THE GENERAL THEORY OF HOUSEHOLD AND MARKET CONTINGENT DEMAND*

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The standard theory of consumer behaviour is based upon the assumption that there can only be a single price for any one commodity. If, however, we wish to understand price-setting behaviour in a market for a homogeneous product, we need to develop the theory of consumer behaviour to allow for a household to face several different prices in a given market. This theory is of particular importance for models with price-setting firms — recent examples include Allen and Hellwig (1986); Brock and Scheinkman (1985); Dasgupta and Maskin (1986); Dixon (1984, 1987); Gelman and Salop (1983); Kreps and Scheinkman (1983). The contingent demand function tells the firm its demand as a function of its price, given the prices and quantities offered by other firms. Existing treatments of contingent demand tend to make very specific assumptions and little is said about the microfoundations of contingent demand. This paper develops a general theory of contingent demand and considers the additional assumptions needed in order to use the standard specifications used in the literature. Although the theory is developed in the context of households who are price-takers, the analysis is also of relevence to strategic market games (for example, Benassy, 1984; Dubey, 1982; Simon, 1984). The general theory of household demand developed in Section II is also applicable to the case of markets with differentiated products.

I COMMON SPECIFICATIONS OF CONTINGENT DEMAND

Before outlining the results of this paper we shall briefly summarize the standard treatments of contingent demand. Suppose that households face several different prices in a given market. Each seller i sets a price p_i,

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and offers to sell up to x_i^0 at that price. The contingent demand function for seller i tells him how much he could sell if he sets p_i, given the prices and offers-to-trade of the other sellers. Thus in its most general form the contingent demand function for the ith seller is $D_i : R_+^{2n-1} \rightarrow R_+$

$$D_i = D_i(p_i, p_{-i}, x_{-i}^0) \qquad \ldots\ldots(1.1)$$

where p_{-i} and x_{-i}^0 are the $n-1$ vectors of the prices and offers-to-trade of the other sellers.

There are two alternative methods of specifying (1.1) employed in the literature. Since most of the literature is concerned with price-setting duopoly, we shall follow suit and consider the case of two sellers, $n = 2$ (this obviously generalizes). The starting point for defining contingent demand is the industry demand function, $F(p)$. All authors agree that the lower-priced seller will have the opportunity to serve the full industry demand. If both sellers set the same price it is often assumed that both sellers face the same contingent demand (the "equal shares" principle), or some other such principle.[1] The important differences arise when we consider the contingent demand for the higher-priced seller (seller 1, say).

The traditional approach since Edgeworth (1897, pp. 111—42) has been to assume that the contingent demand for the higher-priced seller 1 is that proportion of industry demand left unsatisfied by the lower-priced seller:

$$D_1(p_1, p_2, x_2^0) = F(p_1) \cdot \left[1 - \frac{x_2^0}{F(p_2)}\right] \qquad \ldots\ldots(1.2)$$

This specification of contingent demand has been employed by Allen and Hellwig (1986); Bekman (1965); Dasgupta and Maskin (1986); Gelman and Salop (1983); and Shubik (1955, 1959, Chapter 5). As is clear from Edgeworth's account (*op. cit.*), (1.2) naturally accords with the notion that the lower-priced seller serves only a subset of households, so that the contingent demand for seller 1 is constituted by the demands of those households left unserved by the lower-priced seller. Hence we denote the specification (1.2) as *Edgeworthian Demand* (ED).

The second specification is rather more recent, originating in Levitan and Shubik's article (1972), and has been used more recently by Brock and Scheinkman (1985); Dixon (1987); and Kreps and Scheinkman (1983). Under this specification the contingent demand for the higher-priced seller is:

$$D_1(p_1, p_2, x_2^0) = F(p_1) - x_2^0 \qquad (p_1 > p_2) \qquad \ldots\ldots(1.3)$$

As we shall see below, there are a number of rationales for (1.3). One is that

[1] Alternatives include Gelman and Salop's (1983) "lexicographic preferences", by which one firm is preferred by consumers over another, or Allen and Hellwig's principle that demand is proportional to amount offered (1983).

the contingent demand of a single household will yield (1.3) if we ignore the "income effect" resulting from the household's purchase of x_2^0 at a lower price p_2, and interpret $F(p)$ as the household's Marshallian demand (below, Section III, case 1). Hence we denote the specification (1.3) as *Compensated Contingent Demand* (CCD).

These two specifications are dissimilar, and can have very different implications depending on the model in which they are used. The mixed strategy solutions in price-setting oligopoly will be different, for example, as will the upper and lower bounds of the Edgeworth cycle.

The most important difference is the close connection between the CCD specification of contingent demand and the Cournot inverse demand function. From (1.2) taking the inverse of F we obtain:

$$p_1 = F^{-1}(x_2^o + D_1) \qquad \text{......(1.4)}$$

as in the Cournot model. This connection, which we can call the "CCD-Cournot identity" partly explains the close relationship between the Cournot equilibrium and price competition under CCD when firms choose their capacities (x_i^o), as in Kreps and Scheinkman (1983) (see Levitan and Shubik, 1972, also). No such connection with the Cournot demand exists for FCFS, for which the notion that "quantity precommitment and price competition yields the Cournot outcome" is not generally valid.

The choice of contingent demand can thus have very important economic implications.[2] This paper explores the microeconomics of contingent demand, both at the level of the household (Sections I and II) and at the aggregate level of the market (Sections III to V). This analysis not only provides a better understanding of contingent demand but also will help in providing grounds for choosing the appropriate specification. Whilst this paper explores the conditions under which the CCD and ED specifications will be appropriate, we also introduce an alternative specification, the *True Contingent Demand* (TCD), based on the household's contingent demand (Section III, equation (3.13)). The TCD specification includes income effects due to the purchase of output at different prices.

