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STRATEGIC INVESTMENT IN AN INDUSTRY WITH A COMPETITIVE PRODUCT MARKET*

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I. INTRODUCTION

THIS PAPER explores the scope for, and effect of, strategic behaviour by firms in an industry where the product market is "competitive" in the usual sense that firms are effectively price-takers. Traditionally the literature on imperfect competition has concentrated on the product market, employing the notions of the Cournot equilibrium or monopolistic competition. The model presented shows that even if firms are unable to obtain any such market power in the product market, they can still influence the market outcome through investment behaviour which determines their actual cost structure. The ability of firms to thus influence the market outcome is directly related to their market share. This is a result which is not only of theoretical interest. It is sometimes argued that for some industries concentration does not lead to welfare loss because the market may be competitive even with only a few producers (see Fisher *et al.* [1983, pp. 24–25] for the U.S. computer industry and I.B.M.). The message of this paper is that even in such competitive industries concentration still matters since firms can still behave strategically with factors such as investment and R&D.

This paper considers a model of strategic investment in the context of a two stage dynamic equilibrium with two factors of production, capital and labour. In the first "strategic" stage, firms choose their capital stocks. In the second "market" stage the capital stocks are fixed, and a competitive equilibrium occurs. The two stages of the model can be seen as capturing the distinction between the long run and the short run. The equilibrium rests on the reasonable assumption that the capital decisions of firms are irreversible in the market stage. We are thus implicitly assuming that there is no rental market for capital. In the competitive market stage the price equates supply with demand. In essence, the choice of the capital stock in the strategic stage determines the supply function that the firm has in the market stage. Thus firms are able to influence the market equilibrium to their favour through their choice of capital stock. The resultant equilibrium can be interpreted as a Nash equilibrium in supply functions (see for example Grossman [1981], Dixon [1984a]).

In evaluating the welfare consequences of the model, it is necessary to realise that the cost structure of the firm is endogenous. The choice of capital

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determines the firms "short-run" cost function. If we were only to consider the *actual* cost structure of firms, then, since price equals marginal cost in the competitive market stage, the standard welfare analysis would tell us that there was no restriction of output, and no loss of consumer surplus. However, since investment alters the firm's actual cost structure, welfare considerations need to concentrate on possible rather than actual costs. In equilibrium we find that firms are able to restrict investment and output so that the price exceeds least cost, and there is a loss of consumer surplus relative to the socially optimal outcome. This is not the only source of welfare loss.

Because capital is used strategically in this model, there is an asymmetry between capital and labour. In equilibrium the strategic use of investment leads to a bias away from capital, so that there is undercapitalisation and an excessive labour-capital ratio. This results in inefficient production, in the sense that the actual costs of producing the firm's output are in excess of the minimum technically required.

The result of undercapitalisation provides an interesting contrast with existing models of the strategic use of investment. The model which most closely resembles ours is Brander and Spencer's model of R&D expenditure [1983]. The crucial difference between the models is the assumption made about the market stage: they assume that it is Cournot. Under this assumption, they show that there is the opposite factor bias, with overcapitalisation in equilibrium. Furthermore, whereas strategic behaviour improves social welfare in the Brander and Spencer [1983] model as compared with the non-strategic case, in the model presented it leads to a loss in social welfare. Overcapitalisation can also occur in Dixit's model of strategic entry deterrence [1980, pp. 103–4].

The assumption of a competitive market stage, coupled with constant returns to scale gives the model a particularly tractable and intuitive form. In section I of the paper the basic framework is presented. In section II we explore the general properties of the equilibrium. We show that the profit to sales ratio that is, the price-cost margin) is inversely related to the elasticities of demand and supply. In section III two specifications of the general model are presented. First we derive an explicit solution to the model in the case of identical firms with Cobb–Douglas technology and constant elasticity industry demand. This example enables us to relate the strategic investment model with the standard (one stage or non-strategic) Cournot equilibrium. Furthermore, depending on the technical parameter, the strategic investment equilibrium has Cournot and Bertrand outcomes as limiting cases. We also provide a linearised model in section III, which is particularly useful for explicitly analysing the welfare properties of the model. We are able to show, in this case at least, that the welfare loss due to strategic inefficiency may be large relative to the standard welfare "triangle".

The main body of this paper treats the number of firms as given: there is a given set of active incumbents. However, the impact of entry on the model is briefly discussed. Entry adds an additional welfare loss to the model, since sunk set-up costs will in general lead to more firms than are socially optimal in the industry.

I. A MODEL OF STRATEGIC INVESTMENT

We present the general model in this section, and will consider more specific cases in section III. All functions are assumed to be differentiable when required.

Firms have a standard two-input constant-returns production function:

A1: Each firm's production function $f_i : R^2_+ \to R_+$ is homogeneous to degree one and strictly concave in each input:

$$x_i = f_i(k_i, L_i) \qquad f_k, f_L > 0 > f_{LL}, f_{kk}$$

where k_i and L_i are capital and labour inputs of the i^{th} firm.

This assumption is stronger than required, but it simplifies the analysis. We can then define the firm's cost function in the standard manner taking the factor prices w and r as parameters.

(a) *Long-Run Cost function*: $a \cdot x$

$$\begin{aligned} a(x) &= \min_{L,k \in R_+} (w \cdot L + r \cdot k) \qquad \text{subject to} \qquad f(k,L) - x \geqslant 0 \\ &= a \cdot x \end{aligned}$$

where a is (constant) least average cost.

