impose consistency on the firm's conjectures over the whole strategy space, for all $\underline{k}$. Unless we impose $\underline{\phi} = \underline{\phi}(\underline{k})$, then it is very unlikely that exogenously given conjectures will happen to be consistent at any particular (equilibrium) $\underline{k}$.

Indeed, strategic investment models with inconsistent conjectures are rather unsatisfactory. If $\phi_1 \neq \mathrm{d}r_j/\mathrm{d}x_i$, then it is difficult to give a convincing account of the firm's decision making in the two stages of the model. In the strategic stage, when the firm chooses its capital stock, it knows the true structure of the market—its own reaction function and the reaction function of the other firm. Thus when the investment decision is made, the firm is assumed to know the actual slope of the other firms's reaction function. However, when it enters the market stage, the firm chooses its own output according to its exogenously given (and almost certainly incorrect) conjecture about the slope of the other firm's reaction function. Thus in passing from the strategic to the market stage, a veil of ignorance seems to descend on the firm, since it loses its former knowledge of the other firm's reaction function. In essence, there is a conflict between the assumption of perfect foresight which the firm possess in the strategic stage, and inconsistency of conjectures in the market stage. If firms are going to have "rational" expectations in the strategic stage, then surely the conjectures should also be consistent in the market stage.

When we impose the consistency condition on conjectures, so that $\underline{\phi} = \underline{\phi}(k)$, then the outputs given $\underline{k}$ are:

$$x_i(\underline{k}) =_{\text{def}} R_i(\underline{\phi}(\underline{k}), \underline{k}) \tag{2.11}$$

When firm i varies its capital stock k_i, it shifts the reaction functions of *both* firms. Since the conjectures of both firms become more competitive as k_i increases (1.8), both firms reaction functions will move out as in Fig. 2. This makes the overall effect of an increase in k_1 on x_1 ambiguous:

$$\frac{dx_1}{dk_1} = \overset{(-)(-)}{\frac{\partial R_1}{\partial \phi_1}\frac{d\phi_1}{dk_1}} + \overset{(+)(-)}{\frac{\partial R_1}{\partial \phi_2}\frac{d\phi_2}{dk_1}} + \overset{(+)}{\frac{\partial R_1}{\partial k_1}} \tag{2.12}$$

Since the analysis is rather complex, it is useful if we break the overall effect

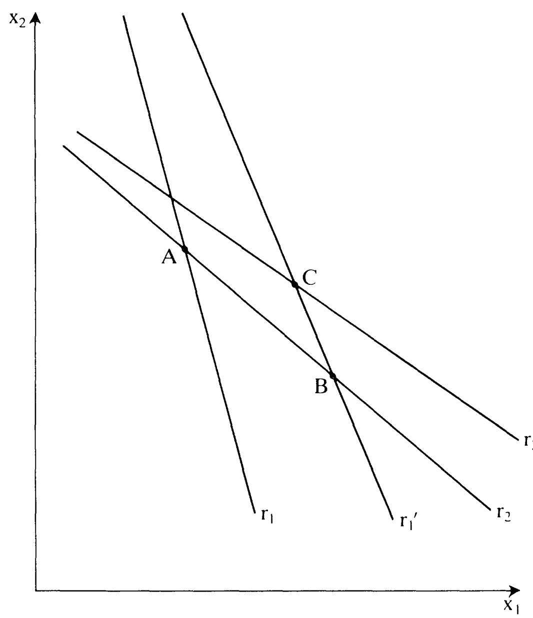

FIG. 2. Output responses to investment by firm 1 with consistent conjectures

into two parts, the first being the effect of k_1 on x_1 holding ϕ_2 constant:

$$\left.\frac{dx_1}{dk_1}\right|_{\phi_2}=\frac{\partial R_1}{\partial \phi_1}\frac{d\phi_1}{dk_1}+\frac{\partial R_1}{\partial k_1}=\frac{1}{\Delta\Delta_\phi}\cdot\frac{2x_1}{k_1^2}\left[\frac{\phi_2\phi_1-\Delta_\phi}{\phi_1}\right]>0 \qquad (2.12)$$

Similarly,

$$\left.\frac{dx_2}{dk_1}\right|_{\phi_2}=\overset{(+)(-)(-)}{\frac{\partial R_2}{\partial \phi_1}\frac{d\phi_1}{dk_1}}+\frac{\partial R_2}{\partial k_1}=\frac{1}{\Delta\Delta_\phi}\cdot\frac{2x_1}{k_1^2}[\phi_1\phi_2-\Delta_\phi]<0 \qquad (2.13)$$

In calculating $dx_i/dk_1|\phi_2$ we are only taking into account the effects of dk_1 in shifting firm 1's reaction function: by holding ϕ_2 constant, we are in effect holding firm 2's reaction function constant. Thus, in terms of Fig. 2, we are considering the move along firm 2's reaction function from point A to B. The crucial point is that since we are moving along firm 2's reaction function:

$$\left.\frac{dx_2/dk_1}{dx_1/dk_1}\right|_{\phi_2}=\phi_1 \qquad (2.15)$$

If we hold ϕ_2 constant, then, the change in ϕ_1 caused by k_1 shifts firm 1's reaction function further out, enhancing the effect with exogenous conjectures (see 2.12).