II The Household's Contingent Demand — The General Case

With the significant exception of Shubik's brief diagrammatic exposition (1959, pp. 82-85), the literature has hardly dealt with the "reconstruction" of an individual's demand. This stems from the fact that since Edgeworth's original 1897 article it has been customary to assume that customers are satisfied either fully or not at all by any one seller.

[2]Another example is given by Dixon (1986), where the existence of approximate equilibria in an industry with price-setting firms is studied in the context of replication. Whilst for CCD it is found that for any $\varepsilon > 0$, an ε-equilibrium will exist if the industry is large enough, this is not found to hold for ED.

In this section of the paper we model the consumer's contingent demand using a slightly adapted version of consumer theory under rationing. Suppose that there are N sellers in the entire economy. We can then treat each seller's output as a separate commodity. If several sellers sell an identical commodity, this simply comes under the extreme case of perfect substitutes in the case of a (weakly) quasi-concave utility function. Let $\mathbf{x}$ be the N-vector of the household's desired consumption, with x_i its purchase from the ith seller. The offer-to-trade of the ith seller is to sell up to x_i^0 at price p_i, $\mathbf{p}$ being the N-vector of prices set. The household has the usual budget constraint so that expenditure $\mathbf{p}.\mathbf{x}$ is less than its money balance m. (Assume throughout that p_i, x_i, $x_i^o \in R_+$.)

The contingent demand function tells the ith seller the largest amount he could sell (were he willing). The relevant programme for deriving the contingent demand is thus where the household maximizes its utility subject to the budget constraint and the offers to trade of the other sellers $j \neq i$. But this is simply the "Benassy" form of effective demand (see Benassy, 1975, 1978), with suitable reinterpretation. The essence of Benassy's formulation is that when a householder visits the ith market (interpreted here as the ith seller) he formulates his effective demand ignoring any constraints in that market, taking into account only (perceived) constraints in other markets (Benassy, 1978, p. 9-10). If we were to include the ith seller's offer to trade as a constraint, then we would have a formulation closer to Drèze (1975). We can write the ith seller's contingent demand as solving the programme:

$$\max \quad W(\mathbf{x}) \qquad \ldots\ldots(2.1)$$

$$\text{s.t.} \quad \mathbf{p}.\mathbf{x} \leqslant m \qquad \ldots\ldots(2.2)$$

$$0 \leqslant x_j \leqslant x_j^0 \qquad j = 1 \ldots N, i \neq j \qquad \ldots\ldots(2.3)$$

where $W : R_+^N \rightarrow R_+$ is the household's utility function, assumed to be weakly quasi-concave. The ith seller's contingent demand is given by the value(s) of x_i that solve (2.1–3). The resultant contingent demand correspondence will be upper hemi-continuous, convex valued, and closed (it will also be bounded for $\mathbf{p} >> 0$).

Consider the following very simple example, where there are two sellers of perfect substitutes, so that $W = x_1 + x_2$, and we restrict ourselves to $p_i > 0$ and $x_i^o < m/p_1$. If we consider the contingent demand for seller 1, we have:

$$\max_{x_1, x_2} \quad x_1 + x_2$$

$$\text{s.t.} \quad p_1.x_1 + p_2.x_2 \leqslant m$$

$$0 \leqslant x_2 \leqslant x_2^0$$

The solution to this is the upper hemi-continuous correspondence $D_1 \colon R_+^N \Rightarrow R_+$:

$$D_1 = \begin{cases} m - p_2.x_2^0/p_1 & p_1 > p_2 \\ [m - p_2.x^0{}_2/p_1, m/p_1] & p_1 = p_2 \\ m/p_1 & p_1 < p_2 \end{cases}$$

This contingent demand correspondence is depicted in Fig. 1. Note that since the contingent demand for each seller solves a different programme, there is no requirement that they "add up" to the total desired purchases at that price. For example, if both firms set the same price, and offer to sell nothing, then $D_1 = D_2 = m/p_1$, and the total value of contingent demand is equal to twice the household's money balance. Evidently, contingent demands do not satisfy Walras's Law.

