The micro foundations of banks..

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1 Financial Intermediation.

- Banks are a special kind of FI

- FI: "Intermediation" - buy and sell financial assets at the same time. Brokers, dealers.

- But: Banks different
  - They offer contracts (for loans and deposits) which are not annonymous and are not easily mrketable.
  - Non-Bank-FIs: often deal in annonymous securites (stocks and bonds) which are easily tradeable.

- The characteristics of the contracts (length, risk etc) differ between fims (borrow from bank) and public (depositors).

- With perfect markets, no transaction costs, firms and households could trade and obtain optimal risk sharing and allocation across time (Arrow-Debreu).

- Banks and FIs can be seen as "coalitions" of depositors who get around non-convexities in technology:

  - economies of scale: need barnch networks, buildings, train staff etc.

  - economies of scope: more efficient to have the same institution doing more than one thing: can use the same staff, can use information from one activity to help another etc..

- Also, provide "informational coalitions", monitoring etc.

2 Liquidity Insurance: Diamond-Dybvig.

- Simpler Version than in book (chapter 2.21).

- Three periods:

  - Period 0: Each farmer has one unit endowment. Has to decide how much to invest I.

  - Period 1: the farmer may consume the quantity not invested.

- Period 2: the farmer consumes the output resulting from then investment.

- Technolgy: $Y_2 = (1 + R) .I : R > 0$.

- Preferences: "liquidity shock".
  - probability π farmer only get utility from period 2 consumption and obtains utility $u(C_1)$ where $C_1 = 1 - I$
  - Probability $(1 - \pi)$ the the farmer only gets utility in period 3 and gets utility $u(C_2) = (1 + R) I$.
  - In period 0, ex ante lifetime utility is

$$U = \pi u(C_1) + (1 - \pi) \rho u(C_2)$$

where $\rho < 1$ and we assume that the technology is "productive" so that $\rho(1 + R) > 1$.

- Note:
  - farmer only gets utility from consumption in one period: either 1 or 2.
  - These are "state contingent" preferences: different from N-M (although look like it if $\rho = 1$).

2.1 Autarky.

- Suppose there is only one farmer: Robinson Crusoe on his Island before man friday arrived!

- In period 0 the farmer solves the following max

$$\max_{I} \pi u(1-I) + (1-\pi)\,\rho u\,(I\,(1+R))$$

- FOC:

$$\begin{aligned} -\pi u_1' + (1-\pi)\,\rho u_2'(1+R) &= 0 \\ \frac{\pi u_1'}{(1-\pi)\,\rho u_2'} &= (1+R) \end{aligned}$$

- Where $\frac{-\pi u_1'}{(1-\pi)\rho u_2'} = MRS$ since

$$\begin{aligned} du &= \pi u_1' dC_1 + (1-\pi)\,\rho u_2' dC_2 \\ MRS &= -\left.\frac{dC_2}{dC_1}\right|_u \\ &= \frac{\pi u_1'}{(1-\pi)\,\rho u_2'} \end{aligned}$$

- This is a tangency condition: the farmer faces a technological trade-off between consuming now and later.

