A unified framework for using micro-data to compare dynamic time-dependent price-setting models.*

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Abstract

This paper develops a statistical framework of steady-state identities which enable us to match the distributions of durations found in the micro-data to generalized Taylor and Calvo models of time-dependent pricing. We illustrate the approach with the UK micro CPI data for 1996-2009, and employ the pricing models in a simple macromodel. We find that the Generalized Taylor Economy generates a hump shaped response function, whilst the Generalized Calvo does not.

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Dynamic pricing and wage-setting models have become central to macroeconomic modelling in the new neoclassical synthesis approach. It has become apparent that different models of pricing have different implications for matters such as the persistence of output and inflation to monetary shocks. Different models of wage or price setting imply different distributions of durations of price-spells (throughout this paper, we will use "price" as a shorthand for "wage and price"). In this paper we focus on the class of time or duration dependent models of pricing, such as Calvo and Taylor, rather than state-dependent models (Dotsey et al 1999, John and Wolman 2008). We formulate a unified framework for consistently understanding and comparing these models and linking them to microdata on prices and wages (in this paper we will talk exclusively about pricing and price-data, but exactly the same approach can be applied to wages as in Dixon and Le Bihan (2011).

We start from the idea of modelling the class of all steady state distributions of durations across a given population (in this case, the firms that set prices). In steady state there are four equivalent ways of describing this. Three of these are well known and standard: the fourth is a new concept, but one that is needed and is developed in this paper to link the data to the economic theory. First, there is the distribution of durations: this treats each price-spell as an individual element in the population. This description ignores the panel structure of the microdata, in which sequnences of price-spells (trajectories) are generated by firms. Second, there is the cross-sectional distribution of *ages*: at a point in time, how long it has been since the current price-spell began. This is like the population census. Third, we can look at the distribution in terms of *hazard rates* or *survival probabilities*: from the cross-section of ages, the probability of progressing from one age to the next one. The main contribution of this paper is to introduce the new concept of the *cross-sectional distribution of completed price-spells* (lifetimes): this corresponds to the average completed price-spell across firms and hence in this context we call it the *Distribution across firms* (DAF). We develop a transparent framework that allows us to move between these concepts. The first three concepts (distribution of durations, cross-section of ages and hazard rates) are of course very well understood in statistics, being basic tools in demography, evolutionary biology and elsewhere. The fourth concept, the cross-sectional distribution of *completed* durations is a new concept. However, this concept is essential if we are to answer questions such as what is the average price-spell across firms and to apply these concepts to understand and compare different models of pricing.

Whilst all four ways of describing the data ared equivalent statistically, only two of them are directly related to modelling price and wage setting. The distribution of durations itself does not link easily to any theory of price setting. However, the hazard function and the the cross sectional DAF have direct application. In the Generalized Taylor Economy (GT) (Taylor 1993, Coenen et al 2007, Dixon and Kara 2010, 2011), there are many sectors with different price-spell lengths, and within each sector there is a simple Taylor process. The simple Taylor economy where all contract lengths are the same is a special case of the GT. The GT is linked to the microdata by looking at the new cross-sectional DAF : any steady-state distribution can be represented by a unique GT. In the Calvo approach, we have a reset probability which may be constant (as in the classical Calvo model) or duration dependent (Wolman 1999, Mash 2003 and 2004, Guerrieri 2006, Sheedy 2007, Paustian and von Hagen 2008). We show that the Calvo model with duration-dependent reset probabilities (denoted as the *Generalized Calvo model* GC) is linked to the microdata through the hazard function: any hazard function can be represented by a unique GC. Hence, both the GT and GC are coextensive with the set of all steady state distributions: each possible steady state distribution has exactly one GC and one GT which corresponds to it. When we look at an economy, we can choose the GT and the GC to exactly match the empirical distribution found in the data.

A great advantage of our approach is that we can move between the four statistical descriptions: if our data is in the form of a "census" (ages at a point in time) we can use the identites in this paper to generate both the hazard function and the cross-sectional DAF. In this paper, we use the estimated hazard function from UK CPI microdata with around 2 million price-spells to construct the corresponding DAF. We are also able to compare the different models of pricing *for a given distribution of durations* of price spells. This enables us to isolate the precise effect of the *pricing model* as opposed to the *differences in the distribution of durations*. For example, if we compare a simple Taylor 4 model with a simple Calvo model with reset probability 0.25, these will have completely different distributions: in one case all price-spells have the same length, whilst in the other there is a distribution of lengths from 1 period to infinity. If we find that the same model with Taylor exhibits different behavior to the corresponding Calvo, it is not clear whether it is because the pricing models differ, or the distribution of spells differs.

The framework in this paper also allows us to directly link microdata to models of wage and price setting. We can take a given distribution of price-spells and model it as either a GT or a GC. We take UK CPI price data for the period 1996-2006 described in Bunn and Ellis (2009, 2010a) and estimate the hazard function from this data which we can use to calibrate a our models of pricing: the resultant GT and GC have *exactly the same distribution of price-spells* as the UK data. We take the sectoral data in Bunn and Ellis (2010a) to model the UK as an 11-sector multiple Calvo model (MC) with sector specific Calvo reset probabilties (as in Carvalho 2006 and Carvalho and Nechio 2008) We are thus able to move directly from the microdata to the three models of pricing. We are able to compare these three pricing models in a simple model economy, enabling us to highlight the differences in the pricing model controlling for the distribution of price-spells. We are also able to compare the GT and GC in the more complicated Smets and Wouters (2003) model. What we find is that for this distribution at least, the three pricing models are quite close in terms of the impulse-response functions they generate in response to a monetary shock. In particular, the GC and MC are quite similar. However, there can be differences: with the UK data we find that the GT has a hump shaped impulse-response for inflation, whilst the GC and MC do not. This reflects the fact that in a GT the firms know how long their price-spell is due to last and in this sense are more "myopic" in their pricing deicisions. We formalize this concept of myopia in a precise concept of *forward lookingnes* and find that on average when firms reset their prices they are less forward looking in the GT model with fixed contract lengths than in the equivalent GC where firms that reset prices do not know how long the price is going to last. We believe that this is the key difference between the Taylor and Calvo approaches to pricing, and that it is this difference that gives rise to different impulse-responses for inflation. In a GT, firms with short price spells of one or two months will just react to what is happening now when they set their prices, whereas all firms in the GC have to look ahead since there is a probability that the price they set now might last a long time.

The existing literature linking price microdata to pricing models has tended to focus on the frequencies of price change (i.e. the proportion of firms changing prices in a given month): see for example the US studies of Bils and Klenow (2004), Klenow and Krystov (2008), Nakamura and Steinsson (2008). This frequency can be linked to the Calvo reset probability at some level. For example, Carvalho (2006) uses the highly disaggregated sectoral data provided by Bils and Klenow (2004). However, whilst the frequency is an interesting statistic, you lose the information about the distribution. Certainly, the UK data does not have a distribution of price spells that corresponds to what would be implied by the Calvo model. The Calvo model has not performed well empirically: when you use it in macroeconomic models it does not give good results in terms of impulse responses[1]. This has given rise to the use of Calvo with indexation (for example Christiano et al 2005, Smets and Wouters 2003). However, indexation just makes a bad model worse. Although it gives better impulse response functions, it does so at the cost being completely at odds with the microdata: with indexation all prices adjust every period. Indeed, other pricing models share the defect of implying all prices change every period: for example sticky information (Mankiw and Reis 2002) and the rational inattention model of Mackowiak and Weiderholt (2009)[2]. The approach adopted in this paper enables the theory to be consistent with the micro evidence.

In section 1 we review the facts about the steady state distribution of durations, ages and hazard rates. We then introduce the new concept of the cross-sectional distribution of durations across firms and show how all four concepts are related by simple formulae which are spreadsheet friendly and provide some simple examples. In section 2, we define our models of pricing and show that the GC and GT are consistent with any distribution of price-spells. In section 3 we study the UK distribution of durations and in section 4 use the UK data to calibrate the pricing models in both a simple macroeconomy and the more complex Smets and Wouters (2003) model. .

1 Steady State Distributions of Durations across Firms.

We will consider the steady-state demographics of price-spells in terms of their durations. The *lifetime* of a price-spell is how long it lasts from its start to its finish, a *completed* duration. There is a continuum of agents f (we will call them firms here), which set prices (or wages), represented by the unit interval $f \in [0, 1]$. Time is discrete and infinite $t \in Z_+ = \{0, 1, 2...\infty\}$ .A

[1] For a more detailed discussion, see Dixon and Kara (2010, 2011).

[2] Both of these models assume that the firm chooses a trajectory of prices (i.e. a sequence of different prices), and that due to "sticky information" or "rational inattention" these plans become more out of date leading to a gap between the current optimal price and what was expected to be the optimal price when the price was planned.

price event (or price-quote) is a price set by a particular firm at a particular time: p_{ft}. A price spell is a duration, a sequence of consecutive periods that have the same price. For every price event pair $\{t, f\}$ we can assign an integer $d(t, f)$ which is the price spell duration of which the price event is part of. Furthermore, we can define the subset of reset price events, when firms set a new price:

$$R = \{(t, f) : p_{ft} \neq p_{ft-1}\} \subseteq Z_{+} \times [0, 1] \tag{1}$$

The distribution of durations is derived from the set R. Let the longest duration[3] be $F < \infty$. Then we can define F subsets of R

$$R(i) = \{(t, f) \in R : d(t, f) = i\} \tag{2}$$

Thus $R(i)$ gives us the subset of durations of length i. The distribution of durations is simply the proportions of all durations having length $i = 1...F$:

$$\boldsymbol{\alpha}^d = \left\{\alpha_i^d\right\}_{i=1}^{F} \in \Delta^{F-1}$$

where Δ^{F-1} is the $F-1$ unit simplex with $\sum_{i=1}^{F} \alpha_i^d = 1$ and $\alpha_i^d \geq 0$. In steady-state this simplifies, since the distribution of durations of new price-spells is the same each period, we can take any "representative" $t > F$ and define

$$\alpha_i^d = \alpha_i^d(t) = \frac{\int_0^1 I((f,t) \in R(i))df}{\int_0^1 I((f,t) \in R)df}$$

Where I is an index function that takes the value 1 if (at our chosen t) price event (f, t) is in the relevant set, 0 otherwise.

In steady-state the distribution of durations is the same as the distribution of durations taken over the subset of reset prices (new price spells). To see why, in steady state each period t a set of new price-spells comes into being, and set of price spells end. In steady state, the new starter price spells will have exactly the same distribution of completed durations as the old price spells finishing. The whole population of price-spells over time will simply be the summation over all of the generations of price-spells. Since each generation has the same distribution, the distribution of price-spell durations taken as a whole is exactly the same as the distribution of new price spells in any one period.

[3]The finiteness of F is merely for convenience and has no importance since it can be set arbitrarily large.

1.1 Ages.

The age of the price-spell of fimr f at time t is defined as the period of time that has elapsed since the price spell started $A(f,t)$. Since we have integer time, we adopt the convention that the minimum age is 1. Let us define the subset of firms at time t that are of age $A = j$.

$$j(t) = \{f \in [0,1] : A(f,t) = j\}$$

Then the proportion of firms aged j at t is for all $t > F$

$$\alpha_j^A = \alpha_j^A(t) = \int_0^1 I\left((f,t) \in j(t)\right).df$$

The steady-state distribution of ages is necessarily monotonic: you cannot have more old price-spells than younger, since to become old you must first be young. Hence the set of all possible steady state age distributions is given by:

$$\Delta_M^{F-1} = \left\{\boldsymbol{\alpha}^A \in \Delta^{F-1} : \alpha_j^A \geq \alpha_{j+1}^A\right\}$$

where the subscript M refers to (weak) monotonicity.

