Introductory Mathematics for Economics MSc's: Course Outline.

Huw David Dixon

Cardiff Business School.

September 2008.

The course will consist of five 2 hour Lectures commencing on . There will also be 4 Tutorials, during which you can work through worksheets that will be given to you.

Reading.

MAIN TEXT : Dowling E.T. 'Introduction to Mathematical Economics' 2nd edition or 3rd edition, Schaum's Outline Series, McGraw-Hill.

References are to this text unless stated otherwise.
Note that the references are specified for the different editions when there is a difference between the two editions.

SUBSIDIARY TEXT: Chiang A. 'Fundamental Methods of Mathematical Economics' McGraw-Hill (any edition, multiple copies in restricted section of library).

Lecture 1: Differentiation and Lecture 2: Optimisation and specific functions.

1. **Concept and Rules** (Dowling ch.3)

   Limits (example 1), continuity, slope, the derivative, notation, (pp 39-43, 2nd ed.; pp 32-36, 3rd ed.).
   Rules of differentiation (pp 44-47, 2nd ed.; pp 37-40, 3rd ed.).

   Solved Problems (pp 47-64, 2nd ed.; pp41-57, 3rd ed.), especially 3.7-3.19 (problem (a) for each).

2. **Uses of the Derivative** (Dowling ch.4)

   Increasing and decreasing functions, concave and convex functions, extrema and points of inflexion, optimisation, the margin, profit maximisation, price elasticity (pp 65-74, 2nd ed.; pp58-64, 3rd ed.).

   Solved Problems
   (2nd ed., pp 74-99), especially 4.9, 4.11, 4.13-4.15, 4.20-4.23.
   (3rd ed., pp 64-81), especially 4.8, 4.10, 4.12-4.14, 4.19-4.22.

3. Exponential Function. Dowling, chapters 7-8.

Lecture 3: Multi-variable functions and Constrained Optimisation.

Functions of More than One Variable (Dowling ch.5)

Partial derivatives, rules, optimisation, constrained optimisation and Lagrange multiplers, total and partial differentials, total derivatives, implicit functions, (pp 100-109, 2^{nd} ed.; pp82-92, 3^{rd} ed.).

Solved Problems (pp 110-126, 2^{nd} ed.; pp 93-109, 3^{rd} ed.), especially 5.1, 5.2a, 5.4a, 5.10a, 5.12a, 5.13, 5.18a, 5.20

Uses of Partial Derivatives (Dowling ch.6)

Marginal productivity, income determination, partial elasticities, differentials, optimisation and constrained optimisation, production functions
(pp 127-135, 2^{nd} ed.; pp 110-119, 3^{rd} ed.).

Solved Problems
(2^{nd} ed., pp 136-161), especially 6.1a, 6.4, 6.18, 6.22, 6.28, 6.38, 6.44a, 6.45.
(3^{rd} ed., pp 119-145), especially 6.1a, 6.4, 6.18, 6.22, 6.28, 6.38, 6.40a, 6.41.

Lecture 4: Linear Algebra and Simultaneous Equations.

Simultaneous Models (Dowling ch.2)
(Chiang ch2, ch3)

Isocost and budget lines, supply and demand analysis, income determination models ISLM models. (Substitution and elimination methods).

Solved Problems
(2^{nd} ed.; pp 30 – 38, although pp24 - 30 on graphs may be useful),
especially problems 2.11a, 2.12 - 2.16, 2.23, 2.24.
(3^{rd} ed.; pp 23 – 31, although pp17 - 23 on graphs may be useful),
especially problems 2.11a, 2.12 - 2.16, 2.23, 2.24.

Introduction to Matrices (Dowling ch.10)

Especially basics (10.1, 10.2), matrix multiplication (10.6), system of equations Operations (Gaussian elimination or Gaussian reduction), (10.9 - 10.12; 2^{nd} ed.; 10.9, 3^{rd} ed.); (Chiang ch. 4.1, 4.2, 4.6).

Solved Problems
(2^{nd} ed.; pp 224 - 243), especially 10.19, 10.20, 10.52, 10.53, 10.54, 10.58.
(3^{rd} ed.; pp 207 - 223), especially 10.19, 10.20.

Matrix Inversion (Dowling ch.11)

2 x 2 determinants (11.1, p 244, 2^{nd} ed.; 11.1, p224, 3^{rd} ed.)

2 x 2 matrix inversion (11.31a,b,c, p 266, 2nd ed.; 11.26a,b,c, p242, 3rd ed.)

2 equation systems (11.32 a, b, pp 267-268, 2nd ed.; 11.27a,b, p244. 3rd ed.)

Lecture 5: Difference Equations.

First Order Difference Equations (Dowling, 2nd ed.,ch.19) (Dowling, 3rd ed.,ch.17)

Concepts, solutions, stability, lagged income determination model, cobweb model, Harrod growth model.
(Chiang ch 16).

Solved Problems
(2nd ed., pp 423-430), especially 19.14-19.21, 19.26, 19.29.
(3rd ed., pp 398-407), especially 17.14-17.21, 17.26, 17.29.