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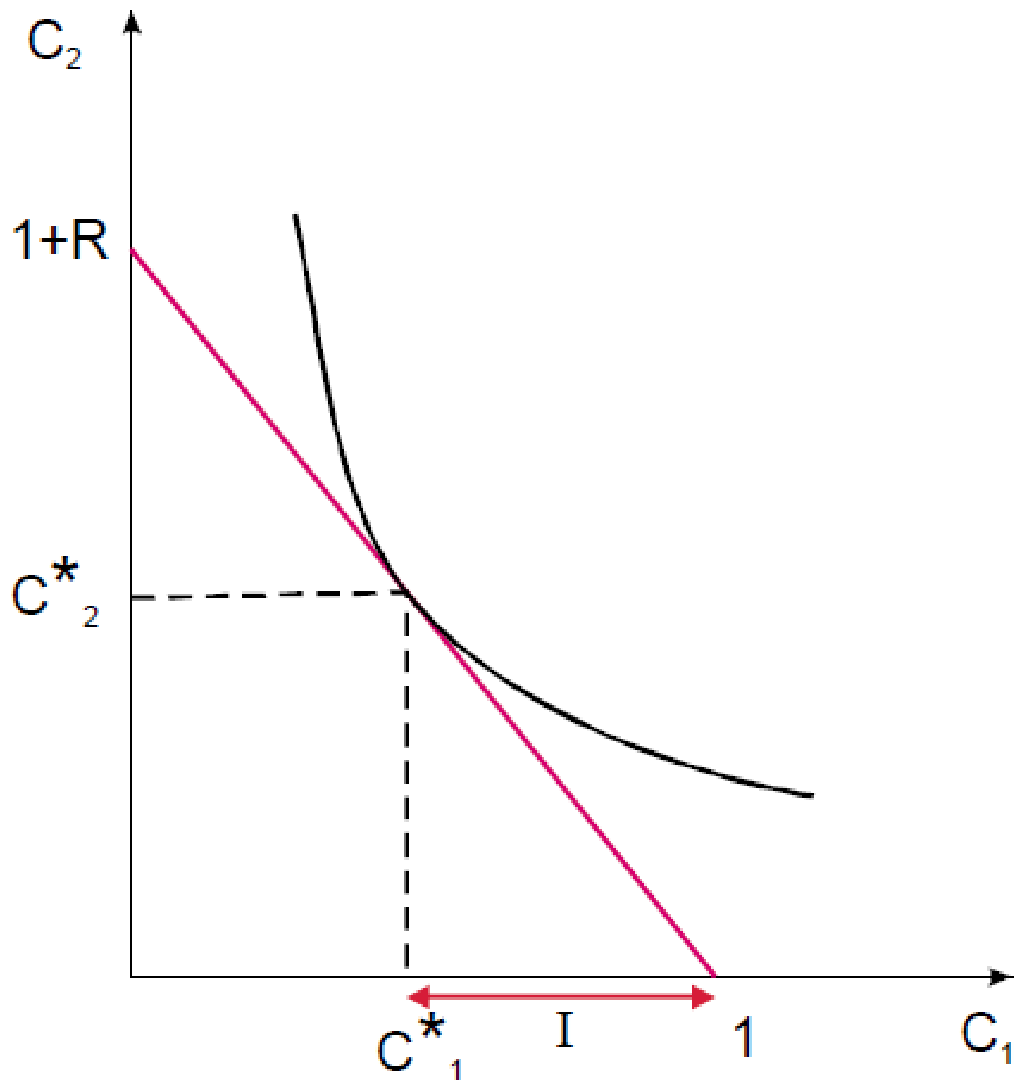
$$\begin{aligned} C_1^A &= 1 - I \qquad (1) \\ C_2^A &= I\,(1+R) \end{aligned}$$



Autarchy.

2.2 Banks 1: provide insurance (pool of piquidity).

- There are lots of farmers. The farmers deposit all their endowment in the bank in period 0.

- The Bank makes loans of size 1 to a proportion $1 - \pi$ of farmers (or just gives the whole lot to one farmer, since constan returns to scale). The loan requires $1 + R_L$ is payed back.

- Bank offers following deposit account:

  – If you withdraw at period 1 you get no interest in period 2.

  – If you leave your money in, you get interest of R_D.

- Profits of Bank:
  - period 1: π will withdraw cash in period 1 The bank has suffient reserves to pay them. No profit.
  - Period 2: $(1-\pi)$ withdraw cash in period 2 and the bank pays receives its loan interest and pays out the cash plus interest to depositors. profit is equal to