1.2 Hazard Rate.

An alternative way of looking at the steady state distribution of durations and the cross-section of ages is in terms of the hazard rate. The hazard rate at a particular age is the proportion of spells at age i which do not last any longer (spells which end at age i, people who die at age i). Hence the hazard rate can be defined in terms of the distribution of ages in steady-state $\boldsymbol{\alpha}^A \in \Delta_M^{F-1}$: the corresponding vector of hazard rates $h \in [0,1)^{F-1}$ (this is called the hazard function or hazard profile) is given by:

$$h_i = \frac{\alpha_i^A - \alpha_{i+1}^A}{\alpha_i^A}; i = 1...(F-1) \tag{3}$$

Since the maximum length[4] is F, without loss of generality we set $h_F = 1$. Corresponding to the idea of a hazard function is that of the *survival*

[4] If $h_i = 1$ for some $i < F$, then i is the maximum duration and subsequent hazard rates become irrelevant. This leads to trivial non-uniqueness. We therefore define F as the *shortest* duration with a reset probability of 1.

probability[5], the probability at birth that the price survives for at least i periods, with $S_1 = 1$ and for $i > 1$

$$S_i = \Pi_{\kappa=1}^{i-1}(1 - h_\kappa) \tag{4}$$

and we define the sum of survival probabilities Σ_S and its reciprocal $\bar{h}$:

$$\Sigma_S = \sum_{i=1}^{F} S_i \quad \bar{h} = \Sigma_S^{-1} \tag{5}$$

The survival function is the F vector of survival probabilities $(S_1, S_2, ...S_F)$.

Clearly, we can invert (3), hence relating the age distribution to the hazard function:

Observation 1 given $h \in [0, 1)^{F-1}$, there exists a unique corresponding age profile $\boldsymbol{\alpha}^A \in \Delta_M^{F-1}$ given by:

$$\alpha_i^A = \bar{h} S_i \quad i = 1...F.$$

Given the flow of new contracts $\bar{h}$, the proportion surviving to age i is S_i : $\bar{h} = \Sigma_S^{-1}$ ensures adding up. From the definition of hazard rates and Observation 1 we can move from an age distribution $\boldsymbol{\alpha}^s \in \Delta_M^{F-1}$ to the hazard profile and vice versa.[6]

Observation 2 given $h \in [0, 1)^{F-1}$, there exists a unique corresponding distribution of durations $\boldsymbol{\alpha}^d \in \Delta^{F-1}$ given by:

[5] The Hazard rate can also be defined in terms of the Survival function.

$$h_i = \frac{S_i - S_{i+1}}{S_i}$$

For the relationship between continouous and discrete time used here see Kiefer (1988) and Fougere et al (2007). In continuous time,

$$h(t) = \frac{S'(t)}{S(t)}$$

whilst in discrete time we take S'as ΔS. Note that whilst in continuous time the hazard rate can be be larger than 1, the discrete time h_i cannot be larger than 1.

[6] This relationship is one of the building blocks of Life Tables (Chiang 1984), which are put to a variety of uses by demographers, actuaries and biologists. Dixon and Siciliani (2009) apply the identity to hospital waiting lists in the UK to derive the hazards and corresponding distribution of completed durations.

$$\alpha_i^d = S_i h_i \quad i = 1...F.$$

The proportion of price-spells of duration i is the proportion surviving i periods and no longer. Hence there is a unique $1-1$ relationship between elements of the set of possible duration distributions and the set of possible hazard profiles.

observation 3. For any $\boldsymbol{\alpha}^d \in \Delta_M^{F-1}$, the corresponding cross-section of ages $\boldsymbol{\alpha}^A \in \Delta_M^{F-1}$ is given by

$$\alpha_i^A = \frac{\bar{h}}{h_i}\alpha_i^d$$

and vice-versa.

1.3 The cross-sectional distribution of Completed Price-spells across Firms.

The steady-state age distribution $\boldsymbol{\alpha}^A \in \Delta_M^{F-1}$, distribution of durations $\boldsymbol{\alpha}^d \in \Delta^{F-1}$ or hazard profile $h \in [0,1]^{F-1}$ are different ways of looking at the same object: a *panel* of price events. Each row of the panel is a trajectory of prices corresponding to a particular firm. Each column is a cross-section of all of the prices set by firms at a point in time. We now introduce a fourth distribution: it is a cross-sectional distribution of *completed* durations or lifetimes across firms $\boldsymbol{\alpha} \in \Delta^{F-1}$. In effect, we take a representative t, and for each firm we see the completed price-spell duration at that time $d(f,t)$. If we define the set of firms at time t who will have a completed price spell of i periods[7]

$$R(i,t) = \{f \in [0,1] : d(t,f) = i\}, \tag{6}$$

then the proportion of firms at time t with a completed duration of i, α_i is defined by:

$$\alpha_i = \alpha_i(t) = \int_0^1 I\left((f,t) \in R(i,t)\right) df$$

Under the steady-state assumption α_i is constant over time, and hence we omit the time indicator.

We can move from the distribution of ages to the distribution of completed contract lengths across firms:

[7] Note that $R(i,t)$ is defined as a subset of firms f at a point in time. It differs from $R(i)$ which from (2) is a subset of pairs (t,f) taken across time and firms.

Proposition 1 *Consider a steady-state age distribution* $\boldsymbol{\alpha}^A \in \Delta_M^{F-1}$. *There exists a unique distribution of lifetimes across firms* $\boldsymbol{\alpha} \in \Delta^{F-1}$ *which corresponds to* $\boldsymbol{\alpha}^A$, *where*

$$\begin{aligned} \alpha_1 &= \alpha_1^A - \alpha_2^A \\ \alpha_i &= i\left(\alpha_i^A - \alpha_{i+1}^A\right) \\ &.. \\ \alpha_F &= F\alpha_F^A \end{aligned} \tag{7}$$

All proofs are in the appendix. Since there is a 1-1 mapping from age to lifetimes, we can compute the distribution of lifetimes from ages:

Corollary 1 Given a distribution of steady-state completed lifetimes across firms, $\boldsymbol{\alpha} \in \Delta^{F-1}$, there exists a unique $\boldsymbol{\alpha}^A \in \Delta_M^{F-1}$ corresponding to $\boldsymbol{\alpha}$

$$\alpha_j^A = \sum_{i=j}^{F} \frac{\alpha_i}{i} \qquad j = 1...F \tag{8}$$

The intuition behind Proposition 1 and the Corollary is clear. In a steady state, each period must look the same in terms of the distribution of ages. This implies that if we look at the i period price-spells, a proportion of i^{-1} must be renewed each period. Thus if we have 10 period contracts, 10% of these must come up for renewal each period. This implies that the proportion of contracts coming up for renewal each period (which have age 1) is:

$$\alpha_1^A = \sum_{i=1}^{\infty} \frac{\alpha_i}{i}$$

The proportion of contracts aged 2 is the set of contracts that were reset last period (α_1^s), less the ones that only last one period (α_1) and so on. The set of all possible steady-state distributions of durations can be characterized either by the set of all possible age distributions: $\boldsymbol{\alpha}^A \in \Delta_M^{F-1}$ or the set of all possible lifetime distributions across firms $\boldsymbol{\alpha} \in \Delta^{F-1}$. They are just two different ways of looking at the same thing.

Proposition 1 and its corollary show that there is an exhaustive and 1-1 relationship between steady state age distributions and lifetime distributions. Now, since we know that there is also a 1-1 relation between Hazard rates and age distributions, we can also see that there will be a 1-1 relationship between completed contract lifetimes and hazard rates. First, we can ask what distribution of completed contract durations corresponds to a given vector of hazard rates. We can simply take observation 1 to transform the hazards into the age distribution, and then apply Proposition 1.

Corollary 2 let $h \in [0,1)^{F-1}$. The distribution of lifetimes across firms corresponding to h is:

$$\alpha_i = \bar{h}.i.h_i.S_i\text{: } i = 1...F \tag{9}$$

The flow of new contracts is $\alpha_1^s = \bar{h}$ each period. To survive for exactly i periods, you have to survive to period i which happens with probability S_i, and then start a new contract which happens with probability h_i. Hence from a single cohort $\bar{h}.h_i.S_i$ will have contracts that last for exactly i periods. We then sum over the i cohorts (to include all of the contracts which are at various stages of moving towards the final period i) to get the expression.

We can also consider the reverse question: for a given distribution of completed contract lengths $\boldsymbol{\alpha}$,what is the corresponding profile of hazard rates? From Corollary 2, note that (9) is a recursive structure relating α_i and h_i: α_i only depends on the values of h_s for $s \leq i$.

Corollary 3 Consider a distribution of contract lengths across firms given by $\boldsymbol{\alpha} \in \Delta^{F-1}$. The corresponding hazard profile that will generate this distribution in steady state is given by $h \in [0,1)^{F-1}$ where:

$$h_i = \frac{\alpha_i}{i}\left(\sum_{j=i}^{F}\frac{a_j}{j}\right)^{-1}$$

Corollary 4. For completeness, we can also ask for a given cross-section DAF $\boldsymbol{\alpha} \in \Delta^{F-1}$, what is the corresponding distribution of durations $\boldsymbol{\alpha}^d \in \Delta^{F-1}$ is:

$$\alpha_i^d = \frac{\alpha_i}{i.\bar{h}} \tag{10}$$

This follows directly form the comparison of (9) and observation 2. Clearly, by definition, the distribution of durations is the same as the distribution across firms resetting prices (new price-spells). The more frequent price setters (shorter price-spells) have a higher representation relative to longer price-spells. Note that the rhs denominator of (10) is the product of the contract length and the proportion of firms resetting price. For the values of $i < \bar{h}^{-1}$, the share of the duration i is greater across contracts than firms: for larger $i > \bar{h}^{-1}$ the share across contracts is less than the share across firms.

1.4 Examples.

The above Propositions and corollaries enable us to move between the four different ways of describing the same underlying distribution of price-spell durations in the panel. We will illustrate this with two simple examples giving $\left(\alpha_i^a, \alpha_i, \alpha_i^d, h_i\right)_{i=1}^F$ for two simple distributions.

Example 1: Simple Taylor 4.

$$\begin{array}{lll} h_1 = 0 & \alpha_1^A = \frac{1}{4} & \alpha_1 = \alpha_1^d = 0 \\ h_2 = 0 & \alpha_2^A = \frac{1}{4} & \alpha_2 = \alpha_2^d = 0 \\ h_3 = 0 & \alpha_3^A = \frac{1}{4} & \alpha_3 = \alpha_3^d = 0 \\ h_4 = 1 & \alpha_4^A = \frac{1}{4} & \alpha_4 = \alpha_4^d = 1 \\ \bar{h} = \frac{1}{4} & \bar{A} = \frac{5}{2} & \bar{T} = \bar{d} = 4 \end{array}$$

A simple lesson can be derived from this example. When all price-spells have the same length, the distribution of durations is the same as the distribution across firms and $\bar{T} = \bar{d}$. In general, the cross-sectional distribution $\{\alpha_i\}$ weights longer price-spells more heavily $\left\{\alpha_i^d\right\}$, so that $\bar{T} \leq \bar{d}$ with a strict inequality if all price-spells do not have the same length.

