Contract Length Heterogeneity and the Persistence of Monetary Shocks in a Dynamic Generalized Taylor Economy*

Huw Dixon† and Engin Kara‡

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Abstract

We develop the Generalized Taylor Economy *(GTE)* in which there are many sectors with overlapping contracts of different lengths. In economies with the same average contract length, monetary shocks will be more persistent when longer contracts are present. Using the Bils-Klenow distribution of contract lengths, we find that the corresponding GTE tracks the U.S. data well. When we choose a *GTE* with the same distribution of completed contract lengths as the Calvo, the economies behave in a similar manner.

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†Corresponding author: Cardiff Business School, Aberconway Building, Column Drive, Cardiff, CF10 3EU. Email: DixonH@cardiff.ac.uk.

‡Economics Department, The University of Bristol, Bristol, BS8 1TN. E-mail: engin.kara@bristol.ac.uk.

1 Introduction

There are two main approaches to modelling nominal wage and price rigidity in the dynamic stochastic general equilibrium (DSGE) macromodels: the staggered contract setting of Taylor (1980) and the Calvo (1983) model of random contract lengths generated by a constant hazard (reset) probability. As in Dixon and Kara (2010), we propose a generalization of the standard Taylor model to allow for an economy with many different contract lengths: we call this a Generalized Taylor Economy - *GTE* for short. The standard approach in the literature has been to adopt a simple Taylor economy, in which there is a single contract length in the economy: for example 2 or 4 quarters[1]. We can use the *GTE* framework to evaluate whether the representative sector approach "is a good approximation to this more complex world" (Taylor (1999)).

An additional advantage of the *GTE* framework is that it includes the Calvo model as a special case, in the sense that we can set up the *GTE* to have the same distribution of contract lengths as the Calvo model. This is an important contribution in itself since the two approaches have until now appeared to be distinct and incompatible at the theoretical level even if they are sometimes claimed to be empirically similar (see for example Kiley (2002) for a discussion). As we shall show, a simple Taylor economy can indeed be a good approximation to a Calvo model, but only if the two are calibrated in a consistent manner.

We develop our approach in a DSGE setting following the approach of Ascari (2000). The issue we focus on is the way a monetary shock can generate changes in output through time, and in particular the degree of persistence of deviations of output from steady-state. Much recent attention has been devoted to the ability of the staggered contract approach of Taylor to generate

[1]This is not to ignore some recent papers: Carvalho (2006), Coenen, Christoffel and Levin (2007) and Carlstrom, Fuerst, Ghironi and Hernandez (2006) which consider economies with multiple contract lengths. See also Taylor (1993) for what we believe to be the first instance. Other papers that allow for two sectors with different contract durations are Aoki (2001), Benigno (2004), Erceg and Levin (2006), Carlstrom, Fuerst and Ghironi (2006) and Mankiw and Reis (2003). However, these studies are mainly concerned with computing optimal monetary policy in a dynamic equilibrium setting. Moreover, recent work by Kara (2010) suggests that there is a limitation in studies like these which use models that have only two sectors and, by using the *GTE*, shows that, a failure to use a model that accounts for the heterogeneity of contract lengths we observe in empirical data can significantly affect policy conclusions.

enough persistence in the sense of being quantitatively able to generate the persistence observed in the data. Two influential papers in this are Chari, Kehoe and McGrattan (2000) (CKM hereafter) and Ascari (2000). Both papers are pessimistic for staggered contracts. CKM develop a microfounded model of staggered price-setting and find that they do not generate enough persistence and conclude that the "mechanism to solve persistence problem must be found elsewhere". Ascari focusses on staggered wage setting, and finds that whilst nominal wage rigidities lead to more persistent output deviations than with price setting, they are still not enough to explain the data. Based on these conclusions, it is commonly inferred that in a dynamic equilibrium framework, staggered contracts cannot generate enough persistence.

We show that by allowing for an economy with a range of contract lengths, the presence of longer contracts can significantly increase the degree of persistence in output following a monetary shock. We calibrate the model in such a way that neither the CKM nor the Ascari setting would generate much persistence. We then show that *even a small proportion of longer contracts can significantly increase the degree of persistence*. The intuition behind this finding is that there is a spillover effect or strategic complementarity in terms of wage or price-setting through the price level. The presence of longer contracts means that the general price level is held back in response to monetary shocks. This in turn means that the wage setting of shorter contracts is influenced and hence they adjust by less than they otherwise would. We also find that the impulse response function in the GTE with the actual distribution of contract lengths for the U.S. based on the Bils and Klenow (2004) data set is very similar to an empirical response function for the U.S.

It has long been observed that in the Calvo setting there can be a significant backlog of old contracts: for example, with a reset probability of $\omega = 0.25$ (a common value used with quarterly data), there is a probability of over 10% that a contract will survive for 8 periods (see for example Erceg (1997), Wolman (1999)). We construct a GTE which has exactly the same distribution of completed contract lengths as the Calvo distribution (as derived in Dixon and Kara (2006)). We find that this Calvo-GTE has similar persistence to the Calvo economy. The remaining difference between the Calvo economy and the Calvo-GTE is in the wage-setting decision. We find that Calvo reset firms are more forward looking on average than in the Calvo-GTE. This is because short contracts are more predominant amongst wage-resetters in the Calvo-GTE than in the economy as a whole, because wage-setters with long contracts reset wages less frequently. However, for the calibrated values this does not make a big difference and indicates that the two approaches of Taylor and Calvo can be brought together within the framework of the GTE.

The outline of the paper is as follows. Section 2 outlines the basic structure of the Economy. The main innovation here is to allow for the GTE contract structure. Section 3 presents the log-linearized general equilibrium and discuss the calibration of the model in relation to recent literature. Section 4 explores the influence of longer term contracts on persistence as compared to the simple Taylor economy and apply this to U.S. data. Section 5 applies our methodology to evaluating persistence in the Calvo model.

2 The Model Economy

The approach of this paper is to model an economy in which there can be many sectors with different wage setting processes, which we denote a Generalized Taylor Economy (GTE). As we will show later, an advantage of the GTE approach is that it includes as special cases not only the standard Taylor case of an economy where all wage contracts are of the same length, but also the Calvo process.

The model in this section is an extension of Ascari (2000) and includes a number of features essential to understanding the impact of monetary shocks on output in a dynamic equilibrium setting. The exposition aims to outline the basic building blocks of the model. However, the novel aspects of this paper only begin with the wage setting process. Firstly, we describe the behavior of firms which is standard. Then we describe the structure of the contracts in a *GTE,* the wage-setting decision and monetary policy.