(b) *Short-Run Cost Function*: $k \cdot c(x/k)$

$$C(K,x) = \min_{L \in R_+} (w \cdot L + r \cdot k) \qquad \text{subject to} \qquad f(k,L) - x \geqslant 0$$

Under constant-returns we can define $k \cdot c(x/k) = k \cdot C(1, x/k)$.

Concavity of f in L implies that c is convex in output-per-unit-capital x/k. Given the technology, we can then define the "short-run" supply function, again taking advantage of constant returns.

(c) *Supply Function*: $k \cdot s(p)$

$S(k,p)$ is the output which solves:

$$\max_{x,L \in R^2_+} [p \cdot x - w \cdot L - r \cdot k] \qquad \text{subject to} \qquad f(k,L) - x \geqslant 0$$

Under A1 this can be written as $k \cdot s(p)$.

Thus $s(p)$ is the profit-maximising output-per-unit-capital. Obviously, the strict convexity and differentiability of $c(x/k)$ imply that s is strictly increasing in p, and its second derivative s'' will be positive or negative depending on c'''. In effect, through its choice of k_i, the i^{th} firm chooses the supply function it will have in the market-stage.

We consider a two stage model. In the first "strategic" stage, firms precommit their capital stock. In the second "market stage", firms' capital stocks are fixed, and a competitive equilibrium occurs. In order to define the competitive price we need to state our assumptions about the industry demand function.

A2: Industry demand $F: R_+ \to R_+$ is bounded, continuous, and strictly decreasing when positive.

The competitive price θ which occurs in the market stage is defined implicitly by the relation:

$$(1.1) \qquad F(\theta) - \sum_{i=1}^{n} k_i \cdot s_i(\theta) = 0$$

We can thus define the implicit function which gives the competitive price θ ruling in the market stage to the n-vectors of capital stocks $\underline{k}$ chosen in the strategic move, $\theta : R^n_+ \to R_+$

$$(1.2) \qquad \theta = \theta(\underline{k})$$

Total differentiation of (1.1) yields:

$$(1.3) \qquad \frac{\partial \theta}{\partial k_i} = \frac{s_i(\theta)}{F' - \sum_{j=1}^{n} k_j \cdot s'_j} < 0$$

where primes denote derivatives with respect to price, which are evaluated at $\theta(\underline{k})$. The numerator s_i will vary between firms, whilst the denominator is common to all firms, and reflects the fact that as the price falls, firms' outputs will fall. If firms are identical then (1.1) and (1.3) simplify, since θ will depend only on the total capital stock Σk_j, as will $\partial\theta/\partial k_i$.

We are now in a position to define the firm's payoff function U_i, giving the profit earned as a function of the capital stocks chosen in the strategic move. First, however, we define the firm's profit-per-unit-capital conditional upon $\theta, \pi : R_+ \to R$

$$(1.4) \qquad \pi(\theta) = \theta \cdot s(\theta) - c(s(\theta))$$

Thus π is the nett or supernormal rate of profit on capital, once the cost-per-unit-capital r has been covered (since r is included in $c(\cdot)$, from definition (b)). We now have the firm's profit-function $U_i : R^n_+ \to R$.

(d) *Profit Function*: $U_i(\underline{k}) = k_i \cdot \pi(\theta(\underline{k}))$.

Having defined firms' payoffs in terms of their choices of k_i in the strategic move, we can define the equilibrium in the two stage model as a Nash equllibrium $\{\underline{k}^*, \theta^*\}$ in the game $[R_+, U_i : i = 1 \dots n]$. The Nash equilibrium assumption seems perfectly natural here, since the capital decision is irreversible, so that firms are unable to react to each other. Whilst the model is formally defined as a Nash equilibrium in capital stocks, it is really a Nash equilibrium in supply functions. We will have more to say about this subsequently.

II. THE PROPERTIES OF THE EQUILIBRIUM

Supposing an equilibrium $\underline{k}^*$ to exist, what does it look like? We will restrict ourselves to those equilibria where all firms are producing, that is, $\underline{k}^* \gg 0$, and we shall denote the equilibrium market price $\theta^* = \theta(\underline{k}^*)$. Then $\underline{k}^*$ will be a Nash equilibrium if for all firms $i = 1 \dots n$:

$$(2.1) \qquad U_{ki}\,|_{k^*} = \pi_i(\theta^*) + k_i^* \cdot \frac{d\pi}{d\theta} \cdot \frac{\partial \theta}{\partial k_i} = 0$$

where U_{ki} is the partial derivative of U_i with respect to k_i (since we shall never need cross-partials, this notation should be unambiguous). We can define the elasticity of θ with respect to k_i as:

$$(2.2) \qquad \varepsilon_i =_{\text{def}} -\frac{k_i}{\theta} \cdot \frac{\partial \theta}{\partial k_i}$$

substituting (2.2) into (2.1), and noting that by Hotelling's Lemma $d\pi/d\theta = s(\theta)$, at $\underline{k}^*$:

$$(2.3) \qquad \frac{\pi(\theta)}{\theta^* \cdot s_i(\theta^*)} = \varepsilon_i$$

Note that the LHS of (2.3) is the i^{th} firm's profit to sales ratio. Hence in equilibrium, each firm's profit to sales ratio will be equal to the elasticity of the market price with respect to its own capital. Clearly, the profit to sales ratio must equal the price-average-cost-margin, and when we expand the LHS of (2.3) we obtain:

$$(2.4) \qquad \frac{\theta^* - \dfrac{c(s(\theta^*))}{s(\theta^*)}}{\theta^*} = \varepsilon_i$$

where $c(s(\theta))/s(\theta)$ is average cost. It should be noted that we have made no assumption of average-cost being equal to marginal cost to derive (2.4): the LHS is in principle a directly observable variable (if we leave aside problems of measuring profits, of course).