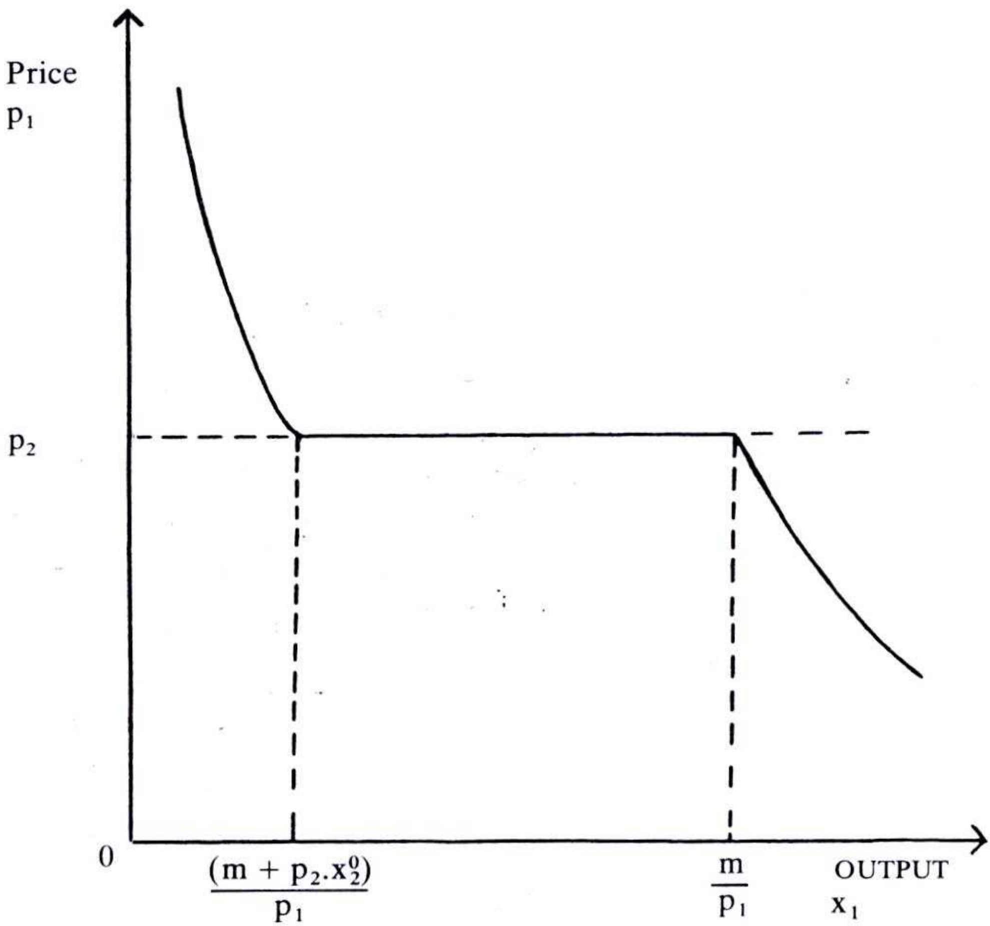


Fig. 1
The First Seller's Contingent Demand

It is thus possible to reformulate the derivation of household contingent demand from the standard theory of effective demand. All that is needed is a little reinterpretation: we treat each seller's output as a commodity itself. As we have shown from our simple example, we can use the definitions of effective demand to derive the specific household contingent demand function for a seller.

Whilst the formulation of contingent demand presented in this section is very general, it might be more useful in some circumstances to specialize it somewhat. In particular, it may be that the very generality of the formulation leaves out some more specific information which we certainly can assume: that is, that some sellers are selling the same product. This imposes a particular (separable) structure on W. We can define a "standard" utility function which takes into account the fact that some outputs are the same commodity. If there are l commodities (in the usual sense $l < N$), we can define utility function $U: R_+^l \to R_+$ such that:

$$W(\mathbf{x}) = U\left(\sum_{i=1}^{n^1} x_i, \sum_{i=n^1}^{n^1+n^2} x_i \ldots \sum^{N} x_i\right)$$

where there are n^j sellers in the jth market.

In the next section we derive the contingent demand correspondence under this and other specific assumptions. More particularly, we relate it to the standard Marshallian demand function. Having thus expressed the contingent demand function in terms of the standard Marshallian demand functions, we are able to consider its properties more closely.

III TRUE CONTINGENT DEMAND AS HOUSEHOLD CONTINGENT DEMAND WHEN UNRATIONED IN OTHER MARKETS

In the previous section, we explored a general formulation of the contingent demand correspondence, noting its similarity to the Benassy formulation of effective demands. We now consider a special case of the general formulation, assuming that the N sellers can be subdivided into l markets ($l \leqslant N$), so that in each market the same commodity is sold. As a reference point, we consider the standard Marshallian demand, which is derived under the assumption that there is one seller per commodity, and no rationing. Defining the household's utility function over these l commodities, $U: R_+^l \to R_+$, programme (2.1) – (2.3) then simplifies to the standard textbook Marshallian demand optimization:

$$\max_{\mathbf{x} \in R_+^l} \quad U(\mathbf{x}) \qquad \text{......(3.1)}$$

$$\text{s.t.} \quad \mathbf{p}.\mathbf{x} \leqslant m \qquad \text{......(3.2)}$$

where $\mathbf{p}$ is the l-vector of prices in each "market". For the rest of the paper we shall assume that U is strictly quasi-concave, so that we can write the solution as a *function* of the constraint parameters,

$$\mathbf{x} = \mathbf{x}^m(\mathbf{p}, m) \qquad (\mathbf{x}^m: R_+^{l+1} \rightarrow R_+^{l}) \qquad \ldots\ldots(3.3)$$

In what follows, we shall suppress the prices in other markets (p_{-j}), and write the solution value x_j as a function of p_j and m, $x_j^m(p_j, m)$ (dropping the j subscript when convenient).

Starting from this standard Marshallian demand, we can derive a special case of the household's contingent demand which can be expressed in terms of the Marshallian demand (3.3) and which we shall denote the household's "True Contingent Demand", TCD. We simply allow for the case of two sellers in the jth market (this is easily generalized to **n**-sellers), and assume that there is no (binding) rationing constraint in the other markets. To formalize this we need to reindex the sellers, so that subscript kj is the kth seller in market j. Formally we have:

Assumption (*TCD*):

(A1) There are two sellers in the jth market, and one seller in markets $i \neq j$ (or, equivalently, only one price set).

(A2) When deriving the contingent demand in market j, the constraints x_{ik}^o are non-binding for all $i \neq j$.

A1 enables us to drop the k subscript in the "other" markets $i \neq j$.