$$(1-\pi)(R_L - R_D)$$

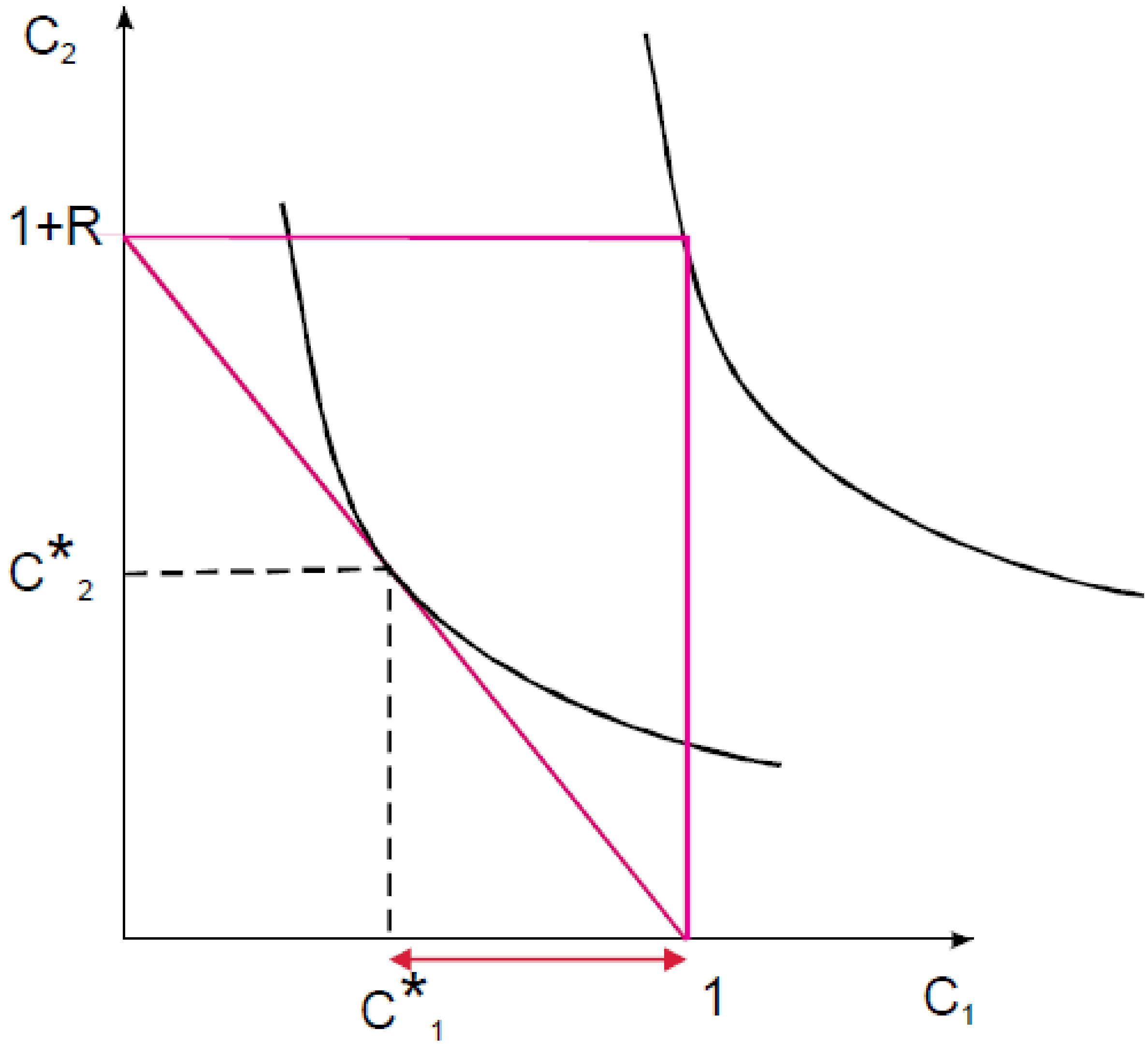
  - Zero profits? Assume $R = R_D = R_L$

- What is the consumption of each farmer?
  - Period 1: $C_1^B = 1$. Put endowment of money in and take it all out in period 1 .

– Period 2: $C_2^B = (1 + R)$. Take it all out plus interest in period 2.

– Utility: much much better! Without Bank each farmer's investment was "wasted" if they had to consume in period 1. If he had to consume in period 2, the portion not invested was "wasted". Inefficient.

$$\begin{aligned} C_1^B - C_1^A &= I \\ C_2^B - C_2^A &= (1 - I)(1 + R) \end{aligned}$$



Simple Bank

- This is also the "market" outcome: set up a market in period 0. Bond B that pays out 1 in period 2. Agents allocate endowment between ivestment and Bond with price p (in period 1).
  - $1 - I = pB$

    $C_1 = 1 - I + pI(1 + R)$: can $I(1 + R)$ bonds at price p.

    $C_2 = (1 + R)I + \frac{1-I}{p}$: gets payoff from bonds.

- Arbitrage: bonds and investment yield same outcome:
  - $(1 + R) = \frac{1}{p} \Rightarrow p = \frac{1}{1+R}$

- In equilibrium, agents idifferent between bonds and investment: market clearing ensures that I is at the optimal level

- Why bank and not bonds: who issues the bonds? In a repeated game, banks can use reputation etc to enforce loan repayments. This happens: microbanks and microfinance amongst farmers (Gramin banks). Bonds only work if perfect frictionless market (e.g. bonds divisible...).

2.3 Compare Autarky with Bank.

- Farmers much better off! In fact the bank can make a profit and charge different loan and deposit rates. Suppose the bank pays zero on deposits $R_D = 0$ and $R_L = R$.

- Banks profits in period 2 are now

$$(1 - \pi) R$$

- $C_2 = I : C_1 = 1$. Farmer can still be better of, since in period 1 can liquidate investment! This may compensate for lower consumption in period 2. From social welfare, the outcome is efficient, since banks shareholders benefit (investment and output the same).

- Banks provide pools of liquidity and "insure" against idiosyncratic shocks. Can model these differently (income or productivity shocks). In book allows for storage (can store good).

2.4 The Social Optimum: Banks can do even better!

- The previous section equilibrium not *ex ante* pareto optimal.

- Take the farmer in period 0. He might need to consume in period 1 or 2. Now, with the previous "market" equilibrium", he is worse off if he is a period 1 type person than if he is a period 2 type person. If $\rho < 1$, for a given lifetime income, might well want $C_1 > C_2$, but in banking solution $C_2 > C_1$.

