NNS models

Monetary Theory and Policy 2010

January 15, 2010

1 Introduction

- we have developed models of wage and price stickiness, nominal rigidity.

- Now put into an MIU model.

- New Keynesian Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma y_t$$

- Walsh Chapter 5 best. It jumps about a bit, but most things are there. Also Woodford chapter 3: can find most things in there if you search long enough.

2 Solving the Basic Taylor model.

- We will set $\beta = 1$, $\nu = 1$.

$$x_t = \frac{x_t^* + x_{t+1}^*}{2} \tag{1}$$

where

$$x_t^* = p_t + \gamma y_t \tag{2}$$

Quantity Theory:

$$y_t = m_t - p_t \tag{3}$$

Money is random walk

$$m_t = m_{t-1} + u_t \tag{4}$$

Prices markup over wages

$$p_t = \frac{x_t + x_{t-1}}{2} \tag{5}$$

could have diminishing marginal product of labour, so that p_t depends on output as well as wages.

- How to solve: make into a second order difference eqaution in x_t. From $(1, 2)$

$$x_t = \frac{p_t + p_{t+1} + \gamma\left(y_t + y_{t+1}\right)}{2}$$

use quantity theory to express output in terms of money and prices

$$x_t = \frac{p_t\left(1-\gamma\right) + E_t p_{t+1}\left(1-\gamma\right) + \gamma\left(m_t + E_t m_{t+1}\right)}{2} \tag{6}$$

- *Strategic complementarity:* Woodford, pp161-2. If the reset wage (or price) is increasing in the current price, then there is a strategic complementarity. This occurs if $0 < \gamma < 1$

$$\frac{dx_t}{dp_t} = \frac{(1-\gamma)}{2}$$

If $\gamma > 1$ we have *strategic substitutability*.

- Express (6) in terms of $x_t' s$

$$\begin{aligned} x_t &= \frac{(1-\gamma)(x_t + x_{t-1})}{4} + \frac{(1-\gamma)(x_t + E_t x_{t+1})}{4} + \gamma m_t \\ &= \left(\frac{1-\gamma}{4}\right)[x_{t-1} + 2x_t + E_t x_{t+1}] + \gamma m_t \end{aligned}$$

$$x_t = A(x_{t-1} + E_t x_{t+1}) - (1-2A) m_t \tag{7}$$

where

$$A = \frac{1}{2}\frac{1-\gamma}{1+\gamma}$$

- How to solve: method of *undetermined coefficients*.(See Romer first edition p.267). Guess that

$$x_t = \lambda x_{t-1} + \nu m_t \tag{8}$$

Note from (2) and the quantity theory (3), in steady state $y_t = 0$ and hence $p_t = m_t$. Since m_t is a random walk $E_t m_{t+1} = m_t$, so that in steady state we have $x = m$ (note, x and m do not return to zero: 0 is the initial steady state, and the new steady state is one with $m = m_0$ where $t = 0$ is the initial shock). For the rule (8) to be consistent with steady state we have the restriction

$$\nu = (1 - \lambda)$$

- Hence rewrite (7) assuming the wage-setting rule is (8)

$$x_t = \left(A + A\lambda^2\right) x_{t-1} + \left(A\left(1 - \lambda^2\right) + (1 - 2A)\right) m_t \qquad (9)$$

- Thus the wage setting rule (8) gives rise to the actual behaviour of wages (9). Comparing the two equations, they are only consistent if the coefficients on x_{t-1} and m_t.

$$\begin{aligned} \lambda &= \left(A + A\lambda^2\right) \\ (1 - \lambda) &= A\left(1 - \lambda^2\right) + (1 - 2A) \end{aligned}$$

  These equalities are identical: hence look only at the first. This is a quadratic in λ with solutions

$$\lambda = \frac{1 \pm \sqrt{\left(1 - 4A^2\right)}}{2A}$$

Hence two solutions: one stable and one unstable: the stable one is

$$\lambda = \frac{1 - \sqrt{\gamma}}{1 + \sqrt{\gamma}}$$

- λ is the stable eigenvalue of the system. Can then solve for the rest of the system

$$y_t = \lambda y_{t-1} + \frac{1+\lambda}{2} u_t$$

λ is the measure of persistence here: y follows an $AR(1)$ process. Larger λ means more persistence. Impulse response just "dies away" with half life given by n where $\lambda^n = 0.5$.