However, the effect of increasing k_1 is to make firm 2's conjecture ϕ_2 more competitive, shifting out firm 2's reaction function. This mitigates the expansionary effect, and the overall effect on x_i seems to be ambiguous (although a negative sign seems unlikely, we haven't been able to rule it out).

$$\frac{dx_1}{dk_1}=\underset{(+)}{\left.\frac{dx_1}{dk_1}\right|_{\phi_2}}+\underset{(+)(-)}{\frac{\partial R_1}{\partial \phi_2}\frac{d\phi_1}{dk_1}}\lesseqgtr 0? \qquad (2.15)$$

$$=\frac{1}{\Delta\cdot\Delta_\phi}\cdot\frac{2\cdot x_1}{k_1^2}\left[\phi_2-\frac{\Delta_\phi}{\phi_1}+\frac{\phi_2\cdot x_1}{\phi_1\cdot x_2}\right] \qquad (2.16)$$

In the symmetric case where $k_1=k_2$, $x_1=x_2=x$, $\phi_1=\phi_2=\phi$ clearly $dx_1/dk_1>0$.

Turning to the overall effect of k_1 on x_2 when we take into account the change in ϕ_2 we have:

$$\frac{dx_2}{dk_1}=\overset{(-)}{\left.\frac{dx_2}{dk_1}\right|_{\phi_2}}+\overset{(-)\ (-)}{\frac{\partial R_2}{\partial \phi_2}\cdot\frac{d\phi_2}{dk_1}} \qquad (2.17)$$

Again, the conjectural effect via ϕ_2 works in the opposite direction to the other effects, and may reverse them. However, in the symmetric case we have $dx_j/dk_i < 0$.

Having related firms' output to investment under consistent conjectures (2.11), we now turn to the firm's strategic investment decision. The payoff function is:

$$U_i(\underline{k}) = x_i(\underline{k})\left(P^0 - \sum_{j=1}^{2} x_j(\underline{k})\right) - \frac{x_i(\underline{k})^2}{k_i} - rk_i \tag{2.18}$$

Setting $\partial U_i/\partial k_i = 0$, we obtain:

$$\frac{\partial_c}{\partial k_1} = x_1 \cdot \frac{dx_1}{dk_1}\left[\phi_1 - \frac{dx_1/dk_1}{dx_2/dk_2}\right] \tag{2.19}$$

If we consider the RHS bracket, this is the difference between the actual output response in market stage (since ϕ_i are consistent), and the tradeoff between x_1 and x_2 which the firm in effect faces in the strategic stage. As is clear from Fig. 2, since an increase in k_1 makes both conjecture more competitive and shifts both reaction functions out, the trade off the firm faces in the market stage must be less negative than the slope of the other firm's reaction function (the slope A–B is more negative than the slope A–C). Combining 2.14), 2.15) and 2.16) we have:

$$\underset{(A-C)}{\frac{dx_2/dk_1}{dx_1/dk_1}} > \underset{(A-B)}{\left.\frac{dx_2/dk_1}{dx_1/dk_1}\right|_{\phi_2}} = \phi_1 \tag{2.20}$$

Hence the RHs bracket of (2.19) must always be negative with *endogenously* consistent conjectures, in contrast to the zero value with *exogenously* consistent conjectures as in Eaton and Grossman (1984). Whether this will give rise to under or overcapitalisation will depend on the sign of dx_1/dk_1:

$$\frac{\partial c}{\partial k_i} > 0 \quad \text{if} \quad \frac{dx_1}{dk_1} < 0 \quad \text{(overcapitalisation)}$$

$$\frac{\partial c}{\partial k_i} < 0 \quad \text{if} \quad \frac{dx_1}{dk_1} > 0 \quad \text{(undercapitalisation)}$$

Efficiency will only occur iff $dx_1/dk_1 = 0$. In the symmetric case, $dx_1/dk_1 > 0$, so that we obtain undercapitalisation. To summarise this:

> *Proposition* 3: Suppose an equilibrium $\underline{k}^*$ exists in the strategic investment model. There will be undercapitalisation if output responds positively to investment. In a symmetric equilibrium there will be undercapitalisation.

Although we have not been able to rule it out, the case of overcapitalisation seems a rather unlikely curiosity.