- Given that feasability means that $I = 1 - \pi$:

$$\max_{C_1, C_2} \pi u(C_1) + (1-\pi)\rho u(C_2)$$
$$st.1 = \pi C_1 + (1-\pi)\frac{C_2}{(1+R)}$$

  – Lifetime budget constraint.

– FOC:

$$\frac{u'\left(C_1^*\right)}{\rho u'\left(C_2^*\right)} = (1+R)$$

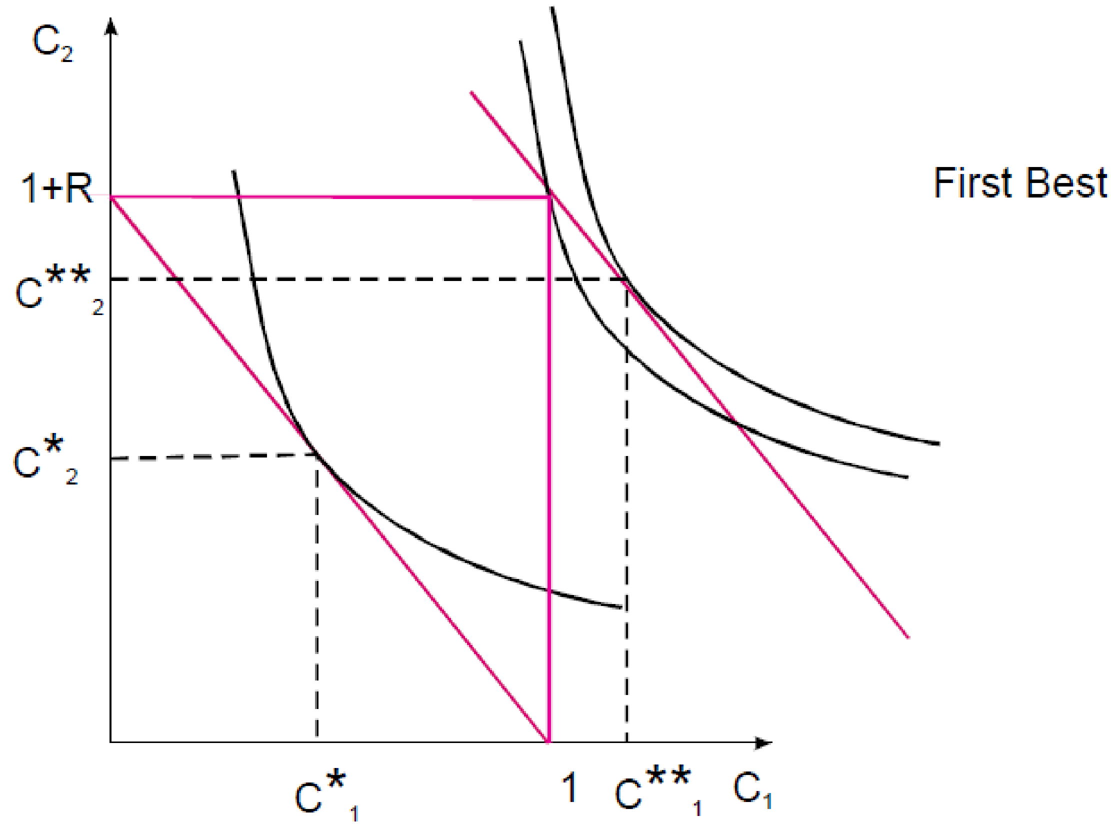
– Same optimality condition as in Autarky, but different budget constraint (no waste): same slope but further out!

- Only chance that $C_1 = 1, C_2 = (1+R)$ is optimal. If $\rho(1+R) \simeq 1$, then certainly want $C_1 \simeq C_2$, and market outcome not first best:

$$1 + R > C_2^* > C_1^* > 1$$

- Intuition: you are worse off if period 1 consumer: hence will probably want to increase period one consumption and reduce period two consumption.

Diamond and Dybvig: if $Cu'(C)$ decreasing in C, then certainly want this!



- So, banks offer contracts of the following form:
  - if you withdraw in period 1 you get interest R_{1D}.
  - if you withdraw in period 2 you get interest R_{2D}
  - $R_{1D} : \pi (1 + R_{1D}) = C_1^*$
  - $R_{2D} : (1 - \pi)(1 + R_{2D}) = C_2^*$
  - Note: $R > R_{2D} > R_{1D} > 0$.
  - Loan rates: $R_L = R$: farmers who invest make no profit.
  - Banks earn zero profits (households get everything).
- Problem of enforcement: if a financial system exists (bonds), then period

two consumers will pull out and buy bonds rahter than invest in bank deposits.