- Since

$$
\begin{aligned}
p_t &= \frac{x_t + x_{t-1}}{2} \\
&= \frac{(1+\lambda)\, x_{t-1} + (1-\lambda) m_t}{2} \\
&= \frac{1+\lambda}{2} x_{t-1} + \frac{(1-\lambda)}{2} m_{t-1} + \frac{(1-\lambda)}{2} u_t
\end{aligned}
$$

- All Taylor 2 models have solutions that look pretty similar to this. Hence to compare different models in this genre, can compute the stable root and relate it to the parameters of the model.

2.1 Ascari 2003: a user's guide.

- Consider the following system

$$
\begin{aligned}
x_t &= \frac{p_t(1-\gamma) + E_t p_{t+1}(1-\gamma) + \gamma(m_t + E_t m_{t+1})}{2} \\
p_t &= \frac{x_t + x_{t-1}}{2} + a y_t \\
y_t &= b(m_t - p_t)
\end{aligned}
\tag{10}
$$

  Ascari relates the constants γ, a, b to the underlying preferences and technology (see page 515). We already know about γ!

$$
\begin{aligned}
a &= \frac{1-\nu}{\nu} \\
b &= \eta_C^{-1}
\end{aligned}
$$

- The solution to the resulting second order difference equation is (after a lot of algebra, but very similar to what we did before)

$$\begin{aligned} \lambda &= \frac{1-R}{1+R} \\ R &= \sqrt{\frac{a+\gamma}{a+\frac{1}{b}}} \end{aligned}$$

- Thus KCM 2000 can be seen as a special case of (10) where $a = 0$ and the γ is the one we derived in lecture 6. Hence the root of KCM is given by

$$\begin{aligned} R^{CKM} &= \sqrt{b\gamma} \\ \gamma^{KCM} &= \frac{\eta_\ell + \nu\eta_c + 1 - \nu}{1 + (1-\nu)\,\theta} \end{aligned}$$

- Ascari goes through several other models (the ones we went through in lecture 6!),. and derives the different values for R in terms of different $\gamma's$ and other parameters. Table 1, p525. Main conclusions of ascari
  - γ captures the notion of real rigidity: smaller γ means more real rigidity: "Real rigidity" means that real wages/real prices are not affected by output. Thus higher η_ℓ or η_c make the economy more persistent. Higher θ less. ν is ambiguous.
  - labour mobility is crucial. in models without labour mobility η_l increases persistence: in models without it the reverse happens.

3 Calvo 1983.

- Calvo model. From lecture 4.

$$x_t = \frac{1}{\sum_{i=0}^{\infty}(1-\omega)^i \beta^i} \sum_{i=0}^{\infty} (1-\omega)^i \beta^i p_{t+i}^* \quad (11)$$

$$x_t = (1-(1-\omega)\beta) p_t^* + (1-\omega)\beta E_t x_{t+1} \quad (12)$$

$$p_t = \omega x_t + (1-\omega) p_{t-1} \quad (13)$$

$$p_t^* = p_t + \gamma y_t \quad (14)$$

- Note: in the Calvo model

$$\frac{dx_t}{dp_t} = \left[\sum_{i=0}^{\infty} (1-\omega)^i \beta^i\right](1-\gamma)$$

So again, the coefficient γ determines whether we have strategic substitutes or complements. If $\gamma < 1$ then we have *strategic complementarity*.

- New Keynesian Phillips curve.
  - Rewrite (13) going one period forward

$$\begin{aligned} Ep_{t+1} &= \omega E_t x_{t+1} + (1-\omega)\, p_t \\ E_t \pi_t &= \omega E_t x_{t+1} - \omega p_t \end{aligned}$$

  - Hence

$$E_t x_{t+1} = \left(\frac{1}{\omega}\right) E_t \pi_t + p_t$$

    which we can use to eliminate $E_t x_{t+1}$ from (12) :

$$x_t = (1-(1-\omega)\,\beta)\, p_t^* + ((1-\omega)\,\beta)\left(\left(\frac{1}{\omega}\right) E_t \pi_t + p_t\right) \tag{15}$$

  - from (13) we have

$$x_t = \frac{1}{\omega} p_t + \left(\frac{1}{\omega} - 1\right) p_{t-1} \tag{16}$$

– We use (16) to eliminate x_t from (15) , and use (14)which after a bit of rearranging :)

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} + \gamma' y_t \\ \gamma' &= \left[\frac{\omega\left(\left(1-\left(1-\omega\right)\beta\right)\right)}{1-\omega}\right]\gamma\end{aligned}$$

– This is the *New Keynesian Phillips* curve. Can get *almost* the same thing from the Taylor model (see Roberts 1995 on the reading list). Empirically it does not fit the data very well; has led to some other work (more about that later!).