2.1 Firms

There is a continuum of firms $f \in [0,1]$, each producing a single differentiated good $Y(f)$, which are combined to produce a final consumption good Y. The production function here is CES with constant returns and corresponding unit cost function P :

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}; \quad P_t = \left[\int_0^1 P_{ft}^{1-\theta} df\right]^{\frac{1}{1-\theta}} \tag{1}$$

The demand for the output of firm f is

$$Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\theta} Y_t \tag{2}$$

Each firm f sets the price P_{ft} and takes the firm-specific wage rate W_{ft} as given. Labor L_{ft} is the only input so that the inverse production function is

$$L_{ft} = \left(\frac{Y_{ft}}{\alpha}\right)^{\frac{1}{\sigma}} \tag{3}$$

Where $\sigma \leq 1$ represents the degree of diminishing returns, with $\sigma = 1$ being constant returns. The firm chooses $\{P_{ft,}Y_{ft}, L_{ft}\}$ to maximize profits subject to (2) and (3) yields the following solutions for price, output and employment at the firm level given $\{Y_t, W_{ft}, P_t\}$

$$P_{ft} = \left(\frac{\theta}{\theta-1}\right)\frac{\alpha^{-1/\sigma}}{\sigma} W_{ft} Y_{ft}^{\frac{1-\sigma}{\sigma}} \tag{4}$$

$$Y_{ft} = \kappa_1 \left(\frac{W_{ft}}{P_t}\right)^{-\sigma\varepsilon} Y_t^{\frac{\varepsilon\sigma}{\theta}} \tag{5}$$

$$L_{ft} = \kappa_2 \left(\frac{W_{ft}}{P_t}\right)^{-\varepsilon} Y_t^{\frac{\varepsilon}{\theta}} \tag{6}$$

where $\varepsilon = \frac{\theta}{\theta(1-\sigma)+\sigma} > 1$, $\kappa_1 = \left(\frac{\theta}{\theta-1}\right)^{-\sigma\varepsilon} \sigma^{-\sigma\varepsilon}\alpha^{-\varepsilon}$, $\kappa_2 = \left(\frac{\theta}{\theta-1}\right)^{-\varepsilon} \sigma^{\varepsilon}\alpha^{\varepsilon\left(\frac{\theta-1}{\theta}\right)}$. Price is a markup over marginal cost, which depends on the wage rate and the output level (when $\sigma < 1$): output and employment depend on the real wage and total output in the economy.

2.2 The Structure of Contracts in a GTE

In this section we outline an economy in which there are potentially many sectors with different lengths of contracts. Within each sector there is a standard Taylor process (i.e. overlapping contracts of a specified length). The economy is called a Generalized Taylor Economy (GTE). Corresponding to the continuum of firms f there is a unit interval of household-unions (one per firm)[2]. The economy consists N sectors $i = 1...N$. The budget shares of

[2]Following Taylor, we will present the model as one of wage-setting. However, the framework also holds for price-setting. The distinction between wage and price-setting rests primarily when we come to calibration, as we discuss in some detail below.

the N sectors with uniform prices (when prices p_f are equal for all $f \in [0,1]$) are given by $\alpha_i \geq 0$ with $\sum_{i=1}^{N} \alpha_i = 1$, the N vector $(\alpha_i)_{i=1}^{N}$ being denoted $\boldsymbol{\alpha}$,where $\boldsymbol{\alpha} \in \Delta^{N-1}$. Without loss of generality, we suppose that in sector i there are $i-$period contracts, so that the longest contracts are N periods. If there are no j period contracts, then $\alpha_j = 0$.

Within sector i there are i equal sized cohorts who move in sequence. If we log-linearise the unit cost function[3], (1) the price in sector i is the average over the prices set in each cohort $j = 1, 2, ...i$ is given by

$$p_i = \frac{1}{i} \sum_{j=1}^{i} p_{ij}$$

Similarly, the average price in the whole economy is the average over each sector

$$p = \sum_{i=1}^{N} \alpha_i p_i$$

or in terms of cohort prices:

$$p = \sum_{i=1}^{N} \sum_{j=1}^{i} \frac{\alpha_i}{i} p_{ij} \tag{7}$$

Note that there is an important property of *CES* technology. The demand for an individual firm depends only on its own price and the general price index (see 2). There is no sense of location: whilst we divide the unit interval into segments corresponding to sectors and cohorts within sectors, this need not reflect any objective factor in terms of sector or cohort specific aspects of technology or preferences. The sole communality within a sector is the length of the wage contract: the sole communality within a cohort is the timing of the contract. This is an important property which will become useful when we show that a Calvo economy can be represented by a *GTE*.

2.3 Household-Unions and Wage Setting

Households $h \in [0,1]$ have preferences defined over consumption, labour, and real money balances. The expected life-time utility function takes the form

[3]To see how the price index decomposes in terms of the unit interval of firms within the *CES* function, see the previous version of the paper ECB working paper 489

$$U_h = E_t \left[\sum_{t=0}^{\infty} \beta^t u(C_{ht}, \frac{M_{ht}}{P_t}, \underbrace{1 - H_{ht}}_{L_{ht}}) \right] \tag{8}$$

where C_{ht}, M_{ht}, H_{ht}, L_{ht} are household $h's$ consumption, end-of period money holdings, hours worked, and leisure respectively, $0 < \beta < 1$ is the discount factor, and each household has the same flow utility function u, which is assumed to take the form

$$U(C_{ht}) + \delta \ln(\frac{M_{ht}}{P_t}) + V(1 - H_{ht}) \tag{9}$$

Each household-union belongs to a particular sector and wage-setting cohort within that sector (recall, that each household is twinned with firm $f = h$). Since the household acts as a monopoly union, hours worked are demand determined, being given by the (6).

The household's budget constraint is given by

$$P_t C_{ht} + M_{ht} + \sum_{s_{t+1}} Q(s^{t+1} \mid s^t) B_h(s^{t+1}) \leq M_{ht-1} + B_{ht} + W_{ht} H_{ht} + \pi_{ht} + T_{ht} \tag{10}$$

where $B_h(s^{t+1})$ is a one-period nominal bond that costs $Q(s^{t+1} \mid s^t)$ at state s^t and pays off one dollar in the next period if s^{t+1} is realized. B_{ht} represents the value of the household's existing claims given the realized state of nature. W_{ht} is the nominal wage, π_{ht} is the profits distributed by firms and $W_{ht} H_{ht}$ is the labour income. Finally, T_t is a nominal lump-sum transfer from the government.