3 Information Sharing (Leyland and Pyke JE 1977).

- Risk averse insiders want to signal quality by retaining equity.
- However, this is inefficient.
- If coalitions of borrowers can form a partnership, the cost to them of signalling rises at a slower rate than the cost of signalling.

3.1 The model.

- ☐ Large number of entrepreneurs.
- ☐ Each has a risky project requiring investment of 1.
- ☐ The net return is random: $\tilde{R}(\theta)$ is distributed normally, $N(\theta, \sigma^2)$.
- ☐ θ is known only to entrepreneur (Adverse selection).

- Distribution of θ is known.
- Investors are risk neutral.
- Entrepreneurs have wealth $W_0 > 1$ but are risk averse.
- Their preferences are: $u(w) = -e^{-\rho w}$.
- $\rho > 0$ is the (constant) absolute index of risk aversion ($-u''(w)/u'(w) = \rho$).

3.1.1 Full information equilibrium.

- ☐ If θ is observable then each entrepreneur will sell project for $p(\theta) = E[\tilde{R}(\theta)] = \theta$.
- ☐ The terminal wealth of type-θ entrepreneur will be $W_0 + \theta$.

Under Full information, each project is valued at expected value: wealth of entrepreneur is equal to initial plus expected value of project.

3.2 Private information.

- Suppose θ is observable only to entrepreneur.
- Price of projects will be the same for all projects: P.
- FACT: If $\tilde{x}$ is normally distributed then:

$$E[-e^{-\rho\tilde{x}}] = -\exp[-\rho(E\tilde{x} - \frac{1}{2}\text{var}(\tilde{x}))].$$

- If self financed, the entrepreneur receives:

$$Eu(W_0 + \tilde{R}(\theta)) = u(W_0 + \theta - \frac{1}{2}\rho\sigma^2).$$

- Utility after selling project is $u(W_0 + P)$.
- The project will be sold only if the utility from selling it is higher than from not

$$W_0 + \theta - \frac{1}{2}\rho\sigma^2 < W_0 + P.$$

Or: if $\theta < \hat{\theta} = P + \frac{1}{2}\rho\sigma^2$.

- □ This is the lemons problem – only bad projects are sold.
- □ In equilibrium the average expected return on equity is P.

$$P = E[\theta \mid \theta < \hat{\theta}].$$

- □ Equilibrium will be a P and a cut-off $\hat{\theta}$.

- Assume θ takes on low value θ_1 with prob. π_1 and high value θ_2 with prob. π_2.
- Since entrepreneurs are risk averse, an efficient allocation implies $\hat{\theta}_2 > \theta_2$.
- In this case, $P = E(\theta) = \pi_1\theta_1 + \pi_2\theta_2$.

- This is possible when: $\pi_1\theta_1 + \pi_2\theta_2 + \frac{1}{2}\rho\sigma^2 \geq \theta_2$.
- Or: $\pi_1(\theta_2 - \theta_1) \leq \frac{1}{2}\rho\sigma^2$.
- Or the risk premium ($\frac{1}{2}\rho\sigma^2$) has to outweigh adverse-selection effect.

3.3 Signalling through self financing.

- High quality entrepreneurs can signal quality by self-financing.

- They keep α of project, and sell off $(1 - \alpha)$. Not first-best, since they keep some of the risk (first best is $\alpha = 0$).

- This is a costyl signal, but crucially, the cost of the signal is lower for the good than the bad.

- The no-mimicking constraint is:

$$u(W_0 + \theta_1) \geq Eu(W_0 + (1-\alpha)\theta_2 + \alpha \tilde{R}(\theta_1)).$$

- The r.h.s. is:

$$u(W_0 + (1-\alpha)\theta_2 + \alpha\theta_1 - \frac{1}{2}\rho\alpha^2\sigma^2).$$

- The conditions simplifies to:

$$\frac{\alpha^2}{1-\alpha} \geq \frac{(\theta_2 - \theta_1)}{\rho\sigma^2}.$$

- ☐ There are a continuum of equilibria characterized by α where only the low quality projects are sold in entirety.
- ☐ The equilibria can be Pareto ranked.
- ☐ The low quality entrepreneurs get utility as full information case.
- ☐ The high quality entrepreneurs receive: $u(W_0 + \theta_2 - \frac{1}{2}\rho\alpha^2\sigma^2)$.
- ☐ The loss to them: $C = \frac{1}{2}\rho\alpha^2\sigma^2$ is increasing in α.
- ☐ The Pareto-dominant equilibrium is where the no-mimicking constraint holds with equality.

$$\frac{\alpha^2}{1-\alpha} = \frac{2(\theta_2 - \theta_1)}{\rho\sigma^2}.$$

- ☐ Look at r.h.s. as σ increases.
- ☐ The derivative of r.h.s. is:

$$\frac{-4\rho\sigma(\theta_2 - \theta_1)}{(\rho\sigma^2)^2} < 0.$$

- ☐ Thus, the l.h.s. must be decreasing in σ.