- Empirically: the New Keynesian Phillips curve does not do well. Much better fit is the hybrid Phillips curve

$$\pi_t = \phi E_t \pi_{t+1} + (1-\phi)\,\pi_{t-1} + \gamma y_t$$

where the estimated value of $\phi = 0.3$ (Fuhrer 1997).

- Gali and Gertler (1999). rather than use y_t, go back to MC. They argue for what they call "Real MC". Recall that nominal marginal cost is:

$$MC = \frac{W}{F_L}$$

Real Marginal Cost is

$$\frac{MC}{P} = \frac{W}{P}\frac{1}{F_L}$$

Note: this is really the "mark-up", not real MC . By definition the markup is

$$\mu \equiv \frac{P}{MC}$$

With Cobb-Douglas technology $Y = AL^{\alpha}K^{(1-\alpha)}$. Hence $F_L = \alpha\frac{Y}{L}$

$$\frac{MC}{P} = \frac{1}{\alpha}\left[\frac{WL}{PY}\right]$$

Gali and Gertler argue that you should use this "Real marginal cost" variable in empirical work and then the importance of lagged inflation much less (on US data maybe 0.25-0.4 coefficient on π_{t-1}).

3.0.1 Solving the Calvo model

- Now. let us set up a simple Calvo macroeconomy!

$$\begin{aligned} p_t &= \lambda p_{t-1} + (1-\lambda)\, m_t \\ y_t &= \lambda y_{t-1} + \lambda u_t \end{aligned}$$

What is λ?

- set $\beta = 1$ (otherwise very complicated algebra!).

- Recall five equations.

$$
\begin{aligned}
x_t &= \omega \sum_{i=0}^{\infty} (1-\omega)^i p^*_{t+i} && (17) \\
x_t &= \omega p^*_t + (1-\omega) E_t x_{t+1} && (18) \\
p_t &= \omega x_t + (1-\omega) p_{t-1} && (19) \\
y_t &= m_t - p_t && (20) \\
m_t &= m_{t-1} + u_t && (21) \\
p^*_t &= p_t + \gamma y_t && (22)
\end{aligned}
$$

  - We derive a second order difference equation in p_t, and solve for the eigenvalues (can use method of undetermined coefficients, but illustrate the method).

– Using the quantity theory (20) to eliminate y_t in (22)

$$p_t^* = (1-\gamma)\, p_t + \gamma m_t$$

– We can move (19) ahead one period, and express $E_t x_{t+1}$ as a function of $E_t p_{t+1}$ and p_t

$$E_t x_{t+1} = \frac{E_{t+1} p_{t+1}}{\omega} - \frac{(1-\omega)}{\omega} p_t \tag{23}$$

– Hence we can use (23) to substitute out $E_t x_{t+1}$ in (18)

$$x_t = \omega\left(p_t\,(1-\gamma) + \gamma m_t\right) + \frac{1-\omega}{\omega} E_t p_{t+1} - \frac{(1-\omega)^2}{\omega} p_t \tag{24}$$

– We can then use (24) to substitute out x_t in (19) results in a second order difference equation in p_t.

$$E_t p_{t+1} - \left[2 + \left(\frac{\gamma\omega}{1-\omega}\right)\right] p_t + p_{t-1} + \frac{\gamma\omega}{1-\omega} m_t = 0$$

– The roots of this equation are

$$\begin{aligned}\lambda_i &= 1 + \frac{1}{2}\left(\frac{\gamma\omega}{1-\omega}\right) \pm \frac{1}{2}\sqrt{\left(2 + \left(\frac{\gamma\omega}{1-\omega}\right)\right)^2 - 4} \\ &= 1 + \frac{1}{2}\left(\frac{\gamma\omega}{1-\omega}\right) \pm \frac{1}{2}\sqrt{\left(\frac{\gamma\omega}{1-\omega}\right)^2 + 4\left(\frac{\gamma\omega}{1-\omega}\right)}\end{aligned}$$