We now state the problem which the household solves in formulating its contingent demand for seller 1 in market j under A1, A2. Let p_{-j} and x_{-j} be the $l-1$ vectors of the price and quantities in markets other than j. The household then solves:

$$\max_{x_{1j},\, x_{2j},\, x_{-j}} \quad U(x_{1j} + x_{2j}, x_{-j}) \qquad \ldots\ldots(3.4)$$

$$\text{s.t.} \quad p_{1j} \cdot x_{1j} + p_{2j} \cdot x_{2j} + p_{-j} \cdot x_{-j} \leqslant m \qquad \ldots\ldots(3.5)$$

$$x_{1j} \geqslant 0 \qquad \ldots\ldots(3.6)$$

$$x_{-j} \geqslant \mathbf{0} \qquad \ldots\ldots(3.7)$$

$$x_2^0 \geqslant x_{2j} \geqslant 0 \qquad \ldots\ldots(3.8)$$

(3.5) is simply the standard budget constraint except that we split up the household's expenditure on commodity j into its expenditure on seller 1 and 2's outputs.

The contingent demand for seller 1 is the solution to (3.4) – (3.8). We consider the solution under three cases (dropping the j subscript on p_{1j} and p_{2j}).

(a) $p_1 < p_2$.

Here, since the household ignores any constraint on seller 1's output, the solution is $x_{2j}^* = 0$, and seller 1's contingent demand is equal to the Marshallian demand at p_1.

(b) $p_1 = p_2$.

The total demand for commodity j is equal to the Marshallian demand: the household is indifferent between the two sellers.

Contingent demand will thus be multivalued.

$$D_1(p_1, p_2, x_2^0) = [\max[0, x^m(p_1, m) - x_2^0], x^m(p_1, m)]$$

(c) $p_1 > p_2$.

This is the more interesting case. The householder will buy from seller 1 only if he cannot obtain all he desires from seller 2 (i.e., $x_2^0 < x^m(p_2, m)$). If this is so, then we can rewrite (3.4) – (3.8) as:

$$\max_{x_{1j},\, x_{-j}} \quad U(x_{1j} + x_2^0, x_{-j}) \qquad \text{......(3.9)}$$

$$p_1 . x_{1j} + p_{-j} . x_{-j} \leqslant m - p_2 . x_2^0 \qquad \text{......(3.10)}$$

plus the usual non-negativity constraints. This reformulation reflects the fact that we know $x_{2j}^* = x_2^0$. We can rewrite the budget constraint (3.10):

$$p_1.(x_{1j} + x_{2j}) + p_{-j}. x_{-j} \leqslant m + (p_1 - p_2). x_2^0 \qquad \text{......(3.11)}$$

But maximizing (3.9) subject to (3.11) simply yields the Marshallian demand evaluated at $(p_1, m + (p_1 - p_2). x_2^0)$. Hence the contingent demand for seller 1 if $p_1 > p_2$ is:

$$D_1(p_1, p_2, x_2^0) = \max[0, x^m(p_1, m + (p_1 - p_2). x_2^0) - x_2^0]$$

The "max" is introduced to reflect the non-negativity constraints.

Putting together cases (a)—(c), seller 1's contingent demand under A1 and A2, its True Contingent Demand, has the form:

$$D_1(p_1, p_2, x_2^0, m) = \begin{cases} \max[0, x^m(p_1, m + (p_1 - p_2).x_2^0) - x_2^0] & p_1 > p_2 \\ [\max[0, x^m(p_1, m) - x_2^0], x^m(p_1, m)] & p_1 = p_2 \\ x^m(p_1, m). & p_1 < p_2 \end{cases} \qquad \text{......(3.12)}$$

This is depicted in Fig. 2. The logic behind case (c) (i.e., $p_1 > p_2$) is quite intuitive. Since the household has been able to purchase a quantity x_2^0 at a lower price p_2, this acts as a "subsidy" if we evaluate the budget constraint assuming that all purchases of commodity j $(x_{1j} + x_{2j})$ are purchased at p_1. The logic of the two-seller case clearly goes through to the case of n_j sellers. If the ith seller sets price p_i, then the "subsidy effect" augments the income term by $\sum_{p_k < p_i} (p_i - p_k).x_k^0$

The rigorous derivation of the household's contingent demand is useful since it gives us the simple TCD specification of contingent demand, which rests on explicit microfoundations (in the next section we show how it can be extended to the case of many households). Furthermore the TCD specification enables us to analyse the properties of contingent demand in terms of standard consumer theory. To turn to the third reason, we can decompose the effects of a rise in price on contingent demand using the Slutsky decomposition. Suppose that $p_1 > p_2$ and $D_1(p_1, p_2, x_2^0) > 0$. Then from (3.12) we have, using the Slutsky decomposition:

$$\frac{\partial D_1}{\partial p_1} = \left.\frac{\partial x^m}{\partial p_1}\right|_{U_0} + \frac{\partial x^m}{\partial y} \cdot \frac{dy}{dp_1} \qquad \text{......(3.13)}$$

where y is income $m + (p_1 - p_2).x_j^o$, so that $dy/dp_1 = - x_j^0$. Hence:

$$\frac{\partial D_1}{\partial p_1} = \left.\frac{\partial x^m}{\partial p_1}\right|_{U_0} - \frac{\partial x^m}{\partial y}. D_1 \qquad \text{......(3.14)}$$