It is easily shown that the equilibrium price θ^* in the market stage will be greater than the long-run (least) average cost a_i for all firms that produce in equilibrium. If $\theta^* < a_i$, then the i^{th} firm will make a loss if it produces anything at all, and if $\theta^* = a_i$ it will earn nothing. Since we restrict ourselves to $\underline{k}^* \gg 0$, the price must be greater than or equal to a_i. But if $\theta^* = a_i$, then the firm could reduce k_i slightly, thus increasing θ and hence earn positive profits.

In the case of identical firms (2.3) simplifies further. Since $\partial\theta/\partial k_i$ is then a function of total capital only, and so is identical across firms:

$$\text{(2.5)} \qquad \frac{\partial \theta}{\partial k_i} = \frac{s(\theta)}{F' - s' \cdot \Sigma k_j} \qquad j = 1 \ldots n$$

so that from (2.2) and (2.5) we have:

$$\text{(2.6)} \qquad \varepsilon_i = \frac{k_i}{\Sigma k} \cdot \frac{1}{\varepsilon_s - \varepsilon_d}$$

where ε_s and ε_d are the familiar price elasticities of supply and demand, $\varepsilon_s = (\theta/s) \cdot s'$, $\varepsilon_d = (\theta/F) \cdot F'$. Hence if all firms have an identical technology, then (2.3) becomes:

$$\text{(2.7)} \qquad \frac{\pi(\theta^*)}{\theta^* \cdot s(\theta^*)} = \frac{1}{n(\varepsilon_s - \varepsilon_d)}$$

Thus the profit to sales ratio equals the reciprocal of the sum of demand and supply elasticities (evaluated at θ^*), divided by the number of firms. In an industry with $\varepsilon_s = -\varepsilon_d = 1$, and fifteen firms, the profit to sales ratio would be $1/30^{\text{th}}$.

Equation (2.7) is clearly similar to the equilibrium condition in the Cournot–Nash model. As we argue in Dixon [1984a], perhaps the most natural and convincing interpretation of the standard Cournot model is that firms pre-commit both factors of production, so that in the strategic move firms choose a capacity, and in the market stage a competitive equilibrium (occurs that is, the price clears the market). Hence the symmetric Cournot equilibrium is given by:

$$\text{(2.8)} \qquad \frac{\theta^c - a}{\theta^c} = \frac{-1}{n \cdot \varepsilon_d}$$

Comparing (2.8) with the equilibrium condition for the strategic investment case, we can see that they differ in two important respects. First, on the LHS of (2.8) the price cost margin is given as a mark-up on a. Second, the RHS of (2.8) and (2.7) are directly comparable, and might lead us to expect that the profit to sales ratio in the strategic investment case will be less than in the Cournot case. However, even if this is so, we cannot infer that the equilibrium price will therefore be lower in the strategic investment case. For any given price θ, the Cournot profit to sales ratio is larger, since average costs are at their minimum. It is therefore quite possible that $\theta^* > \theta^c$ even though the inequality is reversed for the respective profit to sales ratios. No general comparison is presented in this paper.[1] However, in the linear model in section III, the Cournot price will always be the higher of the two. It can also be shown in the case of

[1] For a general treatment see Dixon [1984b, chapter 6, Theorem 6.1].

Cobb–Douglas firms and constant elasticity demand, that a sufficient condition for $\theta^c > \theta^*$ is inelastic demand ($\varepsilon_d > -1$). The presence of ε_s in (2.7) reflects the fact that in choosing its capital stock, the firm takes into account not only the downward slope of the demand curve, but also the reduction in (all) firms' outputs as θ falls. In the Cournot case, of course, these outputs are treated as constant.

An implication of the fact that $\theta^* > a_i$ is that firms will employ an inefficient technology, and incur costs in excess of a_i. The reason for this is that the firm is on its short-run cost curve in the market stage. Hence it will only employ least-cost technology if $\theta^* = a_i$. If $\theta^* > a_i$, then the firm will not be on its long-run cost-curve, and will in fact be *undercapitalised*: the firm's output capital ratio $s(\theta^*)$ is higher than the cost-minimising ratio $s(a)$, so that the firm employs an excessively labour intensive technology. Although the firm is maximising profits, the strategic structure of the model—and the resultant asymmetry between the firm's choice of capital and labour—implies that the firm does not employ the least-cost technology.