- ☐ Differentiate the l.h.s. with respect to α.
- ☐ The derivative is:

$$\frac{2\alpha(1-\alpha) - \alpha^2(-1)}{(1-\alpha)^2} = \frac{\alpha(2-\alpha)}{(1-\alpha)^2} > 0.$$

- ☐ The only way that this can be reconciled is if $\alpha'(\sigma) < 0$.

☐ Look at Equation 6. Rearrange it to get:

$$\frac{1}{2}\rho\sigma^2\alpha^2 = (\theta_2 - \theta_1)(1 - \alpha(\sigma)). \tag{7}$$

- ☐ The l.h.s. of (7) is $C(\sigma)$.
- ☐ Thus to calculate $C'(\sigma)$ differentiate the r.h.s. of (7).
- ☐ This is:

$$((\theta_2 - \theta_1)(1 - \alpha'(\sigma)) > 0.$$

- ☐ The intuition is that if σ increases, then the cost of sending the signal increases, so α decreases as well.
- ☐ By increased diversification, as n increases, the cost of signalling decreases.

- Collateral: by putting up some of their own money, high quality entrepreneurs take on more risk. But, it is less costly for them to do this than it is for the low quality guys. So, get "seperating" equilibrium.

- Self-financing has a welfare cost: so the Pareto optimal is to have as little as possible.

- Banks: pool risk. By getting together, the variabce decreases, so thta the cost of sinalling goes down.

4 Moral Hazard.

- The Behaviour of the borrower is influenced by the terms of the loan.

- Lender's return not monotonic in the rate of interest.

- Two projects $i = A, B$: Two outcomes (success and failure)
  - probability of success p_i, payoff Ri if success, 0 otherwise.
  - $p_a > p_b$; $R_b > R_a > 1$; $p_a R_a > p_b R_b$
  - A is the "safer" project, yields highest expected return.

- But, *if borrower only pays back if succesful, may choose risky project with lower expected return*!

- Loan structure as before $\{L, r, C\}$: returns from A and B for borrower (entrepreneur)

$$
\begin{aligned}
E\pi^A &= p_a[R_a - (1+r)\,L] - (1-p_a)\,C \\
E\pi^B &= p_b[R_b - (1+r)\,L] - (1-p_b)\,C
\end{aligned}
$$

- Now, returns are equated at the level $\bar{r}$

$$\frac{p_a R_a - p_b Rb}{p_a - p_b} = (1 + \bar{r})\, L - C$$

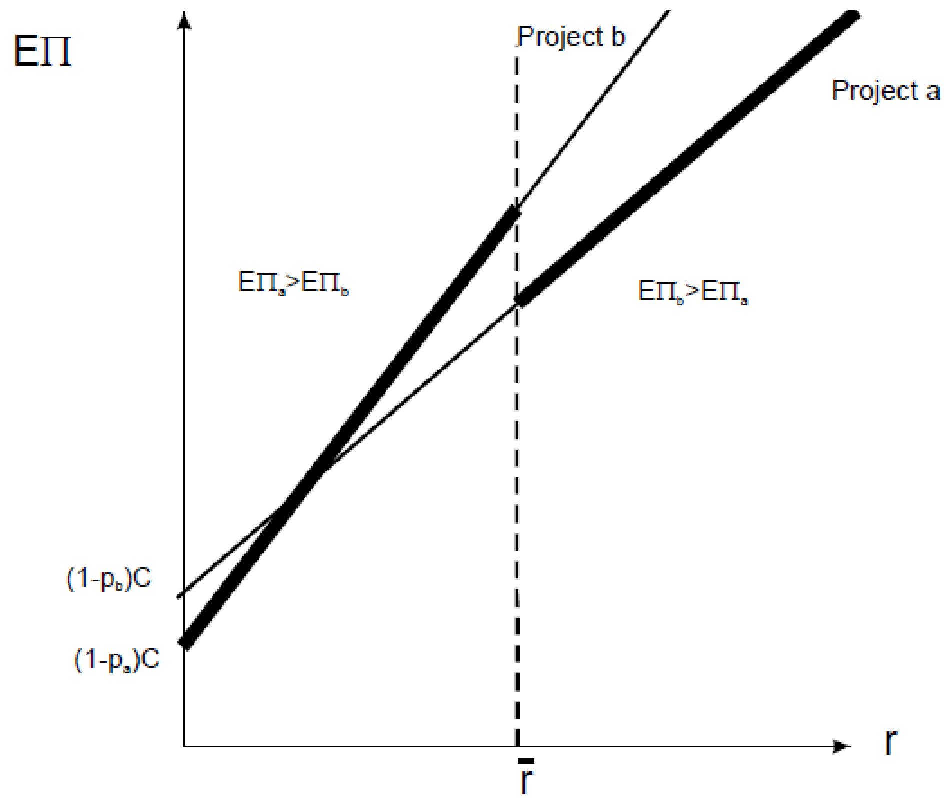
For $r > \bar{r}$ $E\pi^B > E\pi^A$; for $r < \bar{r}$ $E\pi^A > E\pi^B$. Intuition: you only pay the interest if you have a success.