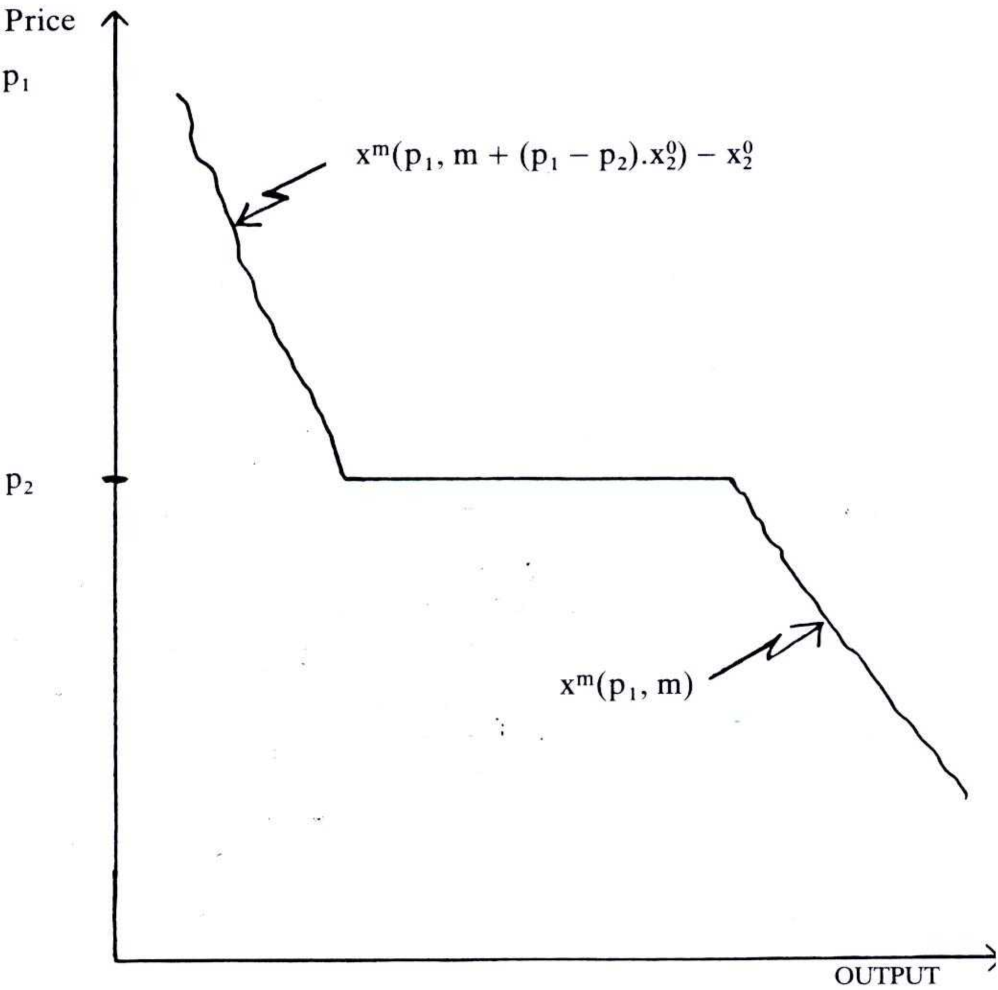


Fig. 2
Seller 1's Contingent Demand under A2.2, A2.3

where derivatives of x^m are evaluated at $(p_1,\ m\ +\ (p_1\ -\ p_2).x_j^0)$. If we compare (3.14) with the slope of the Marshallian demand, we can see that the income effect of the price change is smaller in the case of contingent demand than in the standard Marshallian case, since $D_1 = x^m - x_2^0$. The rise in p_1 does not generate any income effect via the quantity x_j^0 purchased from the lower-priced seller, and hence the income effect of price changes is dampened for the higher-priced seller.

The TCD specification also provides a possible rationale for the CCD specification of contingent demand. Recalling equation (1.3), and comparing this standard CCD specification with (3.12) above, when $p_1 > p_2$, we can interpret CCD in terms of a household's contingent demand compensated for the "subsidy effect". If we equate the industry demand functions (given some suitable m) so that for all p, $F(p) \equiv f(p,m)$, then by the Mean Value Theorem we have:

$$f(p_1, m + (p_1 - p_2).x_2^0) = F(p_1) + \frac{\partial f}{\partial y}.(p_1 - p_2).x_2^0$$

where the derivative $\partial f/\partial y$ is evaluated at some income y between m and $m + (p_1 - p_2).x_2^0$. CCD can either be treated as an approximation to the TCD, or as an exact expression when income effects are compensated or absent. There remains, however, an important difference between CCD and household TCD when $p_1 = p_2$. Under CCD there is (usually) an "equal shares" distribution of demand between sellers setting the same price. As we show in Dixon (1984), the assumption of equal shares can be of crucial importance, although there is no good reason for it suggested with the framework presented. However, so far as the theory of household contingent demand is concerned, contingent demand will be multivalued when firms set the same price.

This section has presented a special case of the household contingent demand as outlined in Section II, giving rise to the TCD specification. This has been useful since it has enabled us to relate contingent demand to the more familiar and standard Marshallian demand. The approach of this section is in fact less restrictive than is suggested by A1 and A2. We can replace the assumption that there is no rationing in other markets (A2) to allow for rationing in other markets. This can be done using the framework employed in Neary and Roberts (1980), replacing the Marshallian demand by its constrained version. Although it is rather more intricate to analyse, the results are similar.