In order to bring out even more clearly the nature of the distortion, we can telescope the firm's decision into a simple optimisation programme:

$$(2.9) \qquad \max_{k_i, L_i, x_i} \theta(k_i, k_{-i}) \cdot x_i - w \cdot L_i - r \cdot k_i \qquad \text{subject to} \qquad f(k_i, L_i) - x_i \geqslant 0$$

The firm chooses x_i and L_i freely as in the standard competitive case. The strategic nature of the firm's investment decision is captured by making θ a function of k_i (and k_{-i}). The first-order conditions yield

$$(2.10) \qquad \frac{w}{r} = \frac{f_L}{f_k - \varepsilon_i \cdot s(\theta)} > \frac{f_L}{f_k}$$

From (2.9) and (2.10) we can see that the dependence of the competitive price in the firm's capital stock reduces the "marginal revenue product" from f_k to $f_k - \varepsilon_i \cdot s(\theta)$, and hence leads to a *lower* capital-labour ratio than required for least-cost production. When choosing k_i, the firm takes into account both its effect on the subgame price, and its effect on costs via the capital-labour ratio. At $\underline{k}^*$, if any firm were to increase its capital stock, the gains from a lower output-capital ratio in terms of reducing average costs would be outweighed by the resultant fall in θ. It should be clear from this simple treatment that the undercapitalisation result is general in this model, and does not depend at all on the assumption of constant returns in A1.

The symmetric industry equilibrium is depicted in Figure 1. The equilibrium price θ^* is determined by the intersection of the industry demand function and the supply function which results from the capital stocks chosen in the strategic stage. The U-shaped industry "short-run" average cost curve is marked AC. The strategic inefficiency due to under capitalisation is represented by the fact that average cost AC* is less than marginal cost in equilibrium. The industry profits $\Sigma k_i^* \cdot \pi(\theta^*)$ are given by the area A. The profit to sales ratio is given by

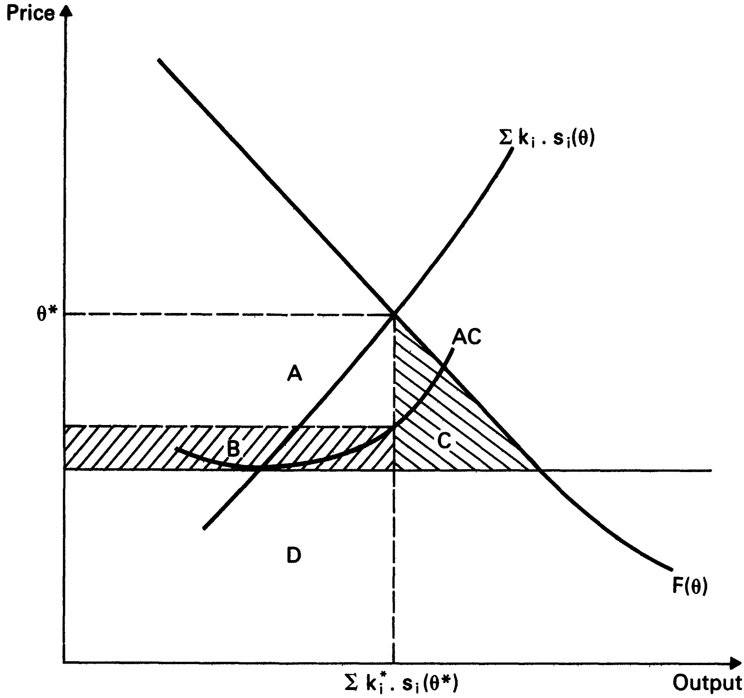

FIGURE 1
Welfare Loss

the ratio of area A to areas A, B, D. We shall now briefly consider the welfare losses in this framework employing the familiar consumer surplus approach, as in the "social costs of monopoly" literature (see Marshall [1920], Harberger [1954], Cowling and Mueller [1978] *inter alia*). Whilst this approach is open to serious criticism, it provides a useful first guide. In this model there will be two sources of lost consumers' surplus. Firstly we have the standard welfare loss given by the "triangle" under the demand curve, area C in the figure. Secondly we have the welfare loss due to strategic inefficiency, which is represented by area B in Figure 1, being the lost profits to shareholders $(AC^* - a) \cdot F(\theta^*)$. It should be noted that this strategic inefficiency is totally different in origin to the X-inefficiency as in Leibenstein [1966] and Comanor and Leibenstein [1969]. In the strategic investment model presented here the inefficiency is due to the dynamic structure of the model, not the fact that the best monopoly profit is a quiet life. In the market stage, the managers minimise costs: the inefficiency stems from the constraint the managers have imposed on themselves by their choice of capital in the strategic stage. In section III below, we show that in a linearised model (Case 2), the welfare loss due to strategic inefficiency may well be larger than the conventional triangle. This is not surprising, since the efficiency loss is $AC^* - a$ multiplied by the entire industry output.

We have now explored the properties of the strategic investment equilibrium. Even if the product market is perfectly competitive—or at least approximately so—the possibility of strategic commitment of capital would lead to prices above least-average cost.