The households optimization breaks down into two parts. First, there is the choice of consumption, money balances and one-period nominal bonds to be transferred to the next period to maximize expected lifetime utility (8) given the budget constraint (10). The first order conditions derived from the consumer's problem are as follows:

$$u_{ct} = \beta R_t E_t \left(\frac{P_t}{P_{t+1}} u_{ct+1} \right) \tag{11}$$

$$\sum_{s_{t+1}} Q(s^{t+1} \mid s^t) = \beta E_t \frac{u_{ct+1} P_t}{u_{ct} P_{t+1}} = \frac{1}{R_t} \tag{12}$$

$$\delta \frac{P_t}{M_t} = u_{ct} - \beta E_t \frac{P_t}{P_{t+1}} u_{ct+1} \tag{13}$$

Equation (11) is the Euler equation, (12) gives the gross nominal interest rate and (13) gives the optimal allocation between consumption and real balances. Note that the index h is dropped in equations (11) and (13), which reflects our assumption of complete contingent claims markets for consumption and implies that consumption is identical across all households in each period $(C_{ht} = C_t)$[4].

The reset wage is for household h in sector i is chosen to maximize lifetime utility given labour demand (6) and the additional constraint that nominal wage will be fixed for T_i periods in which the aggregate output and price level are given$\{Y_t, P_t\}$. From the unions point of view, we can collect together all of the terms in (6) which the union treats as exogenous by defining the constant K_t, where $K_t = \kappa_2 P_t^{\varepsilon} Y_t^{\frac{\varepsilon}{\theta}}$. Since the reset wage at time t will only hold for T_i periods, we have the following first-order condition:

$$X_{it} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \left[\frac{\sum_{s=0}^{i-1} \beta^s \left[V_L \left(1 - H_{it+s}\right) \left(K_{t+s}\right) \right]}{\sum_{s=0}^{i-1} \beta^s \left[\frac{u_c(C_{t+s})}{P_{t+s}} K_{t+s} \right]} \right] \tag{14}$$

Equation (14) shows that the optimal wage is a constant mark-up (given by$\frac{\varepsilon}{\varepsilon-1}$) over the ratio of marginal utilities of leisure and marginal utility from consumption within the contract duration, from t to $t+T_i-1$ When $T_i = 2$, this equation reduces to the first order condition in Ascari (2000).

2.4 Monetary Policy Rule

Following Taylor and Wieland (2008), the central bank follows a Taylor style rule under which the short term interest rate is adjusted to respond to the lagged interest rate, the four-quarter average inflation rate and to the current and lagged output levels:

$$r_t = \phi_r r_{t-1} + \phi_\pi \bar{\pi}_t + \phi_{y_1} y_t + \phi_{y_2} y_{t-1} - \ln \mu_t \tag{15}$$

[4]See Ascari (2000).

where $\pi_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$, $\bar{\pi}_t = \frac{1}{4}\sum_{j=0}^{3}\pi_{t-j}$ and $y_t = \ln Y_t$[5]. We assume that $\ln\mu_t$ follows an AR(1) process:

$$\ln\mu_t = v.\ln(\mu_{t-1}) + \xi_t \tag{16}$$

where $0 < v \leq 1$ and ξ_t is a white noise process with zero mean and a finite variance. The literature suggests that a Taylor rule in this form provides a reasonable description of the U.S. monetary policy (e.g. Orphanides and Wieland (1998), Erceg and Levin (2006), Smets and Wouters (2007), Levin, Onatski, Williams and Williams (2005) among others).

3 General Equilibrium

In this section, we characterize equilibrium of the economy. We follow the standard approach of log-linearizing around the steady state of the model. The steady state which we choose is the zero-inflation steady state, which is a standard assumption in this literature. The linearized version of the equations are listed and discussed below. We follow the notational convention that lower-case symbols represents log-deviations of variables from the steady state.

The linearized wage decision equation (14) for sector i is given by

$$x_{it} = \frac{1}{\sum_{s=0}^{i-1}\beta^s}\left[\sum_{s=0}^{i-1}\beta^s\left[p_{t+s} + \gamma y_{t+s}\right]\right] \tag{17}$$

The coefficients on output in the wage setting equation in all sectors is given by

$$\gamma = \frac{\eta_{LL} + \eta_{cc}(\sigma + \theta(1-\sigma))}{\sigma + \theta(1-\sigma) + \theta\eta_{LL}} \tag{18}$$

Where $\eta_{cc} = \frac{-U_{cc}C}{U_c}$ is the parameter governing risk aversion, $\eta_{LL} = \frac{-V_{LL}H}{V_L}$ is the inverse of the labour elasticity, θ is the elasticity of substitution of

[5] In the pervious version (ECB wp 489), we modeled the monetary policy in terms of the money supply, and obtained similar results as we do here with the Taylor rule formulation.

consumption goods. Using equation (4) and aggregating for sector i, we get

$$p_{it} = w_{it} + \left(\frac{1-\sigma}{\sigma}\right) y_{it} \tag{19}$$

$$\text{with } w_{it} = \frac{1}{i}\sum_{j=1}^{i} w_{ijt}$$

Using equation (2) and aggregating for sector i yields

$$y_{it} = \theta(p_t - p_{it}) + y_t \tag{20}$$

Log-linerazing (11) yields the following

$$y_t = E_t y_{t+1} - \eta_{cc}^{-1}(r_t - E_t \pi_{t+1}) \tag{21}$$

The linearized price index in the economy is a weighted average of the ongoing prices in all sectors and is given by

$$p_t = \sum_{i=1}^{N} \alpha_i p_{it} \tag{22}$$

The Taylor rule is given by

$$r_t = \phi_r r_{t-1} + \phi_\pi \bar{\pi}_t + \phi_{y1} y_t + \phi_{y1} y_{t-1} + \ln \mu_t \tag{23}$$

where $\ln \mu_t = v.\ln(\mu_{t-1}) + \xi_t$

4 The Calibration of Simple Taylor Economies with Wage and Price setting

The utility is additively separable and for simplicity, we approximate $\beta \simeq 1$. The survey by Pancavel (1986) suggests that η_{LL} is between 2.2 and infinity. Following the literature, we set $\eta_{LL} = 4.5$, which implies that intertemporal labour supply elasticity, $1/\eta_{LL}$, is 0.2. Following Ascari (2000) and Huang and Liu (2002), we set $\theta = 6$, $\eta_{CC} = 1$ and $\sigma = 1$. Following Orphanides and Wieland, in the interest rate rule we set $\phi_r = 0.795$, $\phi_\pi = 0.625$, $\phi_{y1} = 1.17$, $\phi_{y2} = -0.97$ and $\nu = 0.8$[6]. Finally, we assume that at time t there is 1%

[6] Smets and Wouters (2005) interpret a persistent change of the inflation target as a serially correlated monetary policy shocks.

shock to the disturbance term in the interest rate rule ξ_t, so that $\xi_t = 1$ and $\xi_s = 0$ for all $s > t$.