Example 2: Simple Calvo The simple Calvo model most naturally relates to the hazard rate approach to viewing the steady state distribution of durations: it has a constant reset probability $\bar{h}$ (the hazard rate) in any period that the firm will be able to review and if so desired reset its price. This reset probability is exogenous and does not depend on how long the current price has been in place. The distribution of ages of price-spells is

$$\alpha_i^A = \bar{h}\left(1 - \bar{h}\right)^{i-1} : \ i = 1 ... \infty$$

which has mean $\bar{A} = \sum_{i=1}^{\infty} \alpha_i^A.i = \omega^{-1}$. Applying Proposition 1(a) gives us the steady-state distribution of completed contract lengths i across firms:

$$\alpha_i = \bar{h}^2 i \left(1 - \bar{h}\right)^{i-1} : \ i = 1 ... \infty \tag{11}$$

which has mean $\bar{T} = \left(2\bar{h}^{-1}\right) - 1$ (see Dixon and Kara 2006 Proposition 1). We illustrate the simple Calvo model with $\bar{h} = 0.25$.

$$
\begin{array}{lll}
\bar{h}_1 = 0.25 & \alpha_1 = 0.0625 & \alpha_1^d = 0.25 \\
\bar{h}_2 = 0.25 & \alpha_2 = 0.09375 & \alpha_2^d = 0.1875 \\
\bar{h}_3 = 0.25 & \alpha_3 = 0.10546875 & \alpha_3^d = 0.1406 \\
\bar{h}_4 = 0.25 & \alpha_4 = 0.10546875 & \alpha_4^d = 0.1052 \\
\bar{h}_i = 0.25 & \alpha_i = (0.25)^2\, i\, (0.75)^{i-1} & \alpha_i^d = \alpha_i^A \\
\bar{h} = 0.25 & \bar{T} = 7 & \bar{d} = \bar{A} = 4
\end{array}
$$

Note that the simple Calvo model has a very special property: the distribution of ages is the same as the distribution of durations: substituting (11) into (10) yields $\alpha_i^A = \alpha_i^d \; i = 1...\infty$, so that the mean *age* of price-spells across firms equals the mean *lifetime* across new contracts and is the reciprocal of the reset probability. Due to the interruption bias inherent in the age distribution, we have in general $\bar{T} > \bar{A}$ unless $F = 1$ in which case $\bar{T} = \bar{A} = 1$. In general, we can have $\bar{d} > \bar{A}$ or $\bar{d} < \bar{A}$. The age distribution is length biased like the DAF, being a cross-sectional distribution, which weights longer price-spells more heavily. The interruption bias acts in the opposite direction, since the age-distribution measures incomplete price-spell durations. In the Calvo model, these two effects exactly offset each other. However, it is easy to see that we can construct examples where this strict equality can be broken in either direction (see Dixon and Siciliani 2009 for examples).

2 Pricing Models with steady state distributions of durations across firms.

Having derived a unified framework for understanding the set of all possible steady state distributions of durations across firms, we can now see how this can be used to understand commonly used models of pricing behavior. Indeed, we can see how each pricing theory relates to the whole set of possible steady-state distributions. There are now several studies using micro data: in particular the *Inflation Persistence Network* (IPN) across the euro area has been particularly comprehensive[8]: Alvarez and Hernando (2006) for Spain, Veronese et al (2005) for Italy, Baudry et al (2007) for France and Hoffman and Kurz-Kim (2006) for Germany. For the US we have Klenow

[8]See Dhyne et al (2006) for a summary of the IPN's findings. Alvarez (2008) also provides a useful summary.

and Krystov (2008), Nakamura and Steinsson (2008) and for the UK Bunn and Ellis (2009, 2010). CPI microdata is in effect a (balanced) panel on the prices of individual products sold at individual outlets. They have *trajectories* for prices: sequences of price spells for a product at an individual outlet. We can think of each trajectory as analogous to the sequence of price spells for an individual "firm" in the context of this paper: of course the analogy is not exact since the retailers will in the most part not be the producers. All of these studies cover the period of the Great Moderation: there is low and stable inflation for which the assumption of steady-state is particularly appropriate: also, the studies span several years so that the seasonal variation in the data is averaged out.

The key insight is that we can use any adequate description of the whole distribution of price-spells. Under the assumption of steady-state we can then derive the others as outlined in the previous section. The way the data is usually described is the following:

1. The Hazard function. This is estimated in different ways (see Appendix 1), but in principle we can relate the hazard profile $\boldsymbol{h} \in [0,1)^{F-1}$ to the estimated hazard function. Most papers include the estimated hazard function for the whole economy and at a more disaggregated level. This can in principle be used to generate the corresponding distribution of durations $\left\{\alpha_i^d\right\}$ and the related cross-sectional distribution $\{\alpha_i\}$.

2. The frequencies of price change: $\bar{h}$. The proportion of firms changing prices per month: this can be aggregated over time or across sectors. Unfortunately, this statistic does not tell us the distribution of durations or the hazard profile. We need to make additional assumptions about the underlying distribution to unpack the distribution of durations from the frequency. In Dixon and Kara (2010), we assume that there is a Calvo distribution within each sector with the Calvo reset probability being the proportion of firms re-setting their price. Applying this to the Bils and Klenow (2004) data appendix, we generated the corresponding cross-sectional distribution $\{\alpha_i\}$.

In general, approach 1 is best: the hazard fucntion encapsulates all of the information about the distribution of price-spells, whilst this information is lost if we have only the sectoral frequencies. In this section we state models of pricing that are sufficiently general to reflect the empirical micro-data on prices: these are essentially generalizations of the standard Taylor and Calvo models.

2.1 The Generalized Taylor Economy GT

Using the concept of the Generalized Taylor economy GT developed in Dixon and Kara (2010, 2011), any steady-state distribution of completed durations across firms $\boldsymbol{\alpha} \in \Delta^{F-1}$can be represented by the GT with the sector shares given by $\boldsymbol{\alpha} \in \Delta^{F-1}:\ GT(\boldsymbol{\alpha})$. In each sector i there is an $i-$period Taylor contract, with i cohorts of equal size. The sector share is given by α_i. Since the cohorts are of equal size and there as many cohorts as periods, there are $\alpha_i.i^{-1}$ contracts renewed each period in sector i. This is exactly as required in a steady-state. Hence the set of all possible GTs is equivalent to the set of all possible steady-state distributions of durations. Note that for the GT we need to know the DAF $\boldsymbol{\alpha}$. Although we can derive the DAF from the distribution of price-spells $\boldsymbol{\alpha}^d$, the latter cannot be applied directly to any price theory. In effect, since the distribution of durations ignores the panel structure of the economy and the role of firms in setting prices, it does not directly relate to firms pricing behavior.

In a GT, the reset price at time t in sector i x_{it} is (in log-linearised form):

$$x_{it} = \left(\frac{1}{\sum_{k=0}^{i-1}\beta^k}\right)\sum_{k=0}^{i-1}\beta^k p^*_{t+k} \tag{12}$$

where p^*_t is the optimal flex-price at time t and β the discount rate. There are F reset price equations, with $i = 1...F$. The F prices in each sector i are simply the average over the i cohorts in that sector:

$$p_{it} = \frac{1}{i}\sum_{k=0}^{i-1} x_{it-k} \tag{13}$$

The aggregate price level is simply:

$$p_t = \sum_{i=1}^{F}\alpha_i p_{it} \tag{14}$$

It is simple to verify that the age-distribution in a GT is given by (8). If we want to know how many price-spells are aged j periods, we look at sectors with lifetimes at least as large as j, $i = j...F$. In each sector i, there is a cohort of size $\alpha_i.i^{-1}$ which set its price j periods ago. We simply sum over all sectors $i \geq j$ to get (8). The GT has been employed by Taylor (1993), Coenen et al (2007), Dixon and Kara (2010, 2011), Kara (2008, 2009)[9].

Note that the GT the "sectors" α_i are defined by the duration of the price-spells, not the industry or CPI category. They are the proportion of all firms (across all sectors) that have i period contracts. The key thing is that *when they set their price, the firm knows how long it will last* (i periods). In fact, if we assume that all firms know how long the price-spells are going to last, individual firms could move between "duration sectors": for example, a firm might set a price for 3 months and be in the $i = 3$ sector, and then put it on sale for one month and be in the $i = 1$ sector that month and so on. The fact that an individual firm may have price-spells of different lengths does not show that it is not Taylor-like in the sense that it might still know how long its price will last.

2.2 The Generalized Calvo model (GC): duration dependant reset probabilities.

The Calvo model most naturally relates to the hazard rate approach to viewing the steady state distribution of durations and it has a constant hazard rate. We now consider generalizing the Calvo model to allow for the reset probability (hazard) to vary with the age of the contract (duration dependent hazard rate). This we will denote the *Generalized Calvo Model GC*. A GC is defined by a sequence of reset probabilities: as in the previous section this can be represented by any $\boldsymbol{h} \in [0, 1)^{F-1}$ where F is the shortest contract length with $h_F = 1$. Thus the empirical hazard rates displayed in the data can be used to calibrate the hazard profile in the GC model. The resultant model can be consistent with any steady-state distribution of durations, including the one found in the data[10].

The GC differs from the GT in that when they reset prices, firms do not know how long the price-spell is going to last. There is not a sector specific reset price, but one economy wide reset price x_t with $x_{it} = x_t$ for all $i = 1...F$.

[9] See also Whelan (2004) for a theoretcial analysis.

[10] Note that an alternative parameterization of the duration dependent hazard rate model is to specify not the hazard rate at each duration, but rather the probability of the completed contract length at birth (see for example Guerrieri 2006).

The log-linearised formula for the optimal reset price at t is

$$x_t = \frac{1}{\sum_{k=1}^{F} S_k \beta^{k-1}} \sum_{k=1}^{F} S_k \beta^{k-1} p^*_{t+k-1} \tag{15}$$

The aggregate price is then given by:

$$p_t = \bar{h} \sum_{k=1}^{F} S_k x_{t-k} \tag{16}$$

with the aggregate price being given by (14) as before, where the α_i are derived from the reset probabilities $\boldsymbol{h} \in [0,1)^{F-1}$ using Corollary 2. The difference between the GT and the GC lies in the whether the duration of the price-spell is known: with the GC only the distribution of price spells is known by the firm. In effect, the firm does not know ex ante which sector it is in. The GC model has been employed by Wolman (1999), Mash (2003,2004), Dotsey and King (2006), Guerrieri (2006), Sheedy (2007) and Paustian and von Hagen (2008), Alvarez and Burriel (2010)[11].

2.3 The Multiple Calvo Model (MC).

One format that the micro-data is presented in is in the form of the proportion of firms changing prices each month, often with a detailed sectoral breakdown: see Bils and Klenow (2004), Nakamura and Steinsson (2008), Klenow and Krystov (2008), similar data in most of the IPN studies and the Bank of England work on the UK (Bunn and Ellis (2010a,b). This naturally suggests a modelling strategy of a multiple sector Calvo model MC. We can define a multiple Calvo process MC as $MC\left(\tilde{\mathbf{h}}, \lambda\right)$ where $\tilde{h} \in (0,1)^n$ gives a sector specific hazard rate[12] $\tilde{h}_k$ for each sector $k = 1, ...n$ and $\boldsymbol{\lambda} \in \boldsymbol{\Delta}^{n-1}$ is the vector of shares λ_k (this might be expenditure or CPI weights). The reset price for each sector $k = 1...n$ is then:

$$x_{kt} = \frac{1}{\sum_{j=1}^{F} \left(1 - \tilde{h}_k\right)^{j-1} \beta^{j-1}} \sum_{j=1}^{F} \left(1 - \tilde{h}_k\right)^{j-1} \beta^{j-1} p^*_{t+j-1} \tag{17}$$

[11] This paper takes a special case of the GC, by having a constant hazard with spikes at 12 monthly intervals.