The second important property of the model leads firms to choose undercapitalised technologies that are economically inefficient. It should be noted that this result only occurs because the firm is forced to separate its choice of capital and labour, and cannot choose them simultaneously. As we discuss in Dixon [1984a], were firms able to precommit both factors of production, and hence their capacity, then there would be no strategic factor bias, and the equilibrium of the two stage game would be Cournot. The strategic inefficiency is also related to the nature of the technology. Even in the case where firms precommit only capital, if the technology is Leontief or putty-clay, then in effect output and labour are thereby precommited, so that there will be a Cournot outcome with no strategic inefficiency. We should also note that the undercapitalisation result stems from the particular nature of the market subgame: had we assumed a Cournot equilibrium in the market stage, we would have obtained an overcapitalisation result, as in Brander and Spencer [1983].[2]

III. SOME EXPLICIT SOLUTIONS TO THE MODEL

In order to explore the mechanisms of the model more closely, it is useful to consider some specific functional forms. We will consider two cases. In case 1 we will explore the model in the case of identical firms with Cobb–Douglas technology, and constant elasticity demand. In case 2 we explore the model when firms may be different, but demand and supply functions are "linear". The second case is particularly simple for exploring the welfare loss due to strategic inefficiency. Consideration of these examples also gives us some ideas about existence in the model, since in both cases an equilibrium will exist under fairly weak conditions.

Case 1: Constant Elasticities: Cournot and Bertrand Outcomes as Limiting Cases of the Strategic Investment Model

In the case of identical firms, we recall the simple equilibrium condition (2.7):

$$\frac{\pi(\theta)}{\theta \cdot s(\theta)} = \frac{1}{n(\varepsilon_s - \varepsilon_d)}$$

Since the RHS of the equation is expressed in terms of ε_s and ε_d, we first consider the model when these are parameters. It will be recalled that ε_s is the elasticity of supply-per-unit capital $s(\theta)$ with respect to the market price.

[2] The market stage can be generalised to allow for a wider range of market outcomes. Yarrow [1985] has allowed for a conjectural variations model.

Cobb–Douglas technologies yield constant elasticity supply functions. Formally, if we specialise A5 to:

$$\text{A1(a)}: \quad x = L^{\alpha} \cdot k^{1-\alpha} : \varepsilon_s = \alpha/1-\alpha \qquad (0 < \alpha < 1)$$

We also specialise A2 to constant elasticity of demand:

$$\text{A2(a)}: \quad F(p) = p^{-\beta} \qquad (\varepsilon_d = -\beta)$$

Under A1(a) and A2(a), the profit to sales ratio will be uniquely determined by the three parameters of the model: n, α and β. Before proceeding to the explicit solution, however, it is necessary to consider whether or not an equilibrium exists: equation (2.8) merely tells us that the first order conditions are satisfied. Under A1–2 we know that there is a unique vector $\underline{k}^*$ which satisfies (2.8), in which all firms capital stocks are the same ($k_i^* = k^*$ for all i). Define ϕ where:

$$(3.1) \qquad \phi = (n-1) \cdot k^*/k_a$$

and k_a is the competitive level of the capital stock ($a = \theta(k_a)$). Given that the other firms choose k^*, firm i will never want to choose its own capital stock so that the price is less than a, so we can restrict our attention to $k_i \leqslant (1-\phi) \cdot k_a$. Under what conditions will $\underline{k}^*$ so defined be an equilibrium? Proposition 1 gives a sufficient condition for the payoff function to be quasi-concave.

Proposition 1 (A1(a), A2(a)): *A unique symmetric equilibrium exists if*

$$\frac{1}{1-\phi} \geqslant \frac{1-\varepsilon_d + 2 \cdot \varepsilon_s}{2 \cdot (\varepsilon_s - \varepsilon_d)}$$

The proof is given in the appendix. A sufficient condition for Proposition 1 to be satisfied is that demand is elastic. If there are more than a few firms, ϕ will be close to unity, so that we can be reasonably assured of existence.

Under A1(a) the profit to sales ratio is given by:

$$(3.2) \qquad \frac{\pi(\theta)}{\theta \cdot s(\theta)} = [1-\alpha]\left[1 - \left(\frac{a}{\theta}\right)^{1/1-\alpha}\right]$$

We can now express (2.7) in terms of our parameters and solve for θ^*.

$$(3.3) \qquad \theta^* = a \cdot \left[1 - \frac{1}{n \cdot (\alpha + (1-\alpha) \cdot \beta)}\right]^{\alpha - 1}$$

(3.3) is particularly useful if we wish to compare the strategic investment equilibrium with the Cournot equilibrium. Recall that in Dixon [1984a] we interpret the Cournot equilibrium as corresponding to the case where the firm precommits both factors of production, and hence chooses its capacity in the strategic move. In this case, firms will always choose a cost minimising capital labour ratio. Since there are constant returns to scale, the resultant Cournot equilibrium is:

$$(3.4) \qquad \theta^c = a \cdot \left(1 - \frac{1}{n \cdot \beta}\right)^{-1}$$

Comparing (3.4) and (3.3) it can be seen that since $0 < \alpha < 1$, a sufficient condition for the Cournot price θ^c to exceed θ^* is that $\beta \leqslant 1$.