The key parameter determining aggregate dynamics is γ (18). The magnitude of γ is important since it governs how responsive household-unions are to current and future changes in output (see equation 17). When there is an increase in aggregate demand, households face higher demand for their labour and therefore the marginal disutility of labour increases. With higher income they consume more and marginal utility of consumption falls. The combination of an increase in the marginal disutility of labour and the fall in the marginal utility of consumption leads household-unions to increase their wage. The coefficient γ determines how wages change in response to changes in current and future output. If γ is large, wages respond a lot to changes in output which implies faster adjustments and a short-lived response of output. On the other hand, if γ is small, unions are not sensitive to changes in current and future output. In response to an increase in aggregate demand, the wage would not change very much and hence wages are more rigid. In the limit, if $\gamma = 0$, there will be no relationship between output and wages, so that shocks are permanent. Hence the smaller γ, the more wages are rigid and hence the more persistent are output responses.

Estimating γ as an unconstrained parameter, Taylor (1980) found that for the U.S. γ is between 0.05 and 0.1. However, in a general equilibrium framework γ is derived so as to conform to micro-foundations. Both CKM and Ascari argue that the microfounded value of γ is too high generate the observed persistence following a monetary shock, hence raising doubts over the Taylor model in this respect.

With staggered *price* setting, CKM find that with reasonable parameter values, the value of γ is bigger than one: in particular with our calibration[7]: $\gamma^{CKM} = \eta_{LL} + \eta_{cc} = 5.5 > 1$. The value of γ with wage-setting is much smaller under our calibration, as in Ascari, we have: $\gamma^{A} = \gamma^{CKM} / \left(1 + \theta \eta_{LL}\right) = 0.2$. The lower value of γ means that in Ascari's wage setting model the aggregate price level changes more slowly than in CKM's price setting model[8]. However, Edge (2002) shows that price-setting is also consistent with lower values of γ if there is a firm-specific labour market (see also Ascari (2003)). In particular,

[7] CKM's own calibration has $\gamma^{CKM} = 1.2$. Huang and Liu (2002) calibrate it at $\gamma^{CKM} = 2$.

[8] In fact, this finding is the main reason behind the conclusion of Huang and Liu (2002), who argue that staggered price setting by itself is incapable of generating sufficient persistence, whilst staggered wage setting has a greater potential.

with firm-specific labour γ is given by (18) under both wage and price setting.

Figure 1 here

The problem is that for the calibrated values of γ, output is not as persistent as in the data. Figure 1 depicts the impulse response functions for $\gamma^{CKM} = 1.22$, $\gamma^{A} = 0.20$ and the estimated value $\gamma = 0.05$ originally used by Taylor (1980). We assume a simple Taylor economy with $T = 2$ (wages last 6 months). All other decisions are made quarterly. We display the impulse-response functions for output after a one percent monetary shock. As we can see, output displays similar patterns in the case of γ^{CKM} and γ^{A}: in both cases, output increases when the shock hits and quickly returns to its steady state level. Output is certainly more persistent with $\gamma = 0.20$, but not enough to match the data.

5 Persistence in a GTE

The existing literature has tended to focus on the value γ in generating persistence. We want to explore another dimension: for a given γ, we allow for different contract lengths in the GTE framework we have developed. In what follows, we show that including longer term contracts can significantly increase persistence. Of course, this is in a sense obvious: longer contracts lead to more persistence, and we can achieve any level of persistence if contracts are long enough (so long as $\gamma > 0$). However, we want to show that even a small proportion of long-term contracts can lead to a significant increase. Throughout this section, we will take the value of $\gamma = 0.2$ and explore how persistence changes when we allow for a range of contract lengths. We do this in three stages: first we simply illustrate our case with a simple two sector example. Second, we use the Bils-Klenow dataset on price-data to calibrated model of the U.S. economy allowing for contract lengths from 1-20 quarters. In the next section we consider the Calvo contract process with the corresponding distribution of contract lengths from 1 to infinity.

5.1 Two-sector GTEs.

First, let us illustrate the main point of the paper by a simple two-sector example. In Figure 2 we have the output response compared in two GTEs with a mean contract length of 2: one is a simple Taylor economy, the other consists of mainly flexible wages and 1/7 are 8 period contracts. The presence of the perfectly flexible one period contracts leads to a dampened impact relative to the 2−period Taylor. However, it is clear that although the economy consists mainly of flexible wages, the output dies away slowly and after the second quarter output is larger in the mixed economy. This is because the 8 period contracts are holding back the general price level and hence influencing the wage-setting of the flexible sector.

Figure 2 here

The intuition behind this finding is that the presence of the longer term contracts influences the wage-setting behaviour of the short-term contracts. This can be seen as a sort of "strategic complementarity". A monetary expansion means that the new steady state price is higher. When setting wages, unions trade off the current price level and the future. The fact that the long-contracts will adjust sluggishly means that the shorter contracts will also react more sluggishly, since their wage setting is influenced by the general price level which includes the prices of the more sluggish sectors. There is a spillover effect from the sluggish long-contract sectors to the short-contract sectors via the price level, a mechanism identified previously in Dixon (1994).

5.2 An Application to U.S. Data.

We now consider an empirical distribution of contract lengths derived from the Bils and Klenow (2004) dataset based on U.S. Consumer Price Index (CPI) microdata[9]. We will then examine the impulse response function and compare it to the actual behaviour of U.S. output taken from CKM.

The data is derived from the U.S. CPI data collected by the *Bureau of Labor statistics.* The period covered is 1995-7, and the 350 categories account for 69% of the CPI. The data set gives the average proportion of prices changing per month for each category. We assume that this is generated by a simple Calvo process within each sector (the proportion of firms changing price per month equals the monthly sector specific reset probability). We then generate the distribution of durations within each sector, and aggregate across sectors to obtain the distribution in the economy[10]. Figure 3 plots the

[9]We could apply exactly the same methodology to the Nakamura and Steinsson (2007a) data on the frequency of price change.

[10]Note, the mean is much longer than stated by Bils and Klenow themselves. This is for

distribution in terms of quarters.