So far we have considered the contingent demand of one household facing two sellers. This is a rather special case. However, it is worth considering when the contingent demand in a market with many buyers will have the same form as the TCD or CCD specification. The vital new ingredient that is introduced when we look at aggregate contingent demand is the rationing regime which operates in the market when there is excess contingent demand for a particular seller. In order to obtain the TCD or CCD specifications of contingent demand there must be a *Proportional Rationing* (PR) mechanism. Under PR, if there is excess contingent demand for a particular seller or group of sellers setting the same price, then each household receives a certain proportion of its contingent demand, that proportion being determined so as to equate purchase with supply. PR is manipulable, since any household can obtain more if it asks for more. However, in the case of identical households, PR is equivalent to the rule where each household receives an *Equal Share* (ES) of available output, which is a non-manipulable rationing scheme. Let the contingent demand of household h for seller j be denoted D_{hj}, which will be given by the household's contingent demand as formulated in equation (3.12). The proportion of their demands which households receive equates supplies $\sum_{p_i = p_j} x_i^0$ with demand $\sum_H D_{hj}$. We can denote this proportion k_j:

$$k_j = \min \ 1, \left[\left(\sum_{p_i = p_j} x_i^0 / \sum_H D_{hj}\right)\right] \qquad \text{......(3.15)}$$

We include the min condition to cover the case where there is excess supply ($\Sigma D_{hj} < \Sigma x_i^0$). Thus the actual purchase of h from seller j (or from sellers setting $p_i = p_j$) is $\bar{x}_{hj} = k_j . D_{hj}$. With identical households and ES, $K_j = 1/ \neq H$.

If we take the case where $p_1 > p_2$ again, and assume that $D_{h1} > 0$, summing over household demands (3.12) using (3.15) yields:

$$D_1(p_1, p_2, x_2^0) = \sum_H [x_h(p_1, m_h + (p_1 - p_2).\bar{x}_{h2})] - x_2^0 \qquad \text{......(3.16)}$$

Comparing (3.16) with (3.12), under PR or ES the market contingent demand is just like the individual household's contingent demand "blown up". The TCD specification of contingent demand will therefore remain valid at the aggregate level under PR and ES, and the properties of the individual household's contingent demand will carry over to the market contingent demand. Therefore the same comments relating to the "weakened" income effect of the contingent demand relative to the Marshallian hold here as well in the individual's case.

IV Market Contingent Demand: First-Come-First-Serve

In this section we consider the market contingent demand when there is first-come-first-serve rationing of households under excess demand. Under FCFS rationing, only some subset s of the set of households H is served, and of this subset all but perhaps one marginal member receives all that they want at that price. This rationing scheme is more attractive than PR in that it is non-manipulable, and also more realistic in that most households will buy a commodity from only one seller. If we again consider the two-seller case, where $p_1 > p_2$, and $x_2^0 < \Sigma x_h(p_2, m_h)$, we can define the corresponding class S of sets s of households which can be served by seller 2 under FCFS.[3] Note that under FCFS all but perhaps the last household served will be able to buy as much as they want. The contingent demand for the higher priced seller 1 then consists of the sum of the Marshallian demands of customers not served at all by seller 2 ($h \varepsilon H-s$), plus the contingent demand of the marginal customer only partly satisfied by seller 2. Hence:

$$D_1(p_1, p_2, x_2^0) = \sum_{H-s} x_h(p_1, m_h) + \sum_{s} D_{h1}(p_1, p_2, x_2^0) \qquad \ldots\ldots(4.1)$$

where $D_{h1} = 0$ for all but at most one member of s, the last to be served by the lower-priced seller.

If there is excess demand for a particular seller, then there may be many possible subsets of households which could be served by the lower-priced seller. We can either assume that FCFS operates with a deterministic rule, which picks out which households will be served, or that it operates randomly. We shall turn first to the properties of FCFS with a deterministic selection mechanism, which is usually explored using the special "unit demand" demand curve originating in Morgenstern (1948, p. 177), where: "... the aggregate demand curve involves a large number of buyers, each of whom desires only one unit, all (or most) having different maximum bids for it, but each willing to buy one unit at any price below this maximum. Each lower bid is added to all the previous bids". Following subsequent usage (e.g., Shubik, 1955; Gelman and Salop, 1983) we shall call Morgenstern's "maximum bid" the household's reservation price, r_h.

The industry demand function $F(p)$ can be written as:

$$F(p) = \sum_{H} \chi_p(h) \qquad \ldots\ldots(4.2)$$

where $\chi_p(h)$ is the characteristic function of the set $h : r_h \leqslant p$, which takes the value 1 if $r_h \leqslant p$, zero otherwise. One popular serving scheme is that those households with the highest reservation prices are served first (this is called "reservation price rationing" by Gelman and Salop, 1983). Suppose for ease that x_2^0 is an integer, then the customers served are those with a reservation price greater than or equal to q where:

$$\Sigma \chi_q(h) = x_2^0 \qquad \ldots\ldots(4.3)$$

(thus q is the price at which industry demand is equal to x_2^0). Then the contingent demand of seller 1 ($p_1 > p_2$) is:

$$\sum_{H} \chi_{p_1}(h) - \sum_{H} \chi_q(h) = F(p_1) - x_2^0 \qquad \ldots\ldots(4.4)$$

Thus firm 1's contingent demand consists of the demands of those consumers with $q \geqslant r_h \geqslant p_2$, and (4.4) is the familiar CCD specification of

[3]Of course, S depends on p_2 and x_2^o.

contingent demand. Thus CCD can arise from FCFS rationing if consumers have Morgenstern unit demands and consumers with the highest price are served first.