[12] The notation here should not be confused: the substrcripts k are sectoral: none of the sectoral calvo reset probabiltities are duration dependent.

The average price in each sector k is then

$$p_{kt} = \sum_{j=1}^{F} \left(1 - \tilde{h}_k\right)^{j-1} \beta^{j-1} x_{kt-j+1} \tag{18}$$

And the aggregate price is then

$$p_t = \sum_{k=1}^{n} \lambda_k p_{kt} \tag{19}$$

The Multiple Calvo model has been employed by Carvalho (2006) and Carvalho and Nechio (2008) and the earlier version of this paper (2006).

It is important to note that the MC does not have the same hazard function as the data: it generates a hazard function with declining hazard which is smooth. In that sense it is a parameteric representation of the data and cannot be an exact representation of the data as we have in the case of the hazard or the three distributions: see Alvarez (2008).

2.4 The Typology of Contracts.

In terms of contract structure (the nature of pricing), we can say that the following relationships hold:

- $GC = GT = SS$. The set of all possible steady state distributions of durations SS is equivalent to the set of all possible GTs and the set of all possible GCs.
- $C \subset MC \subset GC$. The set of distributions generated by the Simple Calvo is a special case of the set generated by MC which is a special case of GC.
- $ST \subset GT = GC$ Simple Taylor is a special case of GT, and hence also of GC.
- $ST \cap MC = \varnothing$ if $F > 1$. Simple Taylor contracts are a special case of GC, but not of MC.

Figure 1: The typology of Contracts

This is depicted in Fig 1. The centrepoint represents the case where all prices are flexible.The GC and the GT are coextensive, being the set of all possible steady-state distributions (Propositions 1 and corollary 3). The Simple calvo C (one reset probability) is a strict subset of the Multiple Calvo process MC which is a strict subset of the GC[13]. The simple Taylor ST and the MC are disjoint except for the case where all prices are flexible. The ST is a strict subset of the GT. The size of the distributions is reflected by the Figure: ST has elements corresponding to the set of integers and is represented by a few dots; Calvo is represented by the unit interval; MC by a slice of the cake. The simple Calvo and Taylor models are only applicable if there is one type of contract and no heterogeneity in the economy. If we believe the Calvo model, but that reset probabilities are heterogenous across price or wage setters, then the MC makes sense. If we do not believe the Calvo model, then either the GC or GT are appropriate.

3 An application to UK price data[14].

In this section, I illustrate the framework of steady-state identities to the UK price data. As described in Bunn and Ellis (2009, 2010a), the ONS micro-data for constructing the CPI is available for use at the VML laboratory: it covers the years 1996-2006 and consists of 11 million price-observations that were collected "locally" by ONS staff, rather than centrally. We have followed Bunn and Ellis's methodology in analyzing the data, except for the estimation of the hazard and survivor function (see Appendix 1). We include sales data (i.e. price spells that represent a temporary discount): excluding sales would reduce slightly the share of short spells, but not the overall picture. The CPI data consists of price-quotes for specific products/services from specific outlets. We are treating interpreting the price of a "product at a particular location" as the price set by a "firm" which produces the product. This is not literally accurate, but is necessary if we are to link the

[13]The MC can be represented by a GC with a decreasing Hazard. See an earlier version of the paper with the same title, ECB working paper 676, Proposition 2 for a derivation in discrete time.

[14]This work contains statistical data from ONS which is Crown copyright and reproduced with the permission of the controller of HMSO and Queen's Printer for Scotland. The use of the ONS statistical data in this work does not imply the endorsement of the ONS in relation to the interpretation or analysis of the statistical data. This work uses research datasets which may not exactly reproduce National Statistics aggregates

macroeconomic theory (in which firms set prices) to the CPI data.

Firstly, we start from the hazard function for the CPI covering all goods and services in the VML dataset: we present the Hazard for the weighted data (see Bunn and Ellis (2010a) for the weighting methodology) . The weighted data puts more weight on services which have a lower hazard in the first few months. We set $F = 44$ months: there are very few spells lasting more than 44 months (less than 0.01%)[15]. We depict the UK hazard rate for the first 37 months.

Fig 2. The UK Hazard Rate: Sorce ONS.

The CPI hazard function is similar to those found in other countries[16]: it declines rapidly for the first few months: there is 12 month "spike" and after that it remains roughly constant. There is a high probability of 36% of changing price before the second month: even if we exclude sales this is still 30%. Note that the Hazard rate is between 10%-15% from month 4 onwards (except for months 12 and 24 and towards the end when very few spells survive). The implications of this are that there is a significant long-tail of price-spells. We can see this if we look at the survival function depicted in Figure 3: the probability of surviving up to 24 months is 2.4%; the probability of surviving up to 36 months is a little under 0.5%. The implied monthly frequency of firms re-setting their prices $\Sigma_S^{-1} = 18.7\%$ which is very close to the direct measurement made in Bunn and Ellis (2010a) of 19.2% in the weighted data[17].

Fig 3. The UK survival function: Source ONS

The survival function implies that there is also a very long tail in the distribution of durations and even more so in the cross-sectional DAF. As we would expect, there are a lot of short durations: 50% of price-spells last only one or two months. There is a 12 month spike, but a long tail (over 2% of price-spells last more than 24 months, 1% more than 30 months). The DAF is much flatter: the share of one and two month spells is only 12%, there is a peak at 12 months and the tail is even longer and fatter: 13.4% last longer

[15] 0.086313% to be exact to 4 s.f.

[16] See Alvarez (2008) page 12 for examples from four EU countries.

[17] See Bunn and Ellis (2010a) Tables A3 and A4. Note that the "direct" measurement is not strictly speaking comparable with the Hazard estimate. This is due to the treatment of censoring and different assumptions about which spells are included.

than 24 months and 3% last longer than 36 months. The mean price-spell duration is $\bar{d} = 5.3$ months (median 2, mode 1): the cross-sectional mean $\bar{T} = 13.2$ months (median 10 months, mode 1 month). The cross sectional distribution is broadly consistent with the survey data (see Alvarez 2008, page 11) where firms are asked how often they change price.

Figure 4. The UK Distributions: Source ONS.

An alternative way of looking at the data is parameterizing it using the Multiple Calvo MC approach. In this we take data on the sectoral frequencies of price-adjustment and assume that within each sector there is a Calvo distribution. We can then sum up across sectors to obtain the aggregate distribution implied by this parameterization. By construction, this has the same frequency of price-change as in the data: 19.2%. However, the Hazard rate is a smoothly declining one and the distribution of durations and the DAF differ significantly from the one found in the data.

4 Pricing Models Compared with a UK calibration.

We will use the UK micro-data to see how the different models of pricing differ in terms of their impulse-response in terms of a monetary shock. We are implicitly assuming that the hazard function is not affected buy the monetary policy shock. We believe that this is reasonable: monetary policy affects the whole economy and is pervasive, but is not that relevant to the individual firm which will be more focussed on sector specific developments. Obviously, if there were a very large monetary shock, then the assumption of an invariant steady-state of distributions might be called into question.

We first use a very simple stripped-down log-linearised "Quantity Theory" macro-model (see Ascari 2003, Dixon and Kara 2010). The advantage of our simple model is that most of the dynamics originates from the pricing models, the only other source being a simple auto-regressive money-supply shock. We will also us the Smets-Wouter's (2003) model following the methodology Dixon and Le Bihan (2011) employ with the French CPI microdata.

4.1 The simple Quantity Theory (QT) model.

To model the demand side, we use the Quantity Theory:

$$y_t = m_t - p_t$$

where (p_t, y_t) are aggregate price and output and m_t the money supply. We model the monetary process as $AR(1)$:

$$\begin{aligned} m_t &= m_{t-1} + \varepsilon_t \\ \varepsilon_t &= \nu \varepsilon_{t-1} + \xi_t \end{aligned}$$

where ξ_t is a white noise error term. We consider the cases of $\nu = 0$ and $\nu = 0.5$ (the latter follows Christiano et al (2005)).

The optimal flexible price p_t^* at period t in all sectors is given by:

$$p_t^* = p_t + \gamma y_t$$

The key parameter γ captures the sensitivity of the flexible price to output[18]: we calibrate $\gamma = 0.2$ as discussed in Dixon and Kara (2010). We have converted the monthly price data into quarterly data.

Given this rudimentary macro-structure, we can then insert the sectoral reset-price equations[19], and sectoral price equations into the model, and aggregate according to (14) or (19). We compare three models: the GT, the GC which both have exactly the same distribution of price-spell durations as found in the UK data, and thirdly the sectoral MC model calibrated with the COICOP weights and frequencies of price-adjustment[20].

Fig 5: Responses to a one-off monetary Shock $(\nu = 0)$

In Figure 5, we depict the responses of output, the reset price[21], the general price level and inflation to a one-off shock. Looking at all the graphs,

[18] This can be due to increasing marginal cost and/or an upward sloping supply curve for labour. See for example Walsh (2003) chapter 5 and Woodford (2003) chapter 3.

[19] For the GTE we have $(12, 13)$, for the GC we have $(15, 16)$, for the MC we have $(17, 18)$.

[20] See Bunn and Ellis (2010a) tables A3 and A4.

[21] The average reset price is the average *conditional on the price being reset.* For the GTE this is:

$$\bar{x}_t = \sum_{i=1}^{F} \left(\frac{\alpha_i}{i}\right) x_{it}.$$

it is striking that the three models of pricing have fairly similar impulse-responses. However, if we compare the GT and GC (which have exactly the same distribution of price-spells), we can see that in the GT the effect on output is consistently bigger than with the GC and likewise the effect on the price level is smaller. This can be explained by the more myopic response of the reset-prices under GT: as we can see from the mean reset-price: the reset price on the GT reponds less than it does for the GC. That is becuase firms that know how long the price-spell is going to last (as in the GT) only need to look ahead for the length of the price-spell, whereas the GC firms have to look F periods ahead. This means that they take into account future price rises over a longer horizon than the GT firms, meaning that they rise prices more in the early stages after the shock. For example, the GT sector $i = 1$ simply sets the best price for the current period: it does not take into account future general price rises at all. It is only the firms with long-contracts that have to look ahead, and of course they do not change their price verty often and so take time to respnd to the shock. The effect on inflation is much dampened for the GT on impact relative to the GC, the level of inflation being lower for the first 4 quarters and higher subsequently. If we turn to the MC, we can see that it is similar to the GC but the effect on output is larger and on the reset price and price level more sluggish. In terms of inflation, this means that there is less inflation initially (for the first six months) and more later.