Figure 3 here.

The mean contract length is 4.4 quarters[11]. The most striking aspect of this distribution is the high share of short-term contracts. The share of 1 and 2 period contracts are about 50% (see Dixon and Kara (2010) for a more detailed discussion).

CKM estimated the dynamic response of output to a policy shock by fitting an AR(2) process to quadratically detrended log of real GDP[12]:

$$y_t = 1.30y_{t-1} - 0.38y_{t-2} + \xi_t$$

The impulse response of output to a unit shock in ξ_t is plotted in Figure 4 (the solid line)[13]. As it is evident from the figure, the estimated output response is persistent: the half life of output is 10 quarters. Another important feature of this response is its hump shape: the response peaks three quarters after the shock. The pattern is consistent with other empirical studies that use an explicit VAR analysis such as Christiano, Eichenbaum and Evans (2005) (CEE hereafter) which have a peak output response of 3-4 quarters and similar half-life[14]. The response is also consistent with the one reported in Smets and Wouters (2007), which uses Bayesian Techniques.

Figure 4 here.

Figure 4 also reports the impulse response functions for output in the $BK - GTE$ and in the simple Taylor Economy for $T = 4$. For comparison

two reasons. First, our mean is the distribution of contract lengths across firms, whereas BK are inferring the average length of contracts; see Dixon (2006) for a full discussion. Second, they are using continuous time: the average allows for firms to reset prices more than once per discrete period.

[11] Note that we are reporting the mean using the full value for each quarter (i.e 1 for the first quarter, i for the ith quarter). If we used the mid-point, then we would simply subtract 0.5 from this mean yielding a mean of 3.9 quarters.

[12] Rather than using a univariate regression, it would have been better to use an explicit VAR analysis to provide identification of a monetary shock (which is clearly absent in CKMs method). However, we are using this for illustrative purposes only and as a concise method of representing US output dynamics in an approximate form.

[13] Note that CKM find little evidence for serial correlation of the residuals.

[14] Mankiw and Reis (2002) also use very similiar features to evaluate the empirical performance of their Sticky Information model.

purposes, the responses are normalized in the sense that the impact is set at 1. As we can see, incorporating empirically relevant contract structure into an otherwise standard DSGE model has a significant effect on dynamic response of output. The response of output in the $BK-GTE$ and the estimated output response have almost identical characteristics. Specifically, the $BK-GTE$ generates a hump-shaped persistent output response and the half life is about 10 quarters. In this sense, the GTE framework with the BK distribution is able to explain the observed pattern of output. The figure further shows that the simple Taylor economy generates much less persistence than the BK-GTE, even though both settings have very similar mean contract lengths. This can be most easily seen by comparing areas under the impulse response functions in both models. The area in the $BK-GTE$ is about the twice the area in the simple Taylor[15].

For robustness, we also explore the implications for allowing a distribution of contract lengths on persistence in terms of two measures of persistence proposed in the literature. One is the "contract multiplier" proposed by CKM, which is defined as the ratio of the half life of output to one-half length of exogenous stickiness. The other one is the "mean lag" measure suggested by Dotsey and King (2006). Mean lag is defined as the ratio of $\sum_{j=0}^{\infty} j * \kappa_j / \sum_{j=0}^{\infty} \kappa_j$, where κ_j is the impulse response coefficient for output at lag j[16].

As Table 1 shows, the both measures of persistence indicate that the $BK-GTE$ generates about 1.6 times more persistence than the simple Taylor contracts. The mean lag of the $BK-GTE$ is less than the mean lag of the CKM response, but it is much closer to the mean lag of the CKM response than the mean lag of the simple Taylor.

	BK-GTE	**Taylor; T = 4**	**CKM IR**
Contract Multiplier	4.4	2.8	
Mean Lag	5.9	3.6	6.6

Table 1: Persistence Measures

[15] Our results are not very sensitive to the assumed value of ϕ_r. We obtain very similar results when we assume a lower $\phi_r \simeq 0.35$. This finding is in line with empirical findings reported in Rudebusch (2002) and Gerlach-Kristen (2004).

[16] We trancate the sum in these experssions at 35 quarters. Adding more terms does not significantly affect the results.

As noted by CKM, the contract multiplier does not vary a lot with the contract length in the simple Taylor. We calculate the contract multiplier in the simple Taylor model for contract lengths $T = 2, 6, 8$. The resulting multipliers are $2.5, 2.8, 2.6$ respectively. The only way to match the observed pattern of output with a simple Taylor model is to assume implausibly long contract lengths or implausible parameter values. In order to get the same degree of persistence in a simple Taylor model, a contract length of 8 quarters is required. Alternatively, if we keep $T = 4, \sigma = \eta_{CC} = 1$ and $\eta_{LL} = 4.5$, then a value of $\theta = 27$ is required, which is implausibly high. As discussed earlier, a reasonable range of θ is from 6 to 10[17]. It is interesting to note that the value of $\theta = 27$ implies that $\gamma = 0.05$, which takes us back to the value of γ put forward by Taylor (1980)[18].

Empirical studies (e.g. CEE) in identifying monetary shocks typically assume that variables such as output, prices and wages do not immediately respond to a monetary policy shock, whereas, so far, we assume that a monetary policy shock at period t can affect these variables at period t . We consider what happens if we impose time delays. More specifically, we perform the same experiment as in Table 1 but, following CEE, assume that a monetary policy shock in period t has no effect on output, prices and wages before period $t + 1$. All the other conditions are the same. Perhaps not surprisingly, we find that, with time delays, the output responses in both the $BK - GTE$ and the simple Taylor economy are more persistent than the case without, although the increases are not large. For example, with time delays, the mean lag of the $BK - GTE$ is around 6.4. The corresponding number in the simple Taylor economy is 4.2.

6 Comparison with a Calvo Economy

It has long been noted that Calvo contracts appear to be far more persistent than Taylor contracts. In this section, we will show that if we focus on the structure of contracts (as opposed to the wage-setting rule), the Calvo

[17] We calculate $T = 8$ and $\theta = 27$ by matching the autocorrelation functions of output in the simple Taylor to that of the estimated output response in CKM.