– Both are strictly positive, one is greater than one, one is less than 1. The stable root can be written

$$\lambda^C = 1 + \frac{1}{2}\left(\frac{\gamma\omega}{1-\omega}\right)\left[1 - \sqrt{\left(1 + 4\left(\frac{1-\omega}{\omega\gamma}\right)\right)}\right]$$

3.0.2 Comparing Calvo and Taylor: Kiley 2002.

- For both Taylor and Calvo models we have an output dynamic of the form $y_t = \lambda y_{t-1} + bu_t$

$$\lambda^T = \frac{1-\sqrt{\gamma}}{1+\sqrt{\gamma}}$$

$$\lambda^C = 1 + \frac{1}{2}\left(\frac{\gamma\omega}{1-\omega}\right) - \frac{1}{2}\sqrt{\left(\frac{\gamma\omega}{1-\omega}\right)^2 + 4\left(\frac{\gamma\omega}{1-\omega}\right)}$$

- We can choose a value of ω that gives the same average contract life as the Taylor contracts: $\omega = 2/3$ implies an average lifetime of 2.. Hence the Calvo stable root becomes

$$\lambda^C = 1 + \gamma - \sqrt{\gamma^2 + 2\gamma}$$

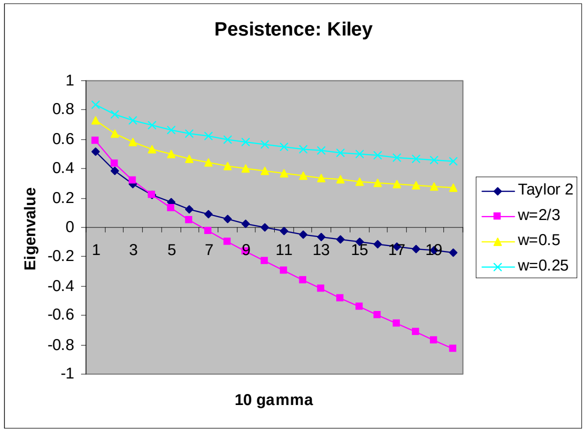
Persistence in Taylor and Calvo.

- If we choose $\omega = 0.5$ (as Michael Kiley does) then

$$\lambda^C = 1 + \frac{1}{2}\left[\gamma - \sqrt{\gamma^2 + 4\gamma}\right]$$

- can see the eigenvalues as a function of γ. As we can see, when we calibrate the eigenvalues for the same average contract length (2 so that

$\omega = 2/3$) the Calvo and Taylor are not so different!



4 Monetary Policy

- Basic idea: put in our model of nominal rigidity in a simple MIU macro-model. All familiar! See Walsh pp.232-247.

$$U = \frac{C^{1-\sigma}}{1-\sigma} + \frac{\kappa}{1-b}\left(\frac{M}{P}\right)^{1-b} - \frac{\chi}{1-\eta}N^{1+\eta}$$

where

$$\begin{aligned} C &= \left[\int_0^1 c_j^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \\ P &= \left[\int_0^1 p_j^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \end{aligned}$$

so that

$$c_j = \left(\frac{p_j}{P}\right)^{-\theta} C$$

Budget constraint (add t now)

$$C_t P_t + M_t + B_t = W_t N_t + M_{t-1} + (1 + i_{t-1}) B_{t-1} + P_t \Pi_t$$

- FOC (see lecture 3)

$$\begin{aligned}
C_t^{-\sigma} &= \beta (1 + i_t) E_t \left(\frac{P_t}{P_{t+1}} C_{t+1}^{-\sigma} \right) \\
\kappa \left(\frac{M_t}{P_t} \right)^{-b} &= C_t^{-\sigma} \frac{i_t}{1 + i_t} \\
\chi N_t^{\eta} &= C_t^{-\sigma} \left(\frac{W_t}{P_t} \right)
\end{aligned}$$

- Firms. Aggregate productivity shock $Z_t, E(Z_t) = 1.$

$$c_{jt} = Z_t N_{jt}$$

CRTS....simplifies γ!

- Define the flexible price equilibrium output at t given the realization of Z : from Labour market equilibrium

$$\frac{W_t}{P_t} = \frac{Z_t}{\mu} = \chi N_t^{\eta} C_t^{\sigma}$$

From the production function $y^f = c^f = n + z$

$$y_t^f = \left(\frac{1+\eta}{\sigma+\eta}\right) z_t$$

Define

$$\tilde{y}_t = y_t - y_t^f$$

Because of the stochastic productivity shocks, what matters is the deviation of actual output from the current "flex price" output. Without productivity shocks this would be constant.