Another possible rule to determine who gets served has been suggested by Shubik: namely that households are chosen to maximize the residual demand at higher prices (1955, p. 419, convention 3). This is equivalent to choosing those customers wishing to be served who have the lowest reservation prices ("lowest first"). The lower-priced seller 2 sets price p_2, so that if $p_1 > p_2$ he will sell x_2^0 units to the first x_2^0 customers with reservation prices over p_2. Suppose that the last customer then served has a reservation price t, then the residual demand for the higher-priced seller 1 is:

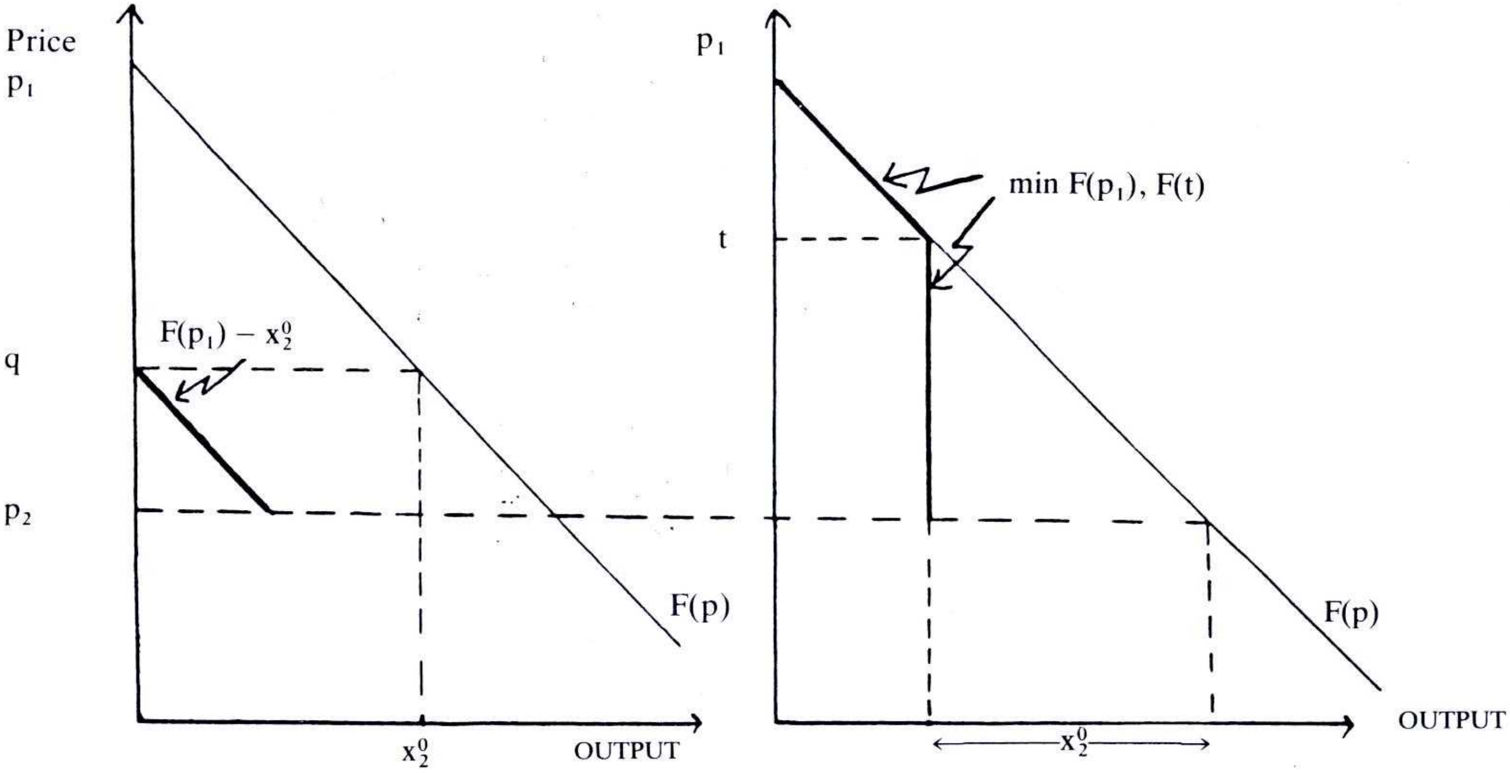
$$\min\,[F(t), F(p_1)] \qquad \text{......(4.5)}$$

If $p_2 < p_1 \leqslant t$, seller 2's contingent demand is thus $F(t)$: if $p_1 \geqslant t$, then contingent demand is given by $F(p_1)$. Clearly, the "lowest first" rationing rule has rather different implications to the conventional "highest first". We depict them side-by-side in Fig. 3.

Whilst the exact nature of the deterministic rationing rule makes a big difference to the resultant contingent demand, there is no obvious criterion to choose between them.[4] If we view the customer's place in the queue as a result of "market effort", then there is no necessary relationship between the reservation price and the disutility of market effort (see Coase, 1934, p. 138). Perhaps if we interpret market effort as leisure foregone, and leisure as a normal good, then it seems plausible that lower income households would have a lower-reservation price and make more market effort than higher-income households. This line of speculation points towards the "lowest first" rule. However, if we abandon the drastic simplification of Morgenstern unit demands, it is not at all clear how we might formulate a simple deterministic rationing rule. In the context of our model, which excludes search and transaction costs, it seems more natural to assume that FCFS operates randomly. It is this possibility to which we next turn.

As Coase commented, under FCFS: "The demand for A's product can be obtained by adding together the demand curves of those individuals who are excluded from purchasing from B. The individuals who are excluded, however, depend on chance. Therefore, the demand curve for A's product depends on chance" (Coase, 1934, p. 138). This randomness has largely been ignored in the literature, as a nuisance to be assumed away rather like

[4]Note that with Equal Shares and unit demands, contingent demand is given by the ED specification. Hence with unit demands ES yields ED, and FCFS can yield CCD: this is the reverse of what holds when households have standard demand functions (ES/PR yield CCD, and FCFS can give rise to ED). Thus the relationship between rationing mechanism and the resultant contingent demand depends very much upon the type of consumer demand which is assumed.