Having computed the equilibrium values $\{k^*, \theta^*\}$ under A1(a) and A2(b), we shall briefly consider how the equilibrium varies with the technological parameter α. When the firm chooses k_i in the strategic move, it chooses the supply function it will have in the market subgame. The parameter α determines the elasticity of the "short run" supply function $s(\theta)$: as $\alpha \to 1$, $\varepsilon_s \to +\infty$; as $\alpha \to 0$, $\varepsilon_s \to 0$. Thus α can be interpreted as representing the flexibility of production in the market stage. With Cobb–Douglas technology the limiting cases of $\alpha = 1$ or $\alpha = 0$ are not in themselves interesting, but they can act as a guide to the intuition for α close to 0 or 1. If $\alpha = 0$, then $x = k$ (for $L > 0$), so that output is totally determined by investment in the strategic move. We would thus expect a Cournot outcome, where the profit to sales ratio is $-1/\varepsilon_d$. If $\alpha = 1$, then $x = L$ $(k > 0)$, so that precommitment does not affect the short run supply function, which would be perfectly elastic at $\theta = w$. In this case we would expect the Bertrand case of a zero profit to sales ratio.

The reasoning from these two cases is indeed correct, since from equations (3.3) and (3.4) it follows that:

$$\alpha \to 1 \quad \text{implies} \quad \frac{\theta^*}{a} \to \left[1 - \frac{1}{n}\right]^0 = 1 \qquad (n \geqslant 2) \text{ "Bertrand"}$$

$$\alpha \to 0 \quad \text{implies} \quad \frac{\theta^*}{a} \to \left[1 - \frac{1}{n\beta}\right]^{-1} \qquad (n \cdot \beta > 1) \text{ "Cournot"}$$

(Note that a itself varies with α.) Thus, depending on the elasticity of supply, the markup of price over least cost has the Cournot and the Bertrand values as limiting cases.

More can be said about the relationship with the Cournot case. If firms have constant returns Leontief technology, for example, then the output (or rather capacity) will be tied down in the strategic stage, and hence the Cournot equilibrium will occur. A further interesting property of the Leontief case is that there is no strategic inefficiency since, given its capacity, the firm will not employ labour over and above the minimum technically required, whatever the market price. Indeed, there will be no inefficiency for any technology for which the capacity is determined by the choice of capital stock. For example, if technology is putty-clay, then the capital labour ratio will be efficient, and the strategic investment equilibrium will be the Cournot equilibrium.

Case 2: The Welfare Loss in a Linear Model

In this section we show how the model works out in the linear case, where the supply and demand functions in the market subgame are linear. Linearity of the firm's supply-per-unit-capital function $s(p)$, can be seen as arising from a symmetric Cobb–Douglas technology (where the exponents on capital and labour are both 0.5). This is a special case of assumption A1 (a), if we allow for non-identical firms:

A1 (b): $x_i = k_i^{0.5} \cdot L_i^{0.5}$

From this specification of the production function we can derive the functions $s(\theta)$, $\pi(\theta)$. These are best expressed using the parameter v, where v is defined as the least-cost or efficient output-capital ratio, so that $s(a) = v$.

$$(3.5) \qquad s(\theta) = \frac{v}{a} \cdot \theta$$

$$(3.6) \qquad \pi(\theta) = \frac{v}{2a}(\theta^2 - a^2)$$

On the demand side we give a special case of A2:

A2 (b): $F(p) = \max[0, \bar{p} - p]$

Thus we assume linearity and normalise so that $F' = -1$. Under these assumptions the market stage equilibrium condition implies the simple relation between capital stocks and price:

$$(3.7) \qquad \theta(\underline{k}) = \frac{\bar{p}}{1 + \Sigma k_i \cdot \omega_i} \qquad \text{where} \qquad \omega_i = v_i/a_i$$

Substitution of (3.7) into (3.6) yields the payoff function:

$$(3.8) \qquad U_i(\underline{k}) = \frac{\bar{P}^2}{2} \cdot \frac{\omega_i \cdot k_i}{1 + \Sigma \omega_j \cdot k_j} - r \cdot k_i$$

Thus U_i is strictly concave in k_i in this linear case. Hence an equilibrium exists. It can be shown that the equilibrium price is less than the Cournot price (Dixon [1984b, Theorems 5.1 and 6.1]).

One of the properties of the strategic investment equilibrium is that production is inefficient. The rest of this section of the paper is devoted to deriving explicit terms for this inefficiency in the linear case.

The excessive labour-capital ratio leads to average-costs in excess of minimum average costs a. Hence the strategic-inefficiency in production can be measured by:

$$(3.9) \qquad \text{AC} - a = \frac{(\theta - a)^2}{2\theta}$$

We are now in a position to derive an expression giving the total welfare loss in the strategic investment game as a function of θ^*. There are, we recall, two sources of "lost" consumer's surplus in the strategic investment model, as depicted in Figure 1. First, we have the standard welfare-loss given by the "triangle" under the demand curve, area A.

$$(3.10) \qquad \Delta w_1 = \frac{(\theta - a)^2}{2}$$

Second, we have the additional efficiency loss given by area B. This reflects the consumer surplus lost to shareholders due to average costs exceeding their minimum. From (3.9) this area is given by:

$$(3.11) \qquad \Delta w_2 = F(\theta) \cdot \left[\frac{c(s(\theta))}{s(\theta)} - a \right] = \frac{(\bar{p} - \theta) \cdot (\theta - a)^2}{2\theta}$$

Hence the total welfare-loss $\Delta w = \Delta w_1 + \Delta w_2$ is given by:

$$(3.12) \qquad \Delta w = \frac{(\theta - a)^2}{\theta} \cdot \frac{\bar{p}}{2}$$