- Calvo model. So, following the arguments above, can summarise Calvo model as New Keynesian Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma' \tilde{y}_t$$

- Use Euler condition to get "IS" curve: log-linearize a around $\pi = 0$ steady state (using $C = Y$) : $u_t = E_t y^f_{t+1} - y^f_t$

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1}) + u_t$$

- Finally, a monetary policy Rule. For $0 < \rho < 1$

$$i_t = \rho i_{t-1} + \nu_t$$

- Voila: we have our EQUILIBRIUM!

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \sigma^{-1} \\ 0 & 0 & \beta \end{bmatrix} \begin{bmatrix} i_t \\ E_t \tilde{y}_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 & 0 \\ \sigma^{-1} & 1 & 0 \\ 0 & \gamma' & 1 \end{bmatrix} \begin{bmatrix} i_{t-1} \\ \tilde{y}_t \\ E_t \pi_t \end{bmatrix} + \begin{bmatrix} \nu_t \\ -u_t \\ 0 \end{bmatrix}$$

Which can be written as

$$\begin{bmatrix} i_t \\ E_t \tilde{y}_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = M \begin{bmatrix} i_{t-1} \\ \tilde{y}_t \\ E_t \pi_t \end{bmatrix} + \begin{bmatrix} \nu_t \\ -u_t \\ 0 \end{bmatrix}$$

where

$$M = \begin{bmatrix} \rho & 0 & 0 \\ \sigma^{-1} & 1 + \frac{\gamma'}{\sigma\beta} & -\frac{1}{\sigma\beta} \\ 0 & -\frac{\gamma'}{\beta} & \beta^{-1} \end{bmatrix}$$

(I repeat Walsh's error of not premultiplying the error vector by the inverse.....:)

4.1 Analysis of equilibrium

- Three variables. There are two forward looking variables: $\tilde{y}_t$ and π_t. Need to have two unstable eigenvalues (Blanchard and Kahn 1980) for a unique SS solution. The monetary policy rule is a stable eigenvalue ρ.

- However, both the other eigenvalues are also less than one in absolute size (inside the unit circle), so this is a sink! Too stable.....

$$\left[\begin{array}{cc} 1+\frac{\gamma'}{\sigma\beta} & -\frac{1}{\sigma\beta} \\ -\frac{\gamma'}{\beta} & \beta^{-1} \end{array} \right]$$

This excessive stability causes indeterminacy. Happens because interest policy is autonomous and does not react to the economy.

- So need to have monetary policy react to output and/or inflation. Bullard and Mitra (2002)

$$i_t = \delta\pi_t + \nu_t$$

This means you can eliminate i and we have two equation system

$$\left[\begin{array}{c} E_t\tilde{y}_{t+1} \\ E_t\pi_{t+1} \end{array} \right] = N \left[\begin{array}{c} \tilde{y}_t \\ E_t\pi_t \end{array} \right] + \left[\begin{array}{c} \sigma^{-1}\nu_t - u_t \\ 0 \end{array} \right]$$

Where

$$N = \begin{bmatrix} 1 + \frac{\gamma'}{\sigma\beta} & -\frac{\beta\delta - 1}{\sigma\beta} \\ -\frac{\gamma'}{\beta} & \beta^{-1} \end{bmatrix}$$

If $\delta > 1$, then one eigenvalue is stable and one unstable. A stable saddlepath. What this means is that the raising the nominal interest raises the real interest: TAYLOR PRINCIPLE. A stable monetary rule at least requires real interest rates to rise when inflation rises.

- Taylor Rule.

$$i_t = \delta\pi_t + \delta_y \tilde{y}_t + \nu_t$$

This requires $\gamma' (\delta - 1) + (1 - \beta) \delta_y > 0$ for a unique equilibrium. Since $\beta = 0.99$ (quarterly data), if $\gamma' > 0$, then $\delta > 1$ (Taylor principle) is almost sufficient for saddle.

5 Conclusion.

- We have looked at the two "basic models" of price or weage setting: Calvo and Taylor.

- We have put them into the "NNS" framework. Calvo: use the NKPC.

- Analysed the dynamics and eigenvalues.