- Banks lender's profit:

$$E\pi^L = \begin{cases} p_a (1 + r)\, L + (1 - p_a)\, C & r \leq \bar{r} \\ p_b (1 + r)\, L + (1 - p_b)\, C & r > \bar{r} \end{cases}$$

$$\frac{dE\pi^L}{dr} = \begin{cases} p_a L & r \leq \bar{r} \\ p_b L & r > \bar{r} \end{cases}$$

- Profits drop when $r = \bar{r}$. Solution?



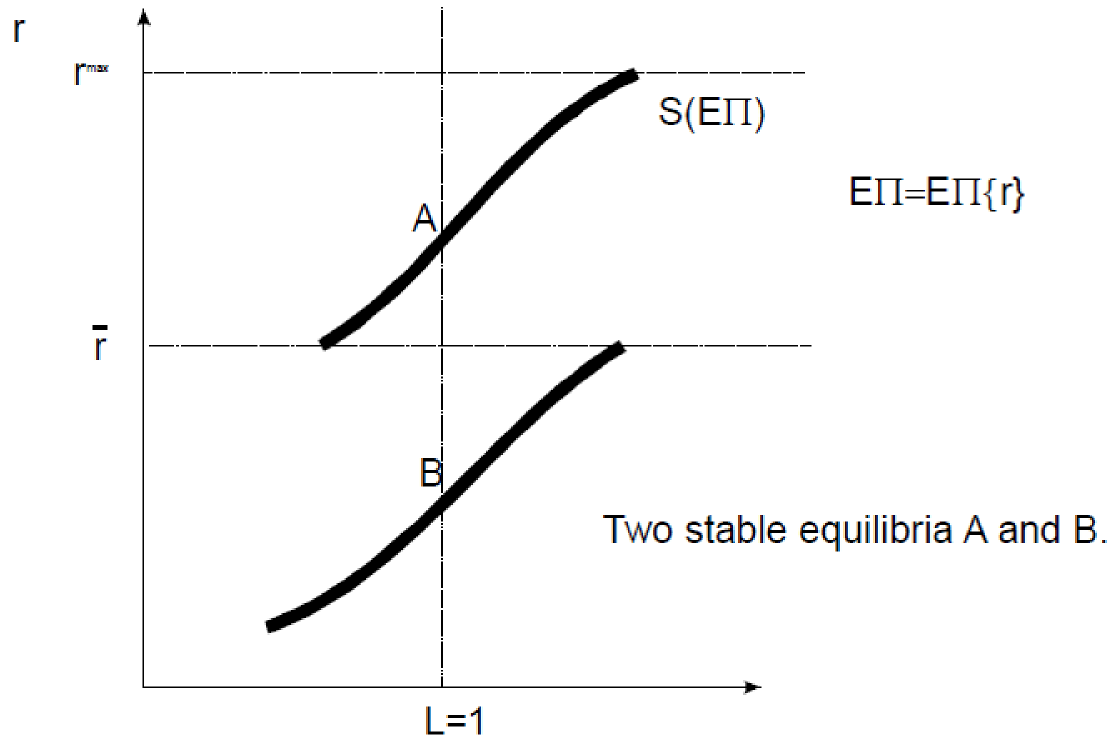
- Equilibrium. Set $L = 1$. Fixed demand: supply of entrepreneurs. Will borrow so long as make non-negative return. Maximum interest rate is one that makes B yield zero

$$
\begin{aligned}
r^{\text{max}} &: \quad E\pi^B = 0 \\
(1 + r^{\text{max}}) &= R_b - \frac{(1 - p_b)}{p_b} C
\end{aligned}
$$

- Supply depends on expected return $\rho = E\pi; S(\rho)$

$$
\begin{aligned}
\rho &= p_a (1 + r) + (1 - p_a) C : r \leq \bar{r} \\
\rho &= p_b (1 + r) + (1 - p_b) C : \bar{r} < r \leq r^{\text{max}}
\end{aligned}
$$

Hence can have mutliple equilibria, or no equilibrium:



- In Equilibrium B, there is a low interest rate and you get only safe low return projects undertaken. In B, there is a high interest rate, and you get only the risky projects undertaken. Lenders prefer equilibrium B. Borrowers prefer equilibrium A: they do not have to pay when project fails.