Fig 6: Serial Correlation in Monetary growth $\nu = 0.5$

In Figure 6 we consider an autoregressive monetary policy shock and find that there is now a more radical difference between the GT and the other two models. If we look at inflation we see that there is a distinct *hump shape which is exclusive to the GT*: the peak impact on inflation appears after the initial monetary shock. The hump we find in the GT results from the myopia of the GT firms on average. Whilst all IRs have a hump in output, both the MC and the GC have a peaked response of inflation on impact. This reflects the finding in Dixon and Kara (2010) that the Calvo model does not capture the characteristic "hump shaped" response indicated by empirical VARS. This "no hump" feature appears to be shared by its generalizations MC and GC. If we turn to output, we can see that the effect of the monetary shock on output (prices) is consistently greater (less) in the GT than the GC. The MC has a more ambiguous relationship. If we compare the GT and MC, we can see that the average reset price and general price levels are higher for the MC up to the 9th quarter, and then lower: this implies that the output response is larger initially for the GT, but less after the 10th quarter.

This example of the $IRFs$ of major variables in a simple macro-model shows how different models of pricing can yield different patterns of behavior even though the distributions of price-spells are exactly the same or similar. This reflects differences in the pricing behaviour of firms under the different models. Using the UK data to calibrate the model, it seems to make a substantial difference to output and inflation responses depending on whether firms know the duration of their price-spells ex ante (as in the GT) or not (as in the GC).

4.2 The Smets-Wouters (2003) model.

In Dixon and Le Bihan (2011), we take the Smets-Wouters model for the euro area and replace the wage and price setting model of Calvo-with-indexation with the GT and GC models that are derived from the French micro-data. We will now perform a similar exercise, but use the UK calibration for the GT and GC models of pricing (we do not have any reliable data for UK wages, so leave wage-setting as in the SW model). This is meant as an illustrative exercise: we do not restimate the model for the UK and use the original SW parameters for the rest of the model (i.e. everything except the price-setting). The model and parameter values are all outlined in Dixon and Le Bihan (2011), so we will not repeat it here. We are not able to consider the MC approach in the context of the SW model as it would need a more thorough reworking if the model, so we restrict our comparison to the GT and GC models.

Fig7. here.

We consider a policy shock to the Taylor rule leading to a higher interest rate[22]. In Figure 7 we see the resultant IRFs in the SW model with the UK calibrated pricing models. Turning first to inflation, we can see that the IRF for inflation under the GT follows a nice hump shape, with little effect

[22] I would like to thank Zhou Peng for running the simulations for me. "Impulse response functions are the expected future path of the endogenous variables conditional on a shock in period 1 of one standard deviation." (Dynare Manual).

on impact but building up to a peak effect at 6 quarters and then dying away. Although our model is purely for illustration, we can compare this IRF to the results in Kamber and Millard (2010) who estimated the VARS and the SW model for the UK data using the original Calvo-with-indexation for pricing. Both the VAR and the IRF for the estimated UK version of the SW model have a hump at 6-8 quarters[23], so in that sense our illustrative GT is not at all out of line with the UK. If we consider the GC, we find that as in the simple QT model, there is no hump at all and the maximum effect of the policy shock. When we look at the output IRFs, the fact that inflation is more sluggish under the GT leads to a bigger output response up to quarter 17. Interestingly, al;though the magnitudes are different, both IRFs peak at 4 quarters..Again, if we compare this with Kamber and Millard (2010), the timing of the peak is consistent with both the estimated VAR and the UK version of the SW model. This confirms what we found in the simple QT model but in the context of a more realistic model. There are many sources of dynamics in the SW model which interact to generate behaviour of the model. However, even though the GT and GC have exactly the same UK distribution of price-spells, the GT still gives rise to a hump-shaped inflation response whilst the GC does not. Whilst the Kamber and Millard results hold for the UK, they are not unlike the results for other economies such as the US (see for example Christiano et al 2005, Uhlig 2005). So, we can see that the GT pricing model appears to offer a framework more in line with the inflation dynamics of the developed world than the GC.

4.3 The Forward Lookingness of pricing rules.

In this subsection we consider how pricing models differ when we control for the distribution of durations (requiring the steady state distributions to be the same) in terms of the "Forward Lookingness" (FL) of the pricing rules which formalizes our notion of the myopic nature of the GT relative to the GC. A pricing rule uses data from the present and future in order to determine the optimal price. In a log-lineraised form, this gives the current price as a linear function of data from each date ahead. The forward lookingness of a pricing rule takes the weights across time (normalised to unity) in the linearised decision rule and is defined as the resultant average over the dates ahead. This simple measure captures the extent to which

[23]See Kamber and Millard (2010) Chart 3, page 29.

the future influences the pricing decision, and is applicable across all pricing rules. For simplicity, we ignore discounting, since it applies to all pricing rules and whilst it would be simple to generalise the formulae to allow for discounting, the no-discount case allows us to understand the differences more clearly. We willl first look at FL in the simple Calvo model with constant rest probability $\bar{h}$. The reset price can be written:

$$\begin{aligned} x_t^C &= \sum_{s=1}^{\infty} C_s p^*_{t+s-1} \\ C_s &= \bar{h}\left(1-\bar{h}\right)^{s-1} \end{aligned}$$

where C_s is the weight put on events s periods in the futire. We can then define Forward-lookingness as

$$FL^C = \sum_{s=1}^{\infty} sC_s = \left(\bar{h}\right)^{-1}$$

Let us now compare the simple-Calvo model to the Calvo-GTE[24], which has exactly the same distribution of durations as the Calvo model (as shown in example 2 above), but where firms know exactly how long the price-spells are going to last. In this case we have the sectoral reset wages x_{it} given by (12) and the resultant mean reset-price is then:

$$\bar{x}_t^{CGT} = \frac{1}{\bar{h}} \sum_{i=1}^{\infty} \frac{\alpha_i}{i} x_{it} = \sum_{i=1}^{\infty} \bar{h}\left(1-\bar{h}\right)^{i-1} x_{it} \tag{20}$$

$$x_{it} = \frac{1}{i} \sum_{j=1}^{i-1} p^*_{t+s-1}$$

Combining (15) with (20) yields:

$$\begin{aligned} \bar{x}_t^{CGT} &= \sum_{s=1}^{\infty} B_s p^*_{t+s-1} \\ B_s &= \bar{h} \sum_{i=s}^{\infty} \frac{(1-\bar{h})^{i-1}}{i} \end{aligned}$$

[24] If we have a Clavo with reset probability $\bar{h}$, the corresponding Calvo-GTE has the cross-sectional distribution $\alpha_i = i.\bar{h}^2.(1-\bar{h})^{i-1}$ as derived in Dixon and Kara 2006).

with the corresponding mean forward-lookingness

$$FL^{Calvo-GT} = \sum_{s=1}^{\infty} sB_s = \frac{1+\bar{h}}{2\bar{h}} = \frac{1}{2}\left[\bar{d}+1\right]$$

Observation 4. *Let $\beta = 1$ and $\bar{h} < 1$. Then $FL^{Calvo-GT} < FL^{C}$.*

The intuition is that when firms know how long the price-spell is going to last (i periods), then the reset price need look no further ahead than the length of the price spell: there is a zero weight on all p^*_{t+s} for $s > i-1$. If the weight on prices after $t+i-1$ is zero, then the weight on periods before will be correspondingly higher. In contrast, the Calvo firms do not know how long the spell will last, and so have to take into account all future periods. On average across the proportion of firms resetting price (which is by construction $\bar{h}$ in both cases), the rest-prices are less forwald looking in the Calvo-GT than the simple Calvo model. We rule out $\bar{h} = 1$, because in that special case $FL^{GT} = FL^{C} = 1$. For example, with the commonly used value of $\bar{h} = 0.25$, the Calvo-reset probability looks forward on average 4 periods, whilst the Calvo-GT looks forward 2.5 periods.

If we turn to the general case of any distribution, the results are similar. In a GC with $\boldsymbol{\omega} \in [0,1)^{F-1}$, the reset price and forward lookingness FL are:

$$x_t^{GC} = \bar{h}\sum_{j=1}^{F-1} S_j P^*_{t+j-1} = \sum_{j=1}^{F-1} \alpha_j^A P^*_{t+j-1} \tag{21}$$

$$FL^{GC} = \sum_{s=1}^{F-1} s\alpha_s^A = \bar{A} \tag{22}$$

In the corresponding GT we have (averaging over all F sectors)

$$\begin{aligned} x_t^{GT} &= \sum_{j=1}^{F} b_s P^*_{t+j-1} \\ b_s &= \frac{1}{\bar{h}}\sum_{i=s}^{F} \frac{\alpha_i}{i^2} \end{aligned}$$

Note that $\sum_{j=1}^{F} b_s = 1$[25] The forward lookingness of the GT is $FL^{GT} = \sum_{s=1}^{F} sb_s$. We can now compare the forward lookingness for any distribution of durations under the alternative models of pricing (the GC and GT).

Proposition 2 *Let $\beta = 1$. Then if we compare and distribution of durations:*

$$FL^{GT} \leq FL^{GC}$$

and $FL^{GT} < FL^{GC}$ if $F > 1$ and $\alpha_i > 0$ for some $i < F$.

Suppose that all price-spells have the same length $\alpha_F = 1$, then $\bar{d} = \bar{T}$. In this case, $\bar{A} = \frac{1+\bar{d}}{2}$ and $FL^{GC} - FL^{GT} = 0$. If there are a variety of lengths ($\alpha_i > 0$ for some $i < F$) then $\bar{A} > \frac{1}{2}\left[1 + \bar{d}\right]$. Hence for any distribution of durations, the GT is less forward-looking than the GC.

5 Conclusions

In this paper we have developed a consistent and comprehensive framework both for analyzing different pricing models (excluding the state-dependent pricing models) and relating the pricing models to the microeconomic data. In particular, the *distribution of completed price-spells across firms* (DAF) is a key perspective which is fundamental to understanding and comparing different models. Any steady state distribution of durations can be looked at in terms of completed durations, which suggests it can be modelled as a GT; it can also be thought of in terms of Hazard rates which suggests the GC approach. Both the GC and the GT are comprehensive: they can represent all possible steady states. We also relate this approach to sectoral frequency data which is widely available and can be modelled as a MC. When we apply this framework to the UK micro-data, we find that the different pricing models imply different macroeconomic behaviour in terms of the impulse response functions to a monetary shock.

[25]

$$\begin{aligned}
\sum_{j=1}^{F} b_s &= \frac{1}{\bar{h}} \sum_{s=1}^{F} \sum_{i=s}^{F} \frac{\alpha_i}{i^2} = \frac{1}{\bar{h}}\left[\alpha_1 1 + 2\frac{\alpha_2}{4} + ..i\frac{\alpha_i}{i^2}..\right] \\
&= \frac{1}{\bar{h}} \sum_{i=1}^{F} \frac{\alpha_i}{i} = 1
\end{aligned}$$

As more empirical micro-data becomes available, it is vital that we adopt a framework which enables us to link the data to our macroeconomic models. Whilst the approach adopted here is limited to steady-state analysis, it does provide a consistent way for linking the micro-data to the macroeconomic models of pricing. It is for future work to see how this analysis can be applied to non-steady-state analysis and state-dependent models.

6 Bibliography

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7 Appendix

7.1 Estimating the survival function.

This section describes how the survival function and related hazard function in discrete time can be estimated using pricing microdata in a way that is consistent and applicable models of pricing. Currently, the methods available in the statistical packages such as SAS and STATA have been designed with other applications in mind (medical data etc) and do not give the most relevant output at first pass. They are based on the Kaplin-Meier (KM) nonparametric estimator Fortunately, it is simple to adjust the output to give the survivor and hazard functions in a manner which is consistent with the framework of this paper.