(a) Highest first (CCD) (b) Lowest first

Fig. 3
FCFS — Two Alternative Rationing Rules

an integer problem. One obvious way to make such randomness harmless is to assume that households are identical, so that it makes no difference who gets served first. This assumption was made by Edgeworth (1897) and has been made since (d'Aspremont and Gabszewicz, 1980). Alternatively, it has been more common to assume that the contingent demand is an expectation, assuming that each household is equally likely to be served (see Shubik, 1955, p. 420; Bekman, 1966, p. 55; and Gelman and Salop, 1983). How valid is it to treat the contingent demand function as an expectation? If a higher-priced seller's payoffs depend upon the actual set s of consumers served by a lower-priced seller, then in general Jensen's inequality tells us that maximizing the expected payoff will be equivalent to maximizing the payoff given the expected realization of demand only if the payoff is linear in the realization of demand. In a Bertrand model with constant unit costs, profit-per-unit output sold is constant given the price set, so that there is no problem. However, in the case considered in Dixon (1984, 1987), where firms' costs can be strictly convex, it will be invalid to treat the contingent demand as an expectation, since profits are then strictly concave in the realization of demand. Whilst we can avoid randomness by assuming households to be identical, or by treating contingent demand as an expectation, the former approach is rather restrictive whilst the latter is not generally applicable.

If we recall the expression for the contingent demand under FCFS rationing in (4.1), and compare it to the ED specification (1.2), we can see that two conditions must hold if the two expressions are to be equivalent for all s: first, there is no marginal customer (for all $s \varepsilon S$, $D_{hi} = 0$ for all $h \varepsilon s$), and, secondly, that for all $s \varepsilon S$,

$$\sum_{H-s} x_h(p_1, m_h) = \sum_{H} x_h(p_1, m_h) . \left[1 - \frac{x_1^0}{\sum_{H} x_h(p_1, m_h)} \right] \qquad \text{......(4.6)}$$

We can ensure the absence of a marginal customer if we assume a continuum of consumers (more precisely an atomless measure space of households). Equation (4.6) will certainly not hold generally since if households are not identical, then $\sum_{H-s} x_h$ will vary with s.

In fact, it turns out that we can weaken the requirement that all households need to be identical to have a non-random contingent demand under FCFS. If households have identical *homothetic* preferences, then whatever the distribution of money balances m_h across households, then (4.6) will be satisfied if there is no "marginal" customer. The reasoning behind this is that if preferences are homothetic, then demand of households is proportional to money balances. Thus all the possible sets $s \varepsilon S$ of households that might be served by the lower-priced seller will have the same total money balances. Hence whichever set of households is served by the lower-priced customer, the total income of those left over to be served by seller 1 will be the same.

If preferences are identical and homothetic, we can write the Marshallian demand of any group of customers $g \subseteq H$ as:

$$\sum_{h \varepsilon g} x_h(p, m_h) = b(p) \,.\, \sum_{g} m_h \qquad \text{......(4.7)}$$

If we consider the class of sets s which may be served by the lower-priced seller 2:

$$b(p_2) \,.\, \sum_{s} m_h = x_2^0 \quad \text{for all } s \varepsilon S \qquad \text{......(4.8)}$$

From (4.7) and (4.8) we can derive the first seller's contingent demand:

$$\sum_{H-s} x_h(p_1, m_h) = b(p_1). \sum_{H} m_h. \left[1 - \frac{x_2^0}{b(p_2). \Sigma x_h(p_2, m_h)}\right] \qquad \text{......(4.9)}$$

which is the non-random ED specification given in (1.2), since industry demand is $b(p). \Sigma x_h(p, m)$. Thus if households have identical homothetic preferences, and there are no "marginal customers", then whichever set of households is served the higher-priced firm faces the same contingent demand as in the ED specification. There is thus no difference in terms of contingent demand between FCFS rationing under alternative deterministic rules and random rationing. Whilst the assumption of identical homothetic preferences is restrictive, it surely provides a more satisfactory solution to the problem than taking expectations or introducing *ad hoc* deterministic rules, as is usual in the literature.

V CONCLUSION

This paper provides a general framework for understanding the theory of contingent demand, both at the level of the individual household and the market. We have provided a general formulation for the household's contingent demand, adapting the standard theory of consumer demand under rationing. This allows a formal analysis of income and substitution effects on contingent demand. Using the Slutsky decomposition reveals that income effects are weaker with contingent demand compared with standard Marshallian demands.

At the level of the market, the rationing mechanism becomes all important, since it determines exactly who gets served what by the lower-priced seller, and hence the nature of the residual demand left over for the higher-priced seller. Since Edgeworth's original paper, the most common assumption has been the all-or-nothing rule of FCFS. The main issue that FCFS raises is that the contingent demand for the higher-priced seller depends on who gets served first. We have shown that if households have identical homothetic preferences, then contingent demand will not depend on exactly who gets served first. Furthermore, the resultant formula for market contingent demand is the original ED specification.

An alternative to FCFS is provided by rationing mechanisms that give something to everyone — PR or ES. With equal shares rationing and identical households, market contingent demand becomes the constituent household's demand writ large. The theory of household contingent demand carries over to the market level. This type of rationing can be seen as giving rise to the CCD specification of contingent demand originating in Levitan and Shubik (1972).

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