[18] When there is strategic substitutability in the wage setting (i.e. $\gamma > 1$), our result that $BK - GTE$ generates a higher degree of persistence than the simple Taylor still holds. However, for the reasons discussed in Section 3.2, the output response in the BK-GTE is considerably less persistent than CKM's estimated output response.

economy is a special case of the *GTE*. Two main features of the Calvo setup stand out as different form the standard Taylor setup. First its "stochastic" nature: at the firm or household level, the length of the wage contract is random. Second, that the model is described in terms of the "age" of contracts (which includes uncompleted durations) and the hazard rate (the reset probability ω). On the first issue, the stochastic nature of the Calvo model at the firm level does affect the wage setting decision. However, apart form the wage setting decision we can describe the Calvo process in deterministic terms at the aggregate level because the firm level randomness washes out. At the aggregate level, the precise identity of individual firms does not matter: what matters is population demographics in terms of proportions of firms setting contracts of particular lengths at particular times. Because there is a continuum of firms, the behavior of contracts at the aggregate level can be seen as a purely deterministic process.

The second difference is one of perspective. As shown in Dixon (2006), any steady state distribution of contracts can be looked at equivalently in terms of the age distribution/hazard rate, or as the distribution of completed contract lengths across firms. In Dixon and Kara (2006) we apply this idea to the comparison of Calvo and simple Taylor contracts. With a reset probability the cross-sectional distribution is represented by the vector of proportions α_i^s of firms surviving at least i periods:

$$\alpha_i^s = \omega \left(1-\omega\right)^{i-1} : i = 1..\infty \tag{24}$$

with mean $\bar{s} = \omega^{-1}$. In demographic terms, i is the *age* of the contract: α_i^s is the proportion of the population of age s; $\bar{s}$ is the average age of the population. The corresponding distribution of completed contract lengths is given by:

$$\alpha_i = \omega^2 i \left(1-\omega\right)^{i-1} : i = 1..\infty \tag{25}$$

with mean $\bar{T} = \frac{2-\omega}{\omega}$. In demographic terms, α_i gives the distribution of ages at death (for example as reported by the registrar of deaths) for the same cross-section: α_i being the proportion of the steady state population who will live to die at age i.

Assuming that we are in steady state (which is implicit in the use of the Calvo model), we can assume that there are in fact ex ante *fixed* contract lengths. We can classify household-unions by the duration of their "contract". The fact that the contract length is fixed is perfectly compatible with the notion of a reset probability if we assume that *the wage-setter does* *not know the contract length*. We can think of the wage-setter having a probability distribution over contract lengths given by α_i^s in (24): Nature chooses the contract length, but the wage-setters do not know this when they have to set the wage (when the contract begins)[19]. Having redefined the Calvo economy in terms of completed contract lengths, we can now define the GTE with exactly the same distribution of completed contract lengths: $GTE\left(\boldsymbol{\alpha}\right)$ where α_i are given by (25) [20].

6.1 Wage-setting in the Calvo-*GTE*

We have defined the Calvo-GTE in terms of the structure of completed contract lengths. The only difference between the Calvo economy and the Calvo-GTE is in the wage-setting decision (exactly the same arguments and observations apply to price-setting). In the Calvo economy, the wage-setter is uncertain of the contract length: the wage-setting decision must be made "ex ante", that is, before the firm knows which length nature has chosen. This yields the standard Calvo wage-setting decision. Once the wage is set, the firm finds out its contract length in due course[21]. By contrast, in the Calvo-GTE, the wage-setters know which sector they belong to when they set the wage. Hence, wages in each sector of the Calvo-GTE will be different. Taking the simple case of $\beta = 1$, from (17) the reset wage in sector i is then the average "optimal" price over the following i periods:

$$x_{it} = \frac{1}{i} \sum_{s=0}^{i-1} \left(p_{t+s} + \gamma y_{t+s}\right) \tag{27}$$

Thus, in sector i, the wage-setter does not need to look forward more than i periods. If we take the mean reset-wage in the Calvo-GTE, we need to

[19]In game-theory terms wage-setting is done under incomplete information.

[20]In the sector i, a proportion α_i/i contracts come to an end. Hence, using (25) and summing across all sectors the total measure of all contracts in the economy coming to an end in any period is ω, since:

$$\sum_{i=1}^{\infty} \frac{\alpha_i}{i} = \sum_{i=1}^{\infty} \omega^2 \left(1-\omega\right)^{i-1} = \omega. \tag{26}$$

[21]It does not matter when: either straight after the pricing decision or at the last moment when it gets the Calvo phone call that it is time to reset the wage.

measure the mean *conditional on the wage being reset*, across the subset of firms who are resetting price[22]:

$$\bar{x}_t = \frac{1}{\omega}\sum_{i=1}^{\infty}\frac{\alpha_i}{i}x_{it} = \sum_{i=1}^{\infty}\omega\left(1-\omega\right)^{i-1}x_{it} \tag{28}$$

There are thus two main differences between the Calvo and the Calvo-GTE wage-setting rules. First, in the Calvo-GTE there is a distribution of sector specific reset wages x_{it} in each period. Hence, in addition to the distribution of wages across cohorts (defined by when they last reset wages) as in the Calvo model, the GTE has a distribution across sectors within the cohort. Second, in the GTE the wage-setters are more myopic: in sector i they only look i periods ahead when setting their wage according to (27). In the Calvo model, the firms do not know how long their contract will last, so that they have to look forward into the indefinite future when they reset their wages. This means that on average the firms are more myopic in the Calvo-GTE than in the Calvo model[23].

6.2 Persistence in the Calvo and Calvo-GTE compared

We now compare the Calvo-GTE and the standard Calvo economy in terms of the impulse-response functions. In theory, the Calvo-GTE and the Calvo economy are exactly the same in terms of contract structure. However, for computational purposes whilst the Calvo economy effectively has an infinite lag structure (via the Koyck transform), the Calvo-GTE has to be truncated. Specifically, we truncate the distribution of contract lengths to 20 quarters $T = 1, ...20.$ with the 20 period contracts absorbing all of the weight from the longer contracts.

Figure 5 compares the impulse response for the Calvo-GTE which has the same distribution of completed contract lengths as the Calvo distribution,

[22] This is different from the unconditional mean, using the sectoral weights: $\hat{x}_t = \sum_{i=1}^{\infty}\alpha_i x_{it}$. Within the sector with i period contracts only i^{-1} reset their wages each period. Hence if we weight each sector using (26), then the less frequent wage setters are under-represented relative to their share in the total population. A union that resets every period ($i = 1$) is counted every period, whilst a union that resets every 10 periods is only counted once every 10 periods.