The first adjustment to the survival function required arises from the fact that we are looking at pricing models in discrete time macroeconomic framework. In these theories, the firm believes that it has a probability of 1 that it its price lasts for at least one period. The way survival rates are usually reported is that the failure occurs during the period: thus the period one survival rate is one minus the period one hazard rate. This is easily adjusted: following the pricing theory, we simply set $S(1) = 1$ and then "lag" the estimated survival rates by one period to get (4). In effect, we are assuming that all of the failures in period 1 occur at the end of the period. We can see this as simply alternative ways of defining the survival rate: in standard survival analysis, $S(i)$ is the "probability of surviving to the end of period i", in this paper we define the rate as the "probability of surviving to the beginning of period i".

The second issue is more complex and has to do with how we reconcile the estimated hazard function with the data on the proportion of firms changing price per month, which is 19.2% for the weighted data. From the theoretical framework of this paper, we know from (5) that the sum of the survival rates should be equal to the proportion of firms changing price each month: $\bar{h} = \Sigma_S^{-1}$. In order to explore this issue, we need to look more closely at how the KM estimator is implemented in packages such as SAS and STATA. First, we define the set of price-spells which we want to include (and hence which to exclude, such as left-censored spells). We also define an event ("failure") which in this case is a price-change. The package then looks at all the price-spells in the panel (in our paper defined by the set R defined by equation (1)). It then prints out the raw data in a column: this lists the number of price-spells that lasted up to i periods $i = 1...F$. The first row is the total of all price-spells. The second row is those that lasted two or more periods etc. Next to this column are two others: "failed" and "lost". Of those spells that did not last more than one period, some ended because of a price-change (which we define as "failure"), and some for another reason (right truncation/censoring or some other reason). If we define the number of number of price-spells that have lasted up to the i^{th} period n_i, these are defined as the spells "at risk" of failure. Of these, f_i fail, L_i are lost and the rest survive to the next period: $n_{i+1} = n_i - f_i - L_i$. The basic KM estimator for the survival probability up to period i is:

$$\hat{S}^1(i) = \prod_{j=1}^{i} \left(1 - \frac{f_i}{n_i}\right) \tag{23}$$

A key assumption of the KM estimator is that *failure and loss are mutually exclusive*: that is if a spell is "lost", then it would not have failed (Kaplin and Meier (1958) page 461 describe this as "the convention that death preceeds loss").

KM recognised that this assumption would not be reasonable in many circumstances: they also considered the "adjusted-observed" estimator, which is

$$\hat{S}^2(i) = \prod_{j=1}^{i} \left(1 - \frac{f_i}{n_i - \frac{L_i}{2}}\right) \tag{24}$$

This estimation method is also found in packages such as SAS and STATA. Many exisiting studies of the micro-price data appear to use either the "death preceeds loss" or the adjusted-observed estimation method: for example Bunn and Ellis (2010a,b) for the UK data, Baudry et al (2007) for the French data.

There is a basic problem with these two estimators when applied to the CPI data: *the survival rates are too high.* For example, with the UK data set, the implied average monthly frequency of price change (the reciprocal of the sum of survival probabilities Σ_S^{-1}) is 11%, which is much smaller than what is observed in the data (19.2%). In this paper, to remedy this problem, we have made the assumption that all of the lost spells represent failures ("loss is failure"), so that we have:

$$\hat{S}^3(i) = \prod_{j=1}^{i} \left(1 - \frac{f_i + L_i}{n_i}\right) \tag{25}$$

This estimator implies a monthly frequency of price change of 18.7% which is much closer to the data. Whilst our preferred KM estimator of the hazard function delivers a result that is closer to the data on monthly frequency of price change, it is an *ad hoc* improvement to the more common methodology and further research on this issue is required to develop a fully appropriate and "optimal" estimator in this context.

7.2 Proofs.

7.2.1 Proof of Proposition 1.

Proof. The proportion of firms that have a contract that last for exactly 1 period are those that are born (age 1) and do not go on to age 2. The proportion of firms that last for exactly i periods in any one cohort (born at the same time) is given by those who attain the age i but who do not make it to $i+1$: this is $\left(\alpha_i^A - \alpha_{i+1}^A\right)$ per cohort and at any time t there are i cohorts containing contracts that will last for i periods.

Clearly, since α_j^A are monotonic, $\alpha_i \leq 1$, and

$$\begin{aligned}\sum_{i=1}^{F} \alpha_i &= \sum_{i=1}^{F} i\left(\alpha_i^A - \alpha_{i+1}^A\right) \\ &= \left(\alpha_1^A - \alpha_2^A\right) + 2\left(\alpha_2^A - \alpha_3^A\right) - 3\left(\alpha_3^A - \alpha_4^A\right) .... \\ &= \sum_{i=1}^{F} \alpha_i^A = 1\end{aligned}$$

Hence $\boldsymbol{\alpha} \in \Delta^{F-1}$.

The relationship between the distribution of ages and lifetimes can be depicted in terms of matrix Algebra: in the case of $F = 4$:

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} \alpha_1^A \\ \alpha_2^A \\ \alpha_3^A \\ \alpha_4^A \end{bmatrix}$$

Clearly, the 4×4 matrix is a mapping from $\Delta^3 \to \Delta^3$: since the matrix is of full rank, the mapping from α^A to α is $1-1$. Clearly, this holds for any F.
■

7.2.2 Proof of Corollary 1:

Proof. To see this, we can rewrite (7):

$$\begin{aligned}\alpha_1 &= \alpha_1^A - \alpha_2^A \\ \frac{\alpha_2}{2} &= \left(\alpha_2^A - \alpha_3^A\right) \\ \frac{\alpha_i}{i} &= \left(\alpha_i^A - \alpha_{i+1}^A\right) \\ \frac{\alpha_F}{F} &= \alpha_F^A\end{aligned}$$

hence summing over all possible durations $i = 1...F$ gives

$$\sum_{i=1}^{F} \frac{\alpha_i}{i} = \sum_{i=1}^{F-1} \left(\alpha_i^A - \alpha_{i+1}^A\right) + \alpha_F^A = \alpha_1^A$$

So that by repeated substitution we get:

$$\begin{aligned}\alpha_2^A &= \alpha_1^A - \alpha_1 = \sum_{i=2}^{F} \frac{\alpha_i}{i} \\ \alpha_j^A &= \sum_{i=j}^{F} \frac{\alpha_i}{i} \qquad j = 1...F\end{aligned}$$

■

7.2.3 Corollary 3.

Proof. Rearranging the $F-1$ equations (9) we have:

$$\frac{\alpha_1}{\bar{h}} = h_1; \frac{\alpha_2}{2\bar{h}} = h_2 (1 - h_1) ... \frac{\alpha_i}{i.\bar{h}} = h_i S_i; ... \frac{\alpha_F}{F\bar{h}} = S_F$$

By repeated substitution starting from $i = 1$ we find that

$$h_i = \frac{\alpha_i}{i} \left(\bar{h} - \sum_{j=1}^{i-1} \frac{\alpha_j}{j} \right)^{-1} \tag{26}$$

$$S_i = \frac{1}{\bar{h}} \left[\bar{h} - \sum_{j=1}^{i-1} \frac{\alpha_j}{j} \right]$$

Since we know that $h_F = 1$, from (26)this means that:

$$1 = \frac{\alpha_F}{F} \left(\bar{h} - \sum_{i=1}^{F-1} \frac{\alpha_i}{i} \right)^{-1} \Rightarrow \bar{h} = \sum_{i=1}^{F} \frac{\alpha_i}{i}$$

Substituting the value of $\bar{h}$ into (26) establishes the result. ■

7.2.4 Proposition 2.

We will first show that $FL^{GT} = \frac{1}{2}\left[1+\bar{d}\right]$.

$$
\begin{aligned}
FL^{GT} &= \sum_{s=1}^{F} s b_s \\
&= \frac{1}{\bar{h}} \sum_{s=1}^{F} s \sum_{i=s}^{F} \frac{\alpha_i}{i^2} \\
&= \frac{1}{\bar{h}} \left[\sum_{i=1}^{F} \frac{\alpha_i}{i^2} + 2 \sum_{i=2}^{F} \frac{\alpha_i}{i^2} .. + s \sum_{i=s}^{F} \frac{\alpha_i}{i^2} + \frac{\alpha_F}{F} \right] \\
&= \frac{1}{\bar{h}} \left[\sum_{i=1}^{F} \frac{\alpha_i}{i^2} \left(\frac{i(i+1)}{2} \right) \right] = \frac{1}{\bar{h}} \left[\sum_{i=1}^{F} \alpha_i \frac{(i+1)}{2i} \right] \\
&= \frac{1}{2\bar{h}} \left[\sum_{i=1}^{F} iS(i)h_i + \sum_{i=1}^{F} S(i)h_i \right] \\
&= \frac{1}{2}\left[1+\bar{d}\right]
\end{aligned}
$$

We have $FL^{GC} = \bar{A}$.There is a simple relationship between the average age and the cross-sectional average completed length. If we look at the cross section, within sector i, there are i cohorts with ages $1...i$. The average age in sector i is thus $\frac{1+i}{2}$. We can then add up the ages across sectors $i = 1..F$:

$$
\begin{aligned}
\bar{T} &= \sum_{i=1}^{F} i\alpha_i \\
\bar{A} &= \sum_{i=1}^{F} \frac{1+i}{2} \alpha_i = \frac{1}{2}\left[1+\bar{T}\right]
\end{aligned}
$$

Hence $\bar{A}$ is approximately half of $\bar{T}$. Since we know that $\bar{T} \geq \bar{d}$ it follows that $FL^{GC} \geq FL^{GT}$. $\bar{T} = \bar{d}$ only when $\alpha_F = 1$.