2. Second Order Difference Equations (Dowling, 2nd ed., ch.20) (Dowling, 3rd ed., ch.18)

[This chapter deals with both differential and difference equations, sometimes in the same section, sometimes in different sections. It is thus important when reading, to take care that the material relates to difference equations.]

(Not 20.1, 2nd ed.; not 18.1, 3rd ed.).

Second order difference equations (20.2, 2nd ed.; 18.2, 3rd ed.).

Characteristic roots (20.3, 2nd ed.; 18.3, 3rd ed.),
(This covers both topics together, see example 9, but only the part relating to example 5 and not to example1!),

Conjugate complex numbers (20.4, 2nd ed.; 18.4, 3rd ed.),
(This covers both topics but not equation 20.20, 2nd ed.; (18.20, 3rd ed.!)).

(Sections 20.5-20.7, 2nd ed.; (18.5-18.7, 3rd ed.), can be avoided).

Stability conditions 20.8, 2nd ed.; (18.8, 3rd ed.).

Solved Problems
(2nd ed.) 20.36 and 20.37 only.
(3rd ed.) 18.36 and 18.37 only.

(Chiang ch 17 (intro., 17.1, 17.2, and in 3rd edition 17.3 (not in 2nd edition)).

Introductory Maths for Economics and Related MScs

Problem Set 2.

1. Find the roots of the following quadratic equations. State the results to four decimal places (you will need to use a calculator).

(a) $-2x^2+5x+5$ (b) $3x^2+3x-2$ (c) $-2x^2+4x-2$

2. For each of the three quadratic functions, find the value of *x* such that the first derivative is zero, and state whether this is a maximum or a minimum. (*optional*: use a spreadsheet to graph the functions).

3. A houseold has the utility function ($0<a<1$)

$$U(x,y)=a\ln x+(1-a)\ln y$$

Setting the price of y equal to 1 (numeraire), so that *p* is the relative price of *x* in terms of *y*, we can write the budget constraint as

$$px+y=I$$

Where *I* is income. The household seeks to maximize utility subject to the budget constraint.

(a) write down the Lagrangean.

(d) derive the first order conditions.

(c) State the optimal values of *(x,y)* for a given income and price *(I,p)*.

(d) Show that the share of *x* in total expenditure is a.

4. A firm has a production function $Y=K^{0.3}L^{0.7}$. The cost of capital is *r* and the cost of labour is *w*. The firm wants to produce 10 units of output as cheaply as possible. What is the best choice of *(K,L)* to achieve this?

5 (*optional: for those who are more advanced students*). Do question 4 (a)-(c), but with the utility function

$$U(x, y) = ax^{0.5} + (1 - a)y^{0.5}$$

(d) what is the elasticity of demand for this utility function?

Introductory Maths for Economics and Related MScs

Problem Set 3.

1. Solve the following simultaneous equations for (x,y):

(a) $$\begin{aligned} 3x - 2y &= 1 \\ 2 - x + y &= 0 \end{aligned}$$

(b) $$\begin{aligned} 9x - 2y + 1 &= 0 \\ 3y + 2x &= 4 \end{aligned}$$

(c) $$\begin{aligned} 5y - x + 2 &= 0 \\ x &= y + 2 \end{aligned}$$

2. Solve the following simultaneous equation system for (x,y,w)

$$6\mathbf{x} + 10\mathbf{y} + 4\mathbf{w} = 65 \qquad (1)$$
$$36\mathbf{x} + 40\mathbf{y} - 16\mathbf{w} = 40 \qquad (2)$$
$$-24\mathbf{x} - 11\mathbf{y} + 16\mathbf{w} = 59 \qquad (3)$$

(Hint: *either* use the formula for a 3x3 determinant and apply cramer's rule, *or* substitute out for w first and then reduce it to a 2x2 system in (x,y) and solve by cramer's rule).

3. Consider the IS/LM system: exogenous variables are (g,M) and endogenous variables are *(Y,r)*.

$$Y = C(Y) + I(r) + g$$
$$M^s = L(r, Y)$$
$$1 > C' > 0 > I'; L_Y > 0 > L_r$$

(a) What is the effect of an increase in the money supply on (Y,r).

(b) What happens if I'=0. What happens if L_y is very large.

(Hint: look at lecture 4 pages 13-14 and follow same steps, but set dg=0 instead of dM).

4. Suppose that the demand and supply of umbrellas depends on rainfall r.

$$x = 1 - p + 0.5r$$

$$x = 0.25p - 0.01r$$

Where x is the quantity of umbrellas, *p* their price and *r* the rainfall. The first equation is demand, the second supply. In equilibrium the price equates supply and demand for a given level of rainfall (the endogenous variables are *x*,*p*, the exogenous variable is *r*.

Find the effect of a change in rainfall on the price and quantity of umbrellas.

Introductory Maths for Economics and Related MScs

Problem Set 4.

1. Consider the following first order difference equations.

$$y_t = 3 + 0.7 y_{t-1}$$
$$3y_t + 5y_{t-1} + 2 = 0$$
$$y_{t+1} - 2y_t - 1 = 0$$

(a) What is the equilibrium output for each equation?

(b) Is the equilibrium globally stable?