[23] In an earlier version of the paper (Cardiff Business School economics working paper 2007/1), this was formalised by the concept of forward lookigness (pp.23-24).

with the standard Calvo economy for $\omega = 0.4$. As the figure shows, the Calvo-GTE has very similar persistence to the Calvo economy. The effect is as little larger for 4 quarters and a little less subsequently, reflecting the less forward looking pricing behaviour. The figure also reports the standard Taylor economy with the same cross-section mean contract length $\bar{T} = 4$. Although the effect is a greater for the first 3 quarters, the effect dies down and is significantly less thereafter. This reflects the fact that although the mean contract lengths are the same, the longer contracts in the Calvo and Calvo-GTE generate the extra persistence.

Figure 5 here

Figure 6 here

To understand the difference between the Calvo and Calvo-GTE, we focus on wage-setting behavior as depicted in Figure 6[24]. In the Calvo economy the wage-resetters are more forward looking and so raise wages more in the initial period in anticipation of the future price rises. This leads to a slightly smaller increase in output in the first few periods. As the new steady state is approached, the Calvo resetters slow down the increase in wages, whilst the more myopic Calvo-GTE wage resetters keep up the momentum of wage increases, so that the output becomes a little larger in the Calvo case.

7 Conclusions

In this paper we have developed a general framework, the GTE which unifies the previously disparate approaches of modelling dynamic price and wage setting: Calvo and Taylor. The approach is a generalization of the simple Taylor model to take into account the presence of a range of different contract lengths. We use this approach to focus on the effect of the presence longer term contracts on the persistence of impulse-response functions generated by a monetary shock.

- A small proportion of long-term contracts can generate a significant increase in persistence.

[24] Given that we truncate the *GTE,* for the sake of comparison, we also truncate the Calvo, at the same number of quarters. We then compare this truncated Calvo with the standard Calvo. We find that there is a perceptible but negligible difference; hence all of the visually apparent differences between the Calvo-GTE and the standard Calvo model are due almost entirely to the difference in wage-setting behaviour.

- We apply the idea to US data using the Bils-Klenow dataset to generate the distribution of contract lengths. We find that the impulse response for this distribution is very similar to an empirical impulse response function.

- In general, if we want to model an economy with many different contract lengths using a simple Taylor economy, we should choose a contract length which is greater than the average. This is because the presence of contracts with longer duration leads to more persistence despite having a similar mean. In the case of the Bils-Klenow distribution (which has a mean of just over 4 quarters), we would need a simple Taylor model to have 8 quarters to generate the equivalent persistence.

- We are able to compare the calvo and Calvo-GTE: the two differ in so much as the firms in the latter know how long their contracts will last. We find that with exactly the same distribution of contract lengths, the two are quite similar in terms of the persistence they generate, but there are small differences in price and wage setting due to the fact that Calvo firms (who do not know how long their contracts will last) are more forward looking on average.

In this paper, we have treated the durations of contracts as a given. However, in practice the duration of contracts will be endogenous and linked to the properties of the market. In particular, we have assumed that all sectors are symmetric in terms of preferences. However, if there are closer links between sectors with similar contract lengths, then this may affect the results. We leave this as a matter for future research.

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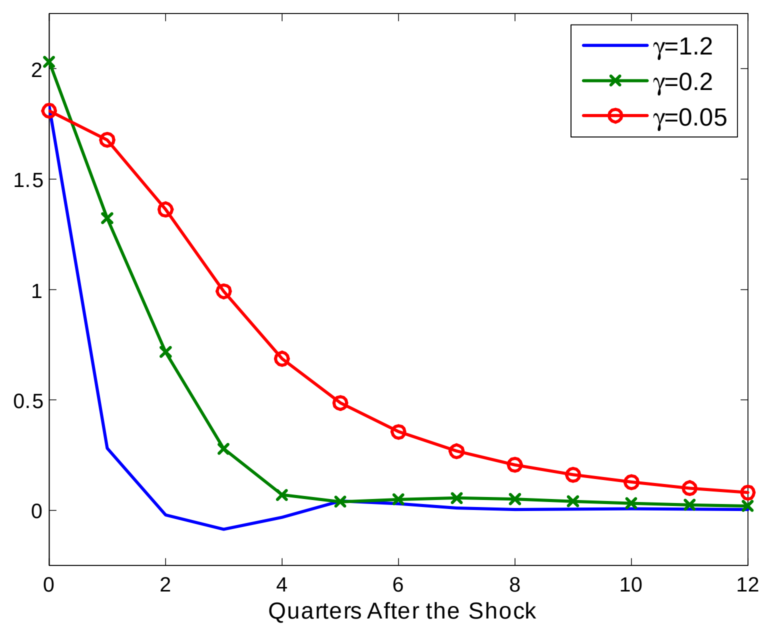