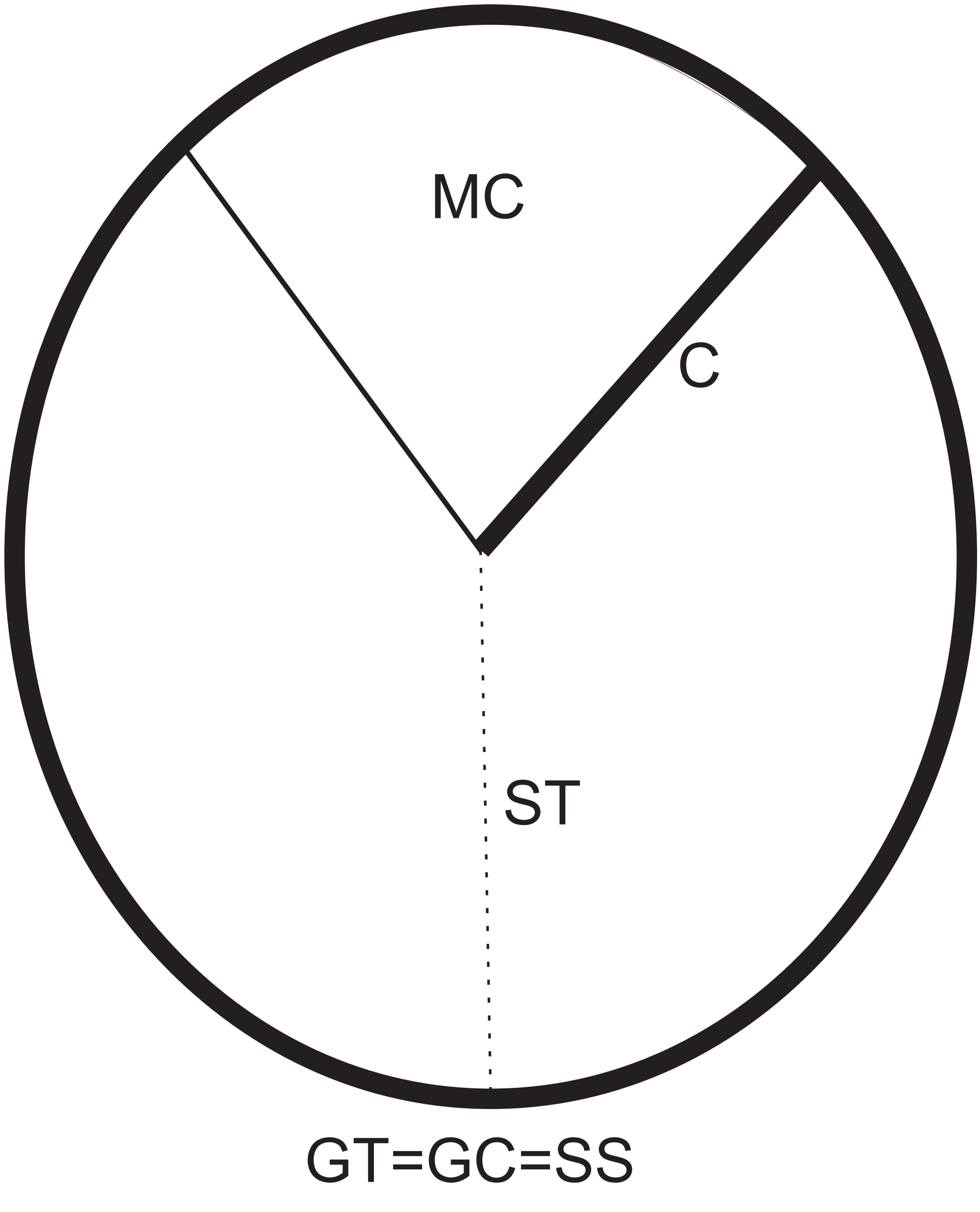


Figure 1: The typology of contracts.

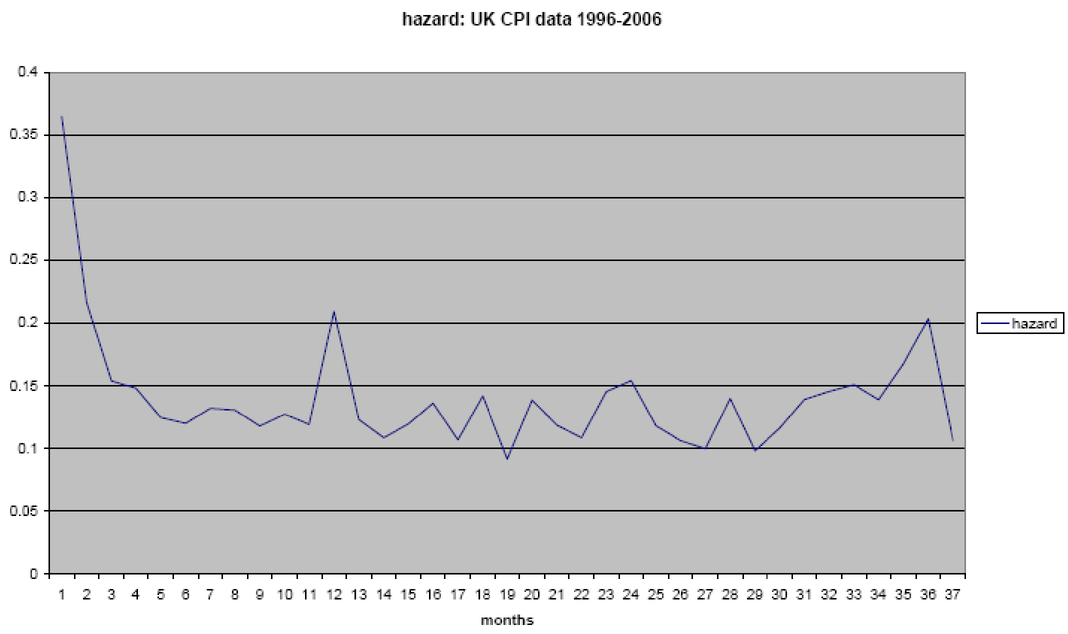


Figure 2: UK Hazard Rate.

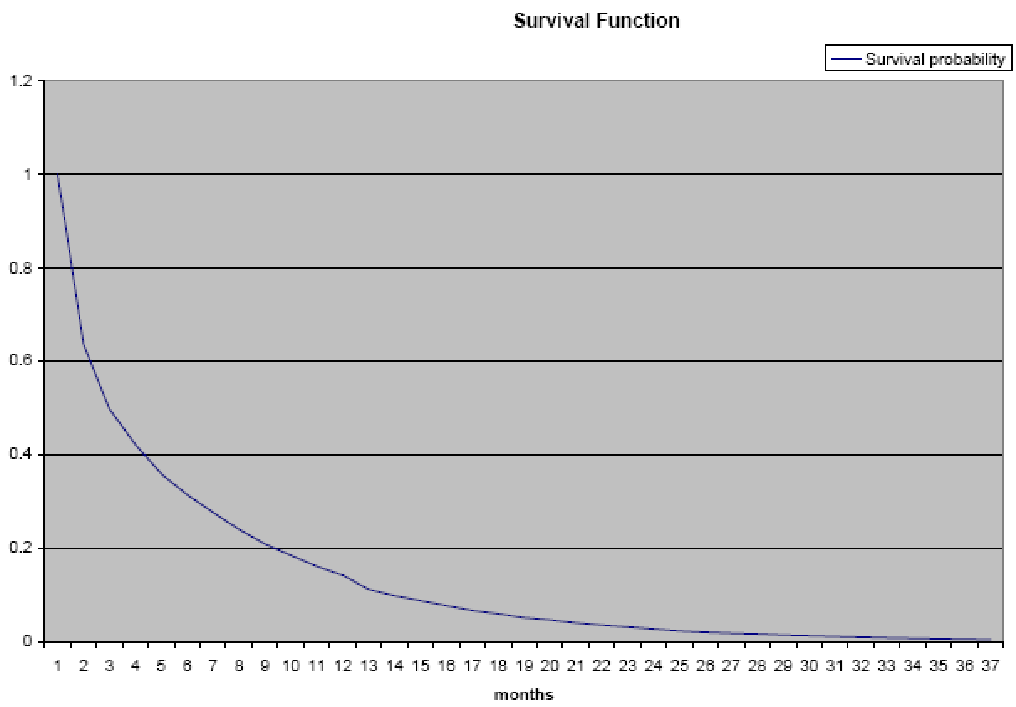


Figure 3: UK Survival Function.

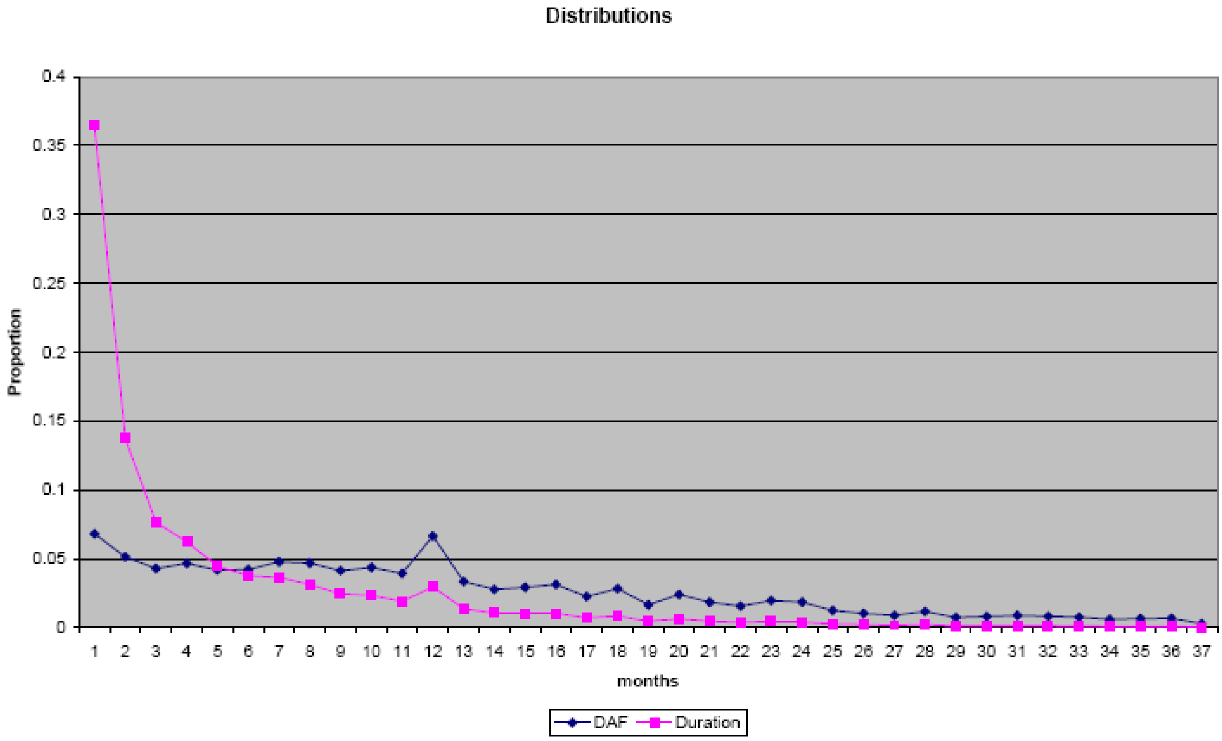


Figure 4: The UK Distributions – Durations and cross-section (DAF).