We have not established whether or not an equilibrium exists: equation (2.19) is merely a necessary condition for firms to be on their reaction functions in the strategic stage. Even under such simple assumptions as A1–2, making conjectures consistent leads to a very complex relation between capitals $\underset{\sim}{k}$ and outputs $\underset{\sim}{x}$. U_i may well not be concave in k_i. However, Proposition 3 at least provides a counter example to Eaton and Grossman's result, and shows how it depends crucially on conjectures being exogenous.

Because it is not possible to formulate useful conditions for the existence and uniqueness of equilibria in this model, we cannot perform meaningful comparative statics on the model using the equilibrium conditions (2.19). However, we can use more general analysis to characterise the properties of any equilibria that exist. We close this section with an analysis of the impact of the rental-wage ratio r on equilibrium conjectures. As r increases, the cost of capital becomes expensive relative to labour. Intuitively, as r becomes very expensive, this will lead eventually to small levels of investment, which implies conjectures close to the cournot value.

Proposition 4: Consider the strategic investment model $[R_+, U_i : i = 1..n]$.

(a) If $r \geqslant (p^0/2)^2$, there exists a unique equilibrium where $\underset{\sim}{\phi}^* = \underset{\sim}{k}^* = \underset{\sim}{0}$.
(b) if $r < (p^0/2)^2$, then for any equilibrium that exists where $\underset{\sim}{k}^* \gg \underset{\sim}{0}$, conjectures are between the Cournot and Bertrand values $\underset{\sim}{\phi}^* \in (-1, 0)$.

Thus relative prices in the factor market eventualy have an influence on the degree of competition in the product market. The basic point to remember is that changes in r have no direct influence on the consistent conjectures, which are determined only by the level of investment by the two firms (this stems from the fact that marginal cost in the market stage is unaffected by r, since capital is a fixed cost). The level of r will, however, influence the degree of investment in the industry in equilibrium.

An immediate implication of this analysis is that in the strategic investment framework, with a technology that has a strictly increasing smooth relationship between the labour input and output given capital, the assumption of either Cournot or Bertrand competition are too extreme. Of course, if the technology were Leontief in the market stage, then the choice of capital would tie down the capacity (i.e. maximum output) of the firm in the market stage, and hence lead to inflexible production (a "vertical" marginal cost at capacity). In this case the *equilibrium* consistent conjectures would be Cournot (out of equilibrium the analysis is more complex, depending on whether the capacity constraint binds). In order to have a Leontief technology in the market stage, we need not assume that the underlying technology is Leontief: it would suffice to have a putty-clay technology, where the capital-labour ratio is freely chosen with the investment decision in the strategic stage, but becomes fixed in the market stage. At the other extreme, if capital and labour are perfect substitutes in production, and cost the same ($r = 1$), then marginal cost is "horizontal" and output is perfectly flexible whatever the level of investment. In this case the equilibrium conjectures are Bertrand.

In the framework presented, firms are only able to influence the flexibility of production through their investment decision. It is possible to widen the firm's choice, for example, by allowing the firm to choose whether to precommit one or both inputs in the strategic stage (as in Dixon (1986)). Alternatively, the firm could choose the type of technology to influence the flexibility of production in the market stage. We leave these possibilities for future research.

Conclusion

This paper explores a model of strategic investment with consistent conjectures. This is a model in which the firm's cost structure and the nature of competition in the product market are endogenous. Thus aspects of both the industry structure (in this case costs) and conduct are determined. Two basic insights are explored. Firstly, that the nature of product market competition depends on the flexibility of firms' production. This is captured by imposing a consistency condition on conjectures. Secondly, the idea that firms can influence the flexibility of production, as in strategic investment models where firms choose their short run cost function through their investment decision. The model combines these two ideas, so that firms take into account the impact of their investment decisions on competition in the product market.

There are three main results in this paper. First, in equilibrium, the degree of competition will lie between the Bertrand and Cournot values. Secondly, there will generally be a factor bias, with undercapitalisation in any symmetric equilibrium. Thirdly, the degree of competition is ultimately sensitive to the wage-rental ratio. As capital becomes very expensive, equilibrium conjectures converge to the Cournot value.

The relationships explored in this paper are ultimately very complex. In order to understand complex phenomenon it is often necessary to construct simple models. This paper is no exception to this tendency. Even a slight relaxation of the A1–2 would make the model intractable and ambiguous. Thus the model should certainly be interpreted as an example rather than a general theory. However, it is hoped that the example is stimulating, and provides a useful first step.

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APPENDIX: PROOFS

The proofs are briefly sketched.