(c) What happens if we start from $y_0 = 1$ (i.e. state the general solution and solve for the arbitrary constant).

(d) Using a spreadsheet, map out the three difference equations over the first 10 periods (or more).

2. Consider the following Second order difference equations.

$$y_{t+1} - 4y_t + y_{t-1} - 2 = 0$$
$$4y_{t+1} - 2y_{t-1} = 2$$
$$y_t = 0.5y_{t-1} - 0.25y_{t-2} + 0.1$$

(a) Calculate the equilibrium value of y* for each equation.

(b) In each case, find out if the equations have two, one or no real roots.

(c) Calculate the values of the real roots for each equation.

(d) Are any of the equilibria globally stable?

(e) Using a spreadsheet, plot out the difference equations when the initial values are $y_0 = 0; y_1 = 1$

3. Consider the cobweb model of the market for potatoes given below:

$Q_{dt} = 400 - 4P_t$
$Q_{st} = -90 + 3P_{t-1}$
$P_o = 220$

(a) Find the Solution giving the time path of the price of potatoes.

(b)Comment on the stability of the model in both the short and the long run.

(c) The government introduces a subsidy on the production of potatoes that changes the slope of the supply function so that the supply function becomes:

$Q_{st} = -90 + 6P_{t-1}$

Determine whether the behaviour of the market changes.

4. Suppose that the supply of potatoes depends on the average price in the preceding two periods.

$$Q^d{}_t = 400 - 4P_t$$

$$Q^s{}_t = -90 + 3\left(\frac{P_{t-1} + P_{t-2}}{2}\right)$$

(a) Show that the resultant cobweb model is a second order difference equation in prices.

(b) what is the equilibrium output and price?

(c) what are the roots of the equation, and what do they tell us about the stability?

MATHEMATICS FOR ECONOMISTS
TEST

FRIDAY 5^{th} OCTOBER 2007

The test has a total of 100 marks. Answer/Attempt as many of the five questions (or parts thereof) as you can in 1 hour. There are 5 marks for each part of each question.

There is no formal pass mark. The test is diagnostic.

Good Luck ☺

Question 1 *(20 Marks)*

A houseold has the utility function ($0 < a < 1$)

$$U(x, y) = x^a y^{(1-a)}$$

Setting the price of y equal to 1 (numeraire), so that *p* is the relative price of *x* in terms of *y*, we can write the budget constraint as $px + y = I$, where *I* is income. The household seeks to maximize utility subject to the budget constraint.

(a) write down the Lagrangean.

(d) derive the first order conditions.

(c) State the optimal values of *(x,y)* for a given income and price *(I,p)*.

(d) Show that the share of *x* in total expenditure is a.

Question 2. *(20 Marks)*

A firm has a production function $Y = K^{0.4} L^{0.6}$. The cost of capital is *r* and the cost of labour is *w*. The firm wants to produce *Y=20* units of output as cheaply as possible.

(a) write down the Lagrangean.

(d) derive the first order conditions.

(c) State the optimal values of *(K,L)* for given factor prices *(r,w)*.

(d) solve for the actual values of *(K,L)* and total cost when *r=2, w=4.* How much would it cost to produce 30 units at these prices?

Question 3. *(20 Marks).*

A firm has the demand curve $P = 10 - x$. and the (total) cost function $C = \frac{x^2}{2}$.

(a) What is the profit of the firm as a function of its output *x*?

(b) Show that profits a strictly concave function of output.

(c) Find the level of output and the price which maximizes profits.

(d) At what levels of output would the firm earn zero profits?

Question 4. *(15 Marks)*

Consider the IS/LM system. *G* is government expenditure, *T* is a lump-sum tax, *M* is the money supply. The exogenous variables are (*G,M,T*) and endogenous variables are *(Y,r)*, output and the interest rate.

$$Y = C(Y - T) + I(r) + G$$

$$M^s = L(r, Y)$$

$$1 > C' > 0 > I'; L_Y > 0 > L_r$$

(a) What is the effect of an increase in government expenditure on (*Y,r*)?

(b) What is the effect of an increase in taxation on output and the interest rate?

(c) Suppose that the government follows a balanced budget policy, so that *G=T.* What is the effect of an increase in *G* in this case? (hint: simply substitute G for T in the first equation.)

Question 5. *(25 Marks)*

Consumption C_t follows the following difference equation:

$$C_{t+1} = \frac{1}{(1+r)} C_t + g$$

Where ?>0 is a constant and *r>0* is the interest rate.

(a) What is the equilibrium level of consumption?

(b) Solve the difference equation when the initial value of consumption is given C_0 >0.

(c) Is the equilibrium stable?

Now suppose that consumption is given by the second order difference equation

$$C_{t+1} = \frac{1}{(1+r)} C_t - \frac{1}{4}\left(\frac{1}{1+r}\right)^2 C_{t-1} + g$$

where the interest rate is $r = 0.05$ and $g = 1$

(d) What is the equilibrium level of consumption?

(e) Calculate the roots of the difference equation. Is the equilibrium stable? What can you say about the time path of consumption?

You can use a calculator for question 5, but it is not necessary.

The End.