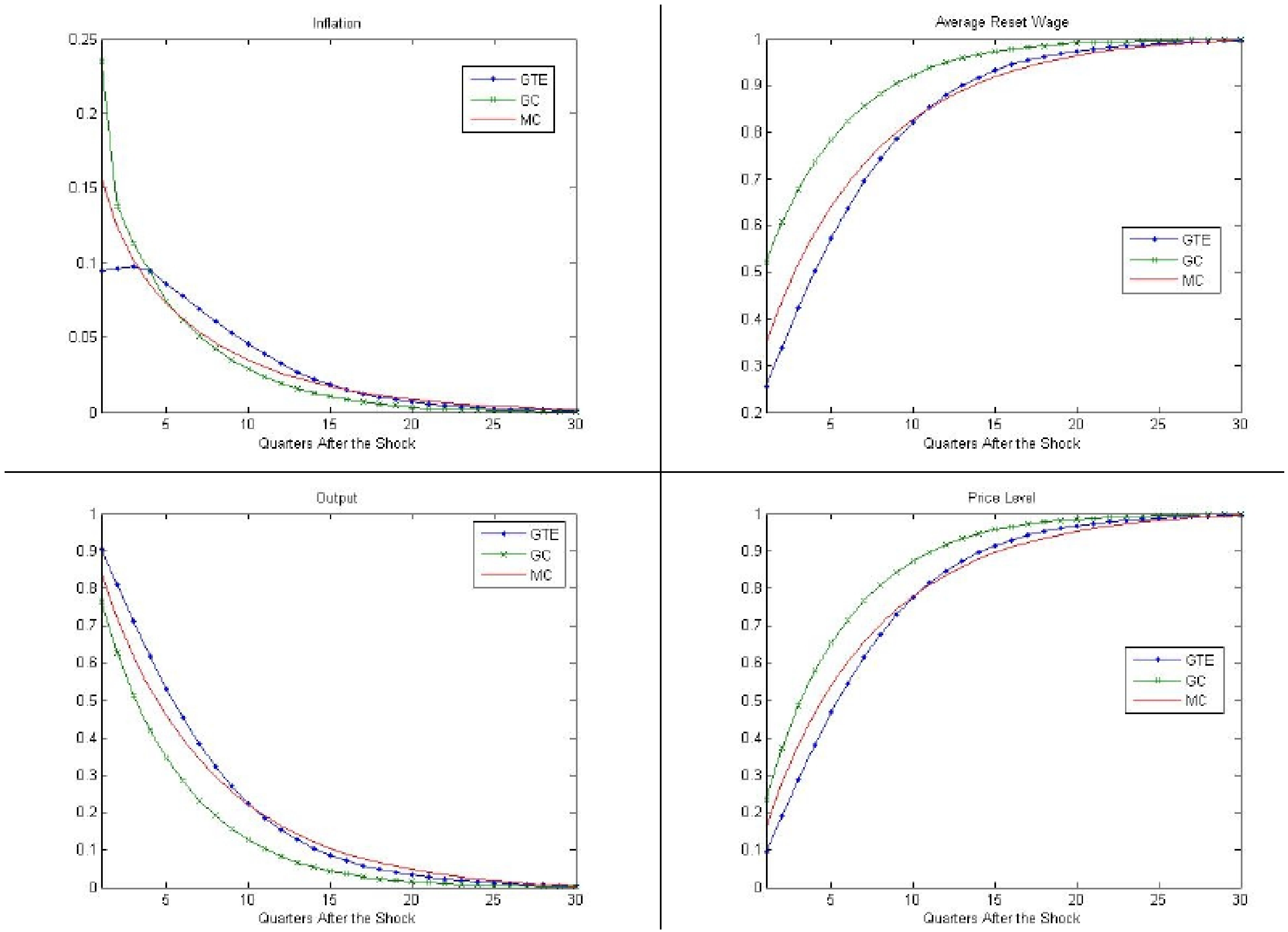


Figure 5: Responses to a one-off monetary Shock (ν=0)

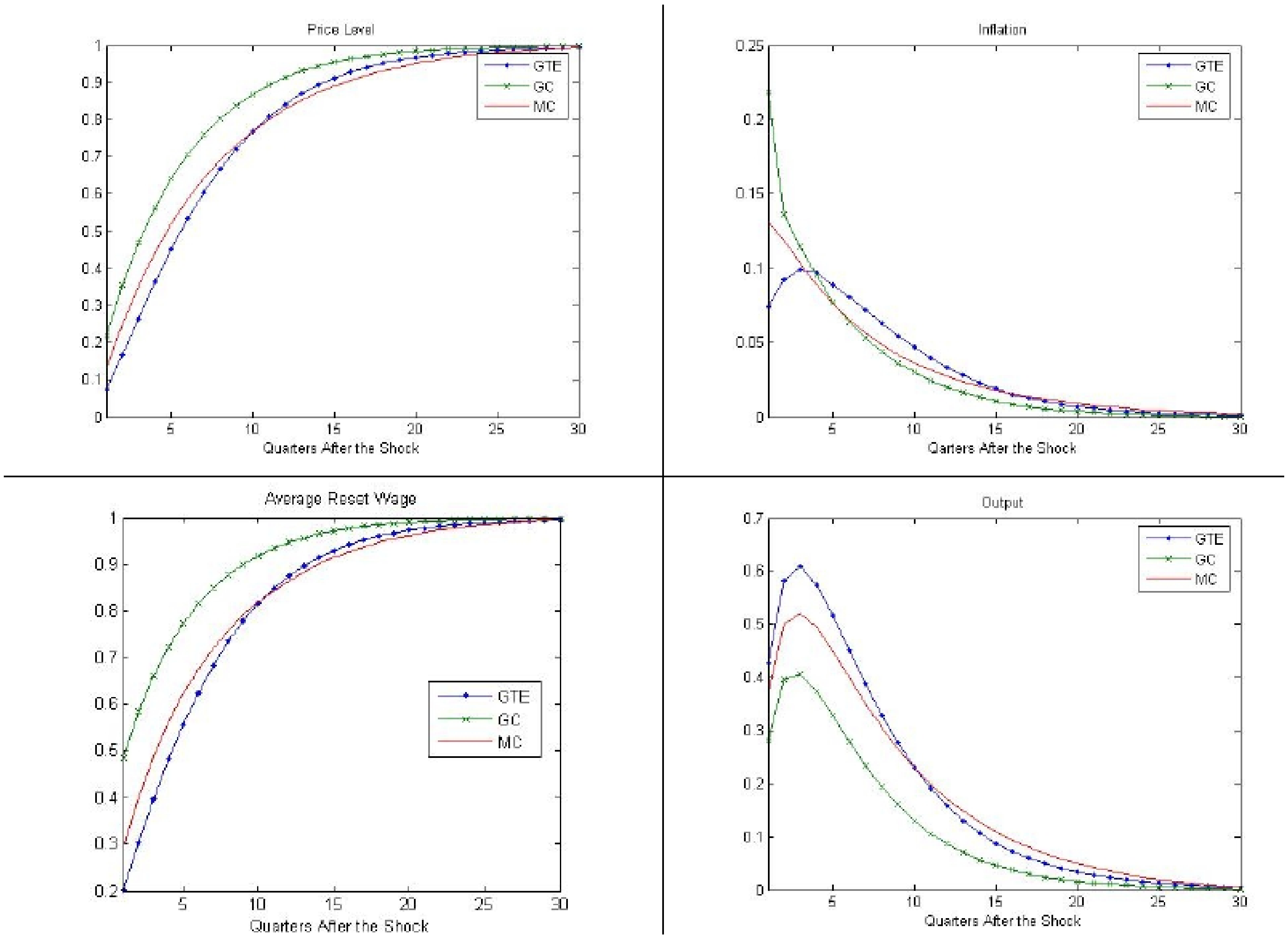
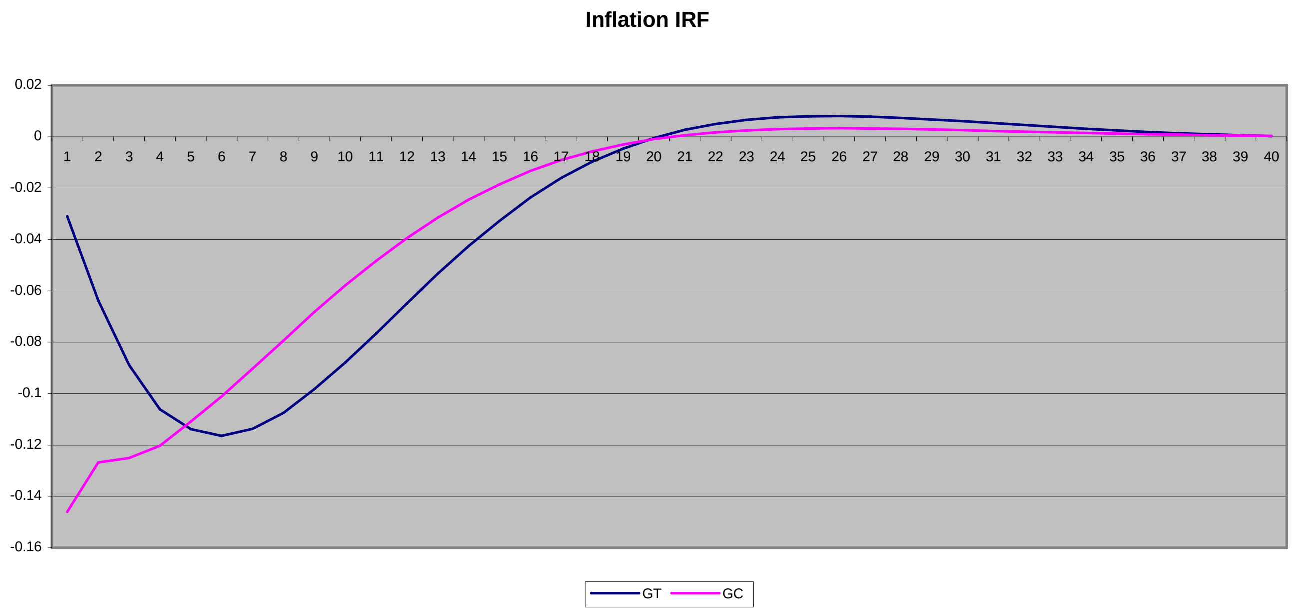


Figure 6: Serial Correlation in Monetary growth ν=0.5



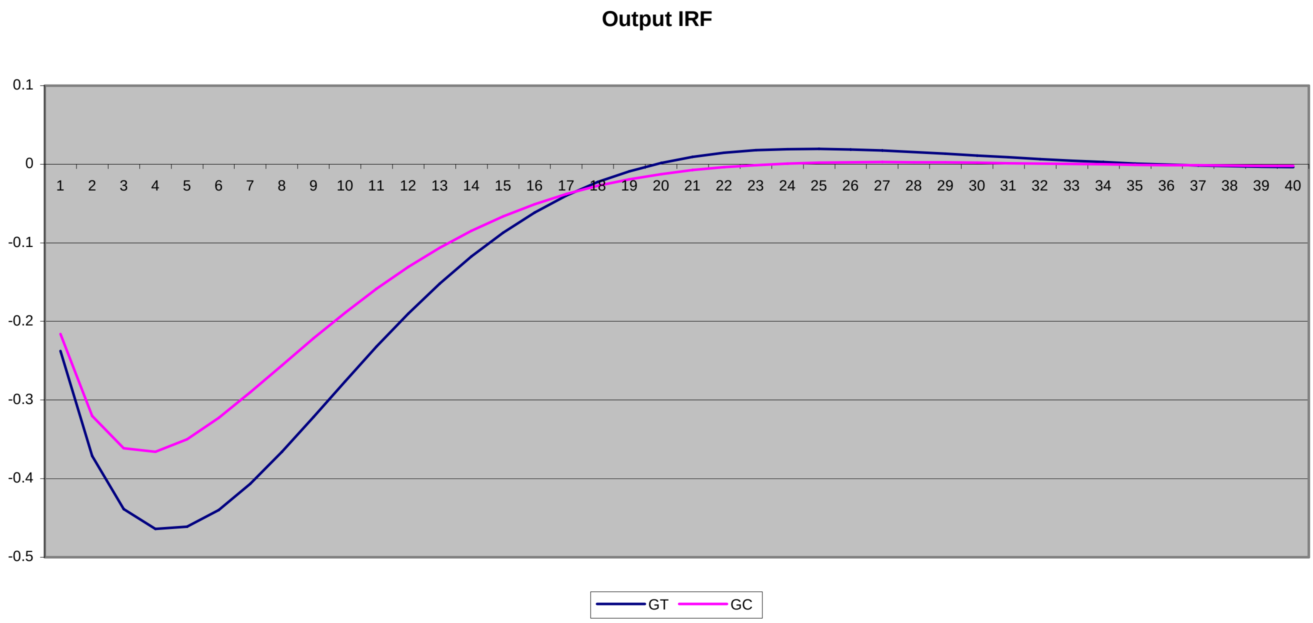


Figure 7: Inflation and Output Responses to a Monetary Shock in the SW model with UK Calibrated Pricing.