Proposition 1: For an explicit solution to the quadratic system, see Bresnehan (1981, pp. 994–5). We give an alternative and more intuitive proof. We demonstrate that: (a) there is a unique solution to 1.4) and 1.5) such that $\phi_i \in (-1, 0)$ $i = 1, 2$; (b) there are no solutions with $\phi_i < -1$.

(a) There exists a unique solution $\underline{\phi} \in (-1, 0)^2$. If $\phi_2 \in (-1, 0)$ then from 1.4)

$$\phi_1 \in [-(1+2/k_2)^{-1}, -(2+2/k_2)^{-1}] =_{\text{def}} C_1$$

Similiarly if $\phi_1 \in (-1, 0)$ then:

$$\phi_2 \in [-(1+2/k_1)^{-1}, -(2+2/k_1)^{-1}] =_{\text{def}} C_2$$

Any consistent conjectures in $(-1, 0)^2$ most lie in the subset $C = C_1 \times C_2$. Expressing both consistency conditions 1.4) so that ϕ_1 is a function of ϕ_2, we have:

$$\phi_1 = Z_1(\phi_2) = -(2 + \phi_2 + 2/k_2)^{-1}$$

$$\phi_1 = Z_2(\phi_2) = -1/\phi_2 - 2(1 + 1/k_1)$$

If conjectures are consitent, then $\phi_1 = Z_i(\phi_2)$ $i = 1, 2$. Restricting ourselves to $\phi_2 \in C_2$, consider Z: $C_2 \rightarrow [-1, 0]$, where $Z(\phi_2) =_{\text{def}} Z_1 - Z_2$.

$$Z(\phi_2) = -(2 + \phi_2 + 2/k_2)^{-1} + 1/\phi_2 + 2(1 + 1/k_1)$$

$$Z(-(2+2/k_1)) < 0 < Z(-(1+2/k_1))$$

Since Z is continuous and strictly decreasing, there exists a unique ϕ' in the interior of C_2 such that $Z = 0$, so that $\phi_1' = Z_i(\phi')$. Hence (ϕ_1', ϕ') are unique consistent conjectures in C.

(b) To see that no other consistent conjectures exist, note that Z is strictly decreasing whenever the second order condition is satisfied.

Proposition 2: $-1 < \phi_i(\underline{k}) < 0$ for all $\underline{k} \gg 0$, and from (1.4):

$$\phi_i > -1/(1 + 2/k_{jn})$$

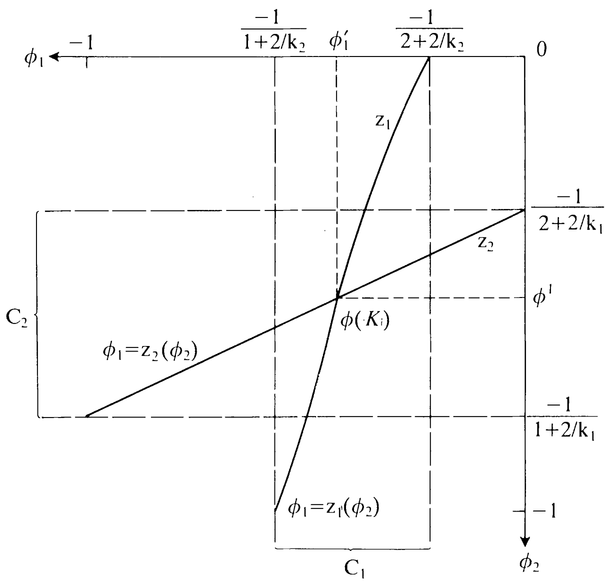

FIG. 3. Illustration of Proposition 1

Hence

$$\underset{k_{jn}\to+\infty}{\text{Liminf}}\ \phi_{in} > \text{Lim}\frac{-1}{1+2/k_{jn}} = -1$$

Since Limsup $\phi_{jn} < -1$, it follows that $\underset{k_n\to+\infty}{\text{Lim}}\ \phi_{jn} = -1$ QED

Proposition 4:
(a) Minimum average cost is $2\cdot\sqrt{r}$. If $\sqrt{r} \geqslant p^0/2$ then firms earn negative profits whenever $k_i > 0$.
(b) If $\sqrt{r} < p^0/2$, then $\underset{\sim}{0}$ cannot be an equilibrium, since either firm can produce a small output efficiently and earn strictly positive profits, hence in any symmetric equilibrium $\phi_i < 0$. To obtain a lower bound on ϕ_i, we define an upper bound on k_i. A necessary condition for $U_i(\underset{\sim}{k}) \geqslant 0$ is that $p^0 - x_i \geqslant 2\cdot\sqrt{r}$. Since with symmetry production is under-capitalised, $x_i \geqslant k_i\cdot\sqrt{r}$. Hence $k_i \leqslant (p^0/\sqrt{r}) - 2$, ensuring $\phi_i < -1$.

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