5 Financial Fragility (Mankiw 1986)..

- Suppose $C = 0$, $L = 1$.

- Return on project is R: pays out $\frac{R}{p}$ with probability p and 0 with $(1-p)$ .R is the expected return on the project.

- Projects (R, p) cannot be observed. (adverse selection).

- Will only take on loan (invest) if expected return is strictly positive

$$R - pr > 0$$

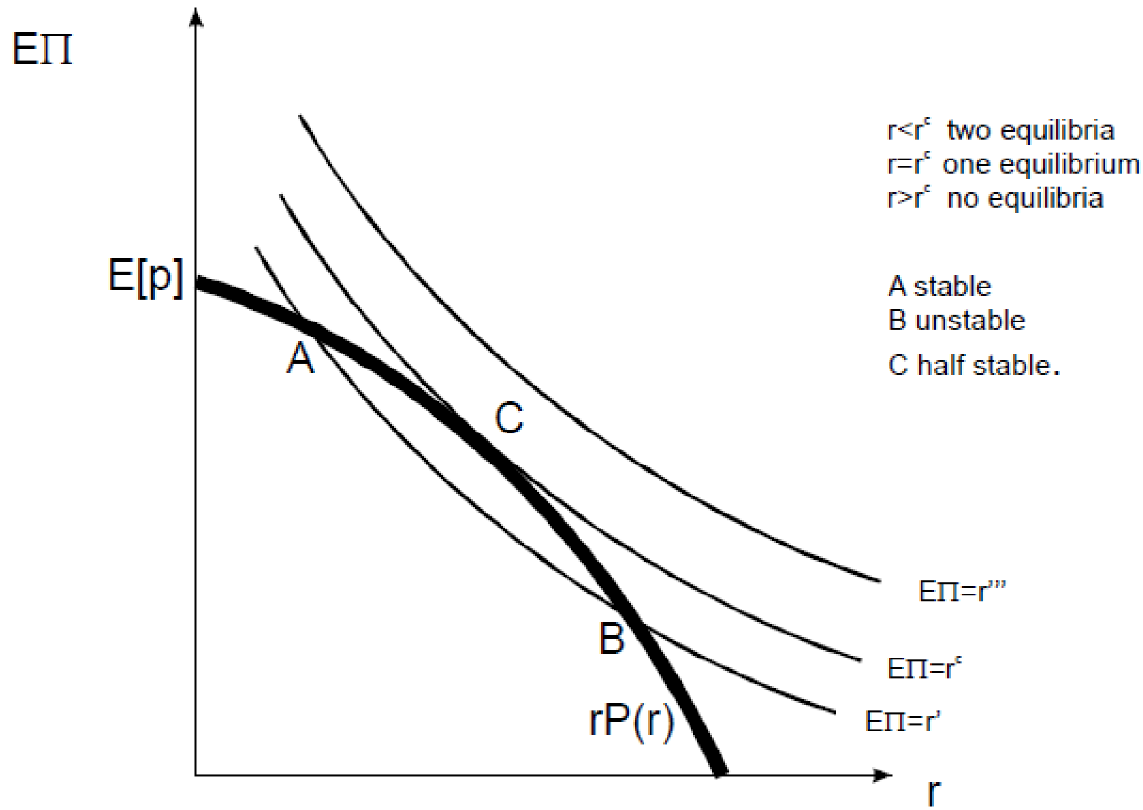
Define average probability of repayment is

$$P(r) = E\left[p : p < \frac{R}{r}\right]$$

$P(0) = E[p]$; for large $\tilde{r}$ $P(\tilde{r}) = 0$. P decreasing in r.

- Banks profits: $E\pi = rP(r)$. Outside option r^*. Will only lend if $rP(r) \geq r^*$.

- can get mutliple equilibria (2, one stable) or 1 or none.



- Market can "collapse" if outside r too big. Intuition? All proects have same expected rate of return R. But, some are riskier than others. As the interest rate increases, adverse selection means that only riskier projects come forward for the loan. The market can collapse if interest rates are too high.

6 Conclusion.

- Why do banks exist?

- Panics: Dymond and Dybvig. Two equilibria: autarky (no one puts money in bank) and equilibrium. Bank functions as "pool of liquidity", can make profit becuase of law of large numbers.

- Banks: coalitions of investors: can reduce informational costs (pooling risk): leland and Pyle.

- adverse selection: riskier projects financed by loans. Can lead to multiple equilibria and even the collapse of the market.