Figure 1: Output response for alternative $\gamma' s$

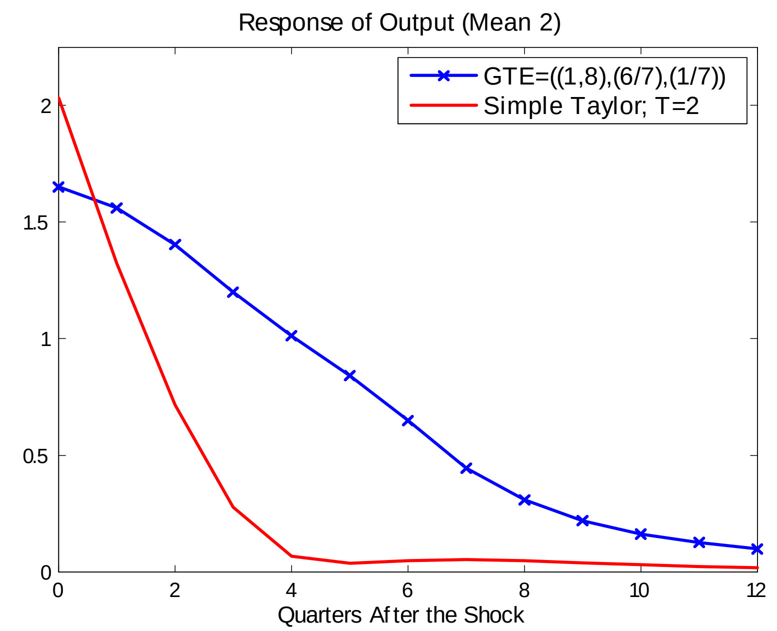

Figure 2: A mean preserving spread increases persistence

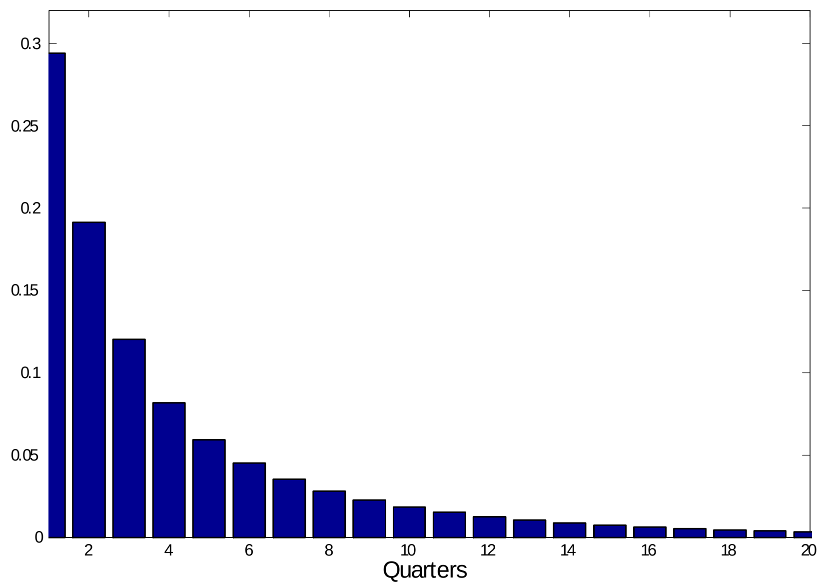

Figure 3: The U.S. distribution of price contract lengths derived from the Bils-Klenow data set

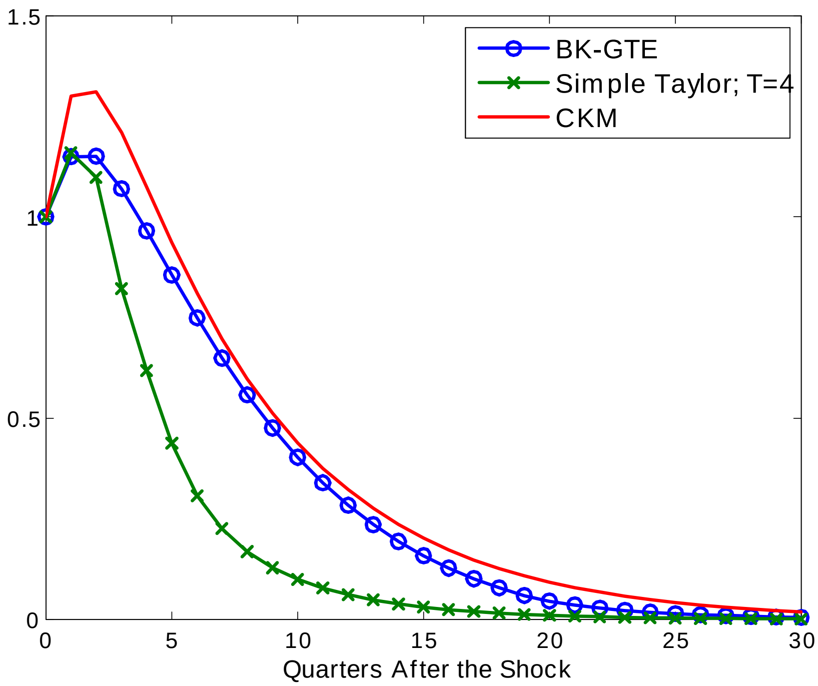

Figure 4: Output response in the $BK - GTE$ and CKM's estimated output response

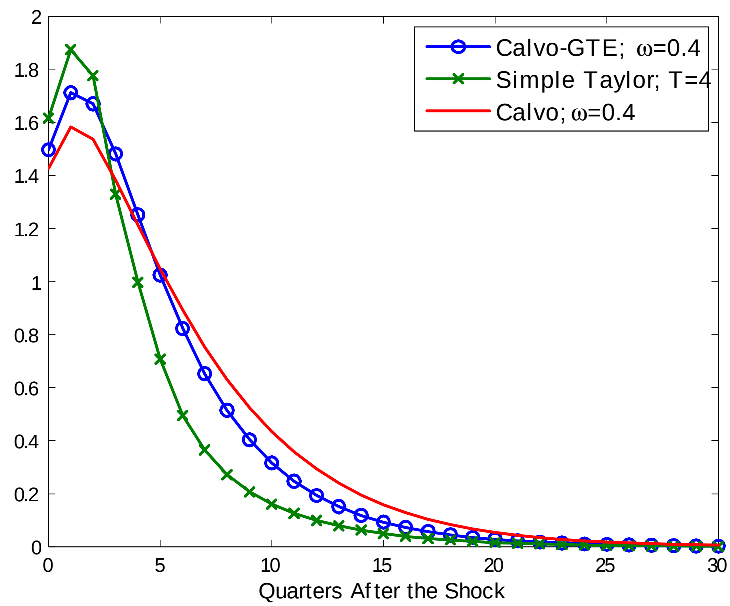

Figure 5: Output responses of the Calvo Economy, the corresponding GTE and the simple Taylor economy with same mean contract length

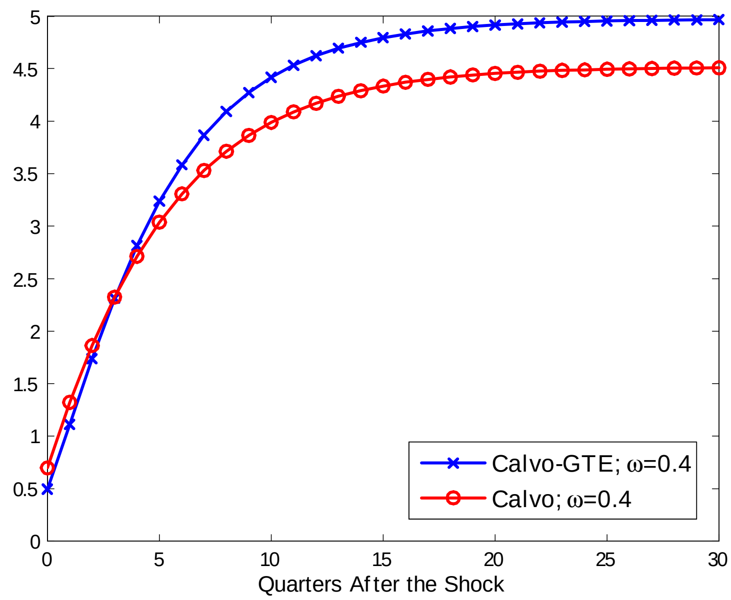

Figure 6: Responses of average reset wage for the Calvo Economy and the Calvo-GTE with $\omega = 0.4$.