It is quite clear that the total welfare loss is strictly monotonic for $\theta \geqslant a$. However, whilst the standard welfare-loss triangle increases with θ, the strategic inefficiency Δw_2 is not monotonic. From (3.11) we can see that the strategic inefficiency is equal to zero both when output is zero $(\theta = \bar{p})$, and when AC $= a$ $(\theta = a)$. We could reasonably expect that the welfare loss due to strategic inefficiency might be rather large relative to the standard welfare loss. If we compare (3.10–3), then:

$$\Delta w_2 = -[\Delta w_1 / \varepsilon_d]$$

Thus the consumer surplus lost through strategic inefficiency equals the surplus lost via the triangle divided by the elasticity of demand. For $\varepsilon_d \geqslant -1$, the strategic inefficiency will represent the more important welfare loss. This is not so surprising, since the welfare loss due to strategic inefficiency is the excess cost multiplied by the entire industry output, so that even if the average cost is only slightly in excess of the minimum, it will be considerably magnified. The importance of strategic inefficiency in this model is not peculiar to this specification.

IV. STRATEGIC INVESTMENT AND ENTRY

The main body of this paper has assumed that there is a fixed set of active incumbents in the industry, with no entry or exit. This section provides a simple example of how entry might be modelled, and its welfare implications. It needs to be made quite clear from the outset that this section is more an outline than a detailed formal analysis. The special case considered provides an interesting illustration that is reasonably typical.

There are many ways of treating entry, depending on when entry is allowed.[3] We shall consider the simplest case: we add an extra entry move prior to the

[3] In effect, the treatment presented does not allow for entry after the investment decisions have been taken. This contrasts with models of entry deterrence where entry may occur after the incumbents have made their investment decision (e.g. Dixit [1980]).

strategic stage of the model. Firms incur sunk set-up costs $\mu > 0$. There is an infinite set of identical firms, which make their entry decision in some pre-ordained order. Given the number of firms that decide to enter, a strategic investment equilibrium occurs. No entry occurs after the initial entry move. In evaluating the profits from entry, firms have perfect foresight.

With strictly positive set-up costs and constant returns to scale production, the model is of course the rather special case of natural monopoly (since the industry cost structure is super-additive (see Baumol *et al.* [1982]). Whilst this makes the analysis very simple, the qualitative results are generally valid.

Consider first the social optimum in this industry, which we shall take to be the Ramsey solution, which maximises the consumer surplus subject to the self-financing constraint. The Ramsey solution is the triplet $\{n_r, a_r, k_r\}$, the optimal number of firms, price, and capital stock respectively. Since the industry is a natural monopoly, the analysis is very simple, since we know that $n_r = 1$. Furthermore, it will be socially optimal to produce efficiently, with the output capital ratio v. Since demand is downward sloping, the Ramsey optimal price and capital stock are those which maximise output subject to the finance constraint:

$$(4.1) \qquad \max \quad F(a_r)$$

$$(4.2) \qquad \text{subject to} \qquad k_r \cdot v \cdot (a_r - a) \geqslant \mu$$

$$(4.3) \qquad F(a_r) = k_r \cdot v$$

How does the strategic investment equilibrium with entry compare with the Ramsey optimum? Let us denote this equilibrium by the triplet $\{n^*, K^*, \theta^*\}$, where K^* is the industry capital stock, and $\theta = \theta(K^*)$ (since firms are identical, the equilibrium is symmetric). These values are determined by the equilibrium condition (2.7) between the active incumbents, plus the entry condition. Both of these conditions can be summarised using the "outcome function" $\Omega(n)$, giving profits net of entry costs when there are n firms in the industry. For a broad class of standard models Ω is a uniquely defined, strictly decreasing function of n, and from (2.7) $\Omega \to 0$ as $n \to \infty$. The number of firms n^* is determined by $\Omega(n^*) \geqslant \mu \geqslant \Omega(n^*+1)$. This will in turn define the equilibrium capital stock in the industry.

Given this skeletal outline, we have enough information to compare the strategic investment equilibrium with the Ramsey optimum. There are four points of contrast which are of interest:

(i) *Excess Entry*: $n^* \geqslant n_r$. Unless entry costs are large, $n^* > n_r$.

To see why, note that if $\Omega(1) > \mu$, there will be more than one firm. Entry in excess of the social optimum will occur unless entry costs are large, where "large" means that $\mu > \Omega(2)$.

(ii) *Inefficiency in Production*: We know from the analysis in section II that the method of producing output will be inefficient, being under-capitalised. This is unchanged by entry, since $\theta^* > a$ for all finite n.

(iii) *Excessive Price*: $\theta^* > a_r$.

This must be so if the incumbents are to cover their entry costs μ. From (i), fixed costs in the industry $n^* \cdot \mu$ are at least as great as in the Ramsey case. But for $\theta > a$, production is inefficient by (ii). For any given price, therefore, industry profits are less than in the efficient Ramsey case (see (4.2)). Hence the equilibrium price θ^* must exceed the Ramsey price.

(iv) *Underinvestment*: $K^* < K_r$. There is less investment in the industry than in the Ramsey case.

This follows from (ii) and (iii). Since $\theta^* > a_r$, industry output is less than optimal. But $s(\theta^*) > v$ from (ii), so that $K^* < K_r$.

In the strategic investment model with entry, there are too many firms which underinvest and are inefficient. In essence there are too many firms producing too little output with too little capital. As should be quite clear from the analysis, the conclusions (i)–(iv) will apply beyond the case of fixed set-up costs plus constant returns technology. However, the case of natural monopoly is rather special.

What happens if firms have the more standard U-shaped long run average cost curves? These can be seen as arising from a diminishing returns to scale production function with strictly positive fixed or sunk costs. All comparisons

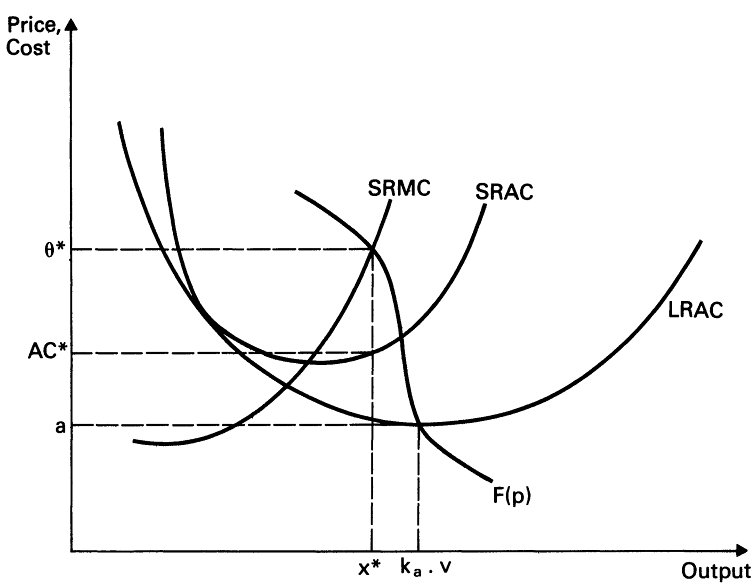

FIGURE 2
Chamberlinian Excess Capacity

(i)–(iii) will still be directly valid. The only important modification concerns comparison (iv). If we ignore the integer problem, in the Ramsey solution each firm will produce at the bottom of its "long run" average cost curve. This implies a certain capital stock for each firm, denoted k_a. However, in the strategic investment equilibrium the zero profit entry condition will mean that firms will produce at below the least cost capacity, as depicted in Figure 2, which depicts a symmetric equilibrium as in Figure 1, except that the LRAC is U-shaped. Thus we can extend (iv) to include the standard "Chamberlinian" undercapacity result. Not only are firms off their LRAC as in the constant returns case, but the output is below the minimum economic scale ($k^* < k_a$). With the introduction of a U-shaped cost function, we have a rather paradoxical result. Given the capital stocks chosen in equilibrium, there is overutilisation of capacity, an excessive output-capital ratio. This is due to strategic inefficiency. The entry condition, however, implies that the capital stock is too small, being less than the "capacity" level k_a.

V. CONCLUSION

There may still be a relationship between concentration and welfare loss even in an industry where conditions in the product market are competitive. Firms can employ such variables as investment to influence the market outcome, and their ability to do so is directly related to their "market share", interpreted as the firm's share in the industry capital stock. This leads to a price which is higher than the price that occurs when investment is not employed strategically, and there is a corresponding welfare loss due to the resultant restriction of output. The strategic use of investment in this way also gives rise to an additional inefficiency in production, since the technology will be too labour intensive. In the simple linear model presented in section III, the welfare loss due to this strategic inefficiency may be larger than the loss due to the price being too high. This suggests that the evaluation of monopoly power needs to go beyond a simple analysis of the conditions in the product market and the actual cost structure of firms, and to take into account such strategic behaviour and the distortions to which it gives rise.

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APPENDIX

Proof of Proposition 1

We show that U_i is strictly quasi-concave in k_i. A necessary and sufficient condition for quasi-concavity is that *if* $U_{ki} = 0$ then the second partial U_{kki} is non-positive. Now,

$$U_{ki} = \pi(\theta) + k_i \cdot s(\theta) \cdot \frac{\partial \theta}{\partial k_i} = \pi(\theta) - \frac{1}{\varepsilon_s - \varepsilon_d}(\theta \cdot s(\theta) \cdot \sigma)$$

where

$$\sigma = \frac{K_i}{\Sigma k_j} \qquad (j = 1 \ldots n),$$

σ being the i^{th} firm's capital share. If $U_{ki} = 0$, then

$$\frac{\pi(\theta)}{\theta \cdot s(\theta)} = \frac{\sigma}{\varepsilon_s - \varepsilon_d}$$

If we evaluate U_{kki} when $U_{ki} = 0$ we find after some manipulation that:

$$\left.\frac{\partial U_i^2}{\partial k_i^2}\right|_{U_{ki}=0} = -\frac{1}{\varepsilon_s - \varepsilon_d} \cdot \frac{\pi(\theta)}{\Sigma k_j} \cdot \left[\varepsilon_s \cdot 2 \cdot \frac{(1-\sigma)}{\sigma} - \varepsilon_d \cdot \left(\frac{2-\sigma}{\sigma}\right) - 1\right]$$

This will be non-positive if

$$\frac{1}{\sigma} \geqslant \frac{1 - \varepsilon_d + 2 \cdot \varepsilon_s}{2 \cdot (\varepsilon_s - \varepsilon_d)}$$

But $\sigma \leqslant 1 - \phi$ (for profits to be positive).