Models of Pricing.

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1 Models of Pricing

- Paradox of perfect competition. Everyone acts as a price-taker, nobody sets prices. In order to understand price-setting behaviour, you need to have price setters.

- In general, price-setters will have market power (except in the case of Bertrand competition with a homogeneous product and constant returns). Hence the equilibrium with price-setters will be imperfectly competitive.

- Unlike the perfectly competitive equilibrium, the imperfectly competitive equilibrium does not maximize social welfare even in the representative agent model. This is a good thing! Changes in output and employment can have first order effects on welfare even in the absence of technology shocks.

- imperfect competition does not give rise to nominal price rigidity per se. However, you cannot understand price rigidity unless you have a model of how prices are set.

1.1 Types of Pricing rules.

- The behaviour of prices can be though of in different ways.

- The decision process: at a particular time the firm gathers information and decides on a pricing strategy for the future. This strategy is then implemented for a particular period. Then, the firm again gathers information and decides on a pricing strategy and so on.

- We can differentiate pricing theories by two main features
  - How the duration of the pricing strategy is determined.
  - What the pricing strategy is.

- The duration of the pricing strategy can be determined in three ways.
  - determined randomly: each period there is a probability that the current pricing strategy will end. Calvo (1983) this probability is constant; can also have duration dependent models where the probability varies with the how long the strategy has been in place.
  - Fixed: as in Taylor model, the pricing strategy lasts for a fixed period which is known ex ante.

  - Chosen optimally ex ante (or ex post). State-dependent pricing (menu cost) models fall into this category.

- What the pricing strategy is.

  - The price is set at a particular level for the duration of the plan (the most common assumption: Calvo and Taylor).

  - A pricing plan is chosen: the firm chooses a trajectory of prices over the life of the plan (Fischer (1977) , Mankiw and Reis).

  - The initial price is set at the outset of the plan, and then updated according to a simple rule such as indexation. This is quite [popular at the moment!

- This lecture series will not do State-dependant pricing (much...).

1.2 Basic Intertemporal pricing decision.

- Monopolistic competition: CES consumer preferences gives constant elasticity of demand.

$$
\begin{aligned}
Y_t &= \left[\int_0^1 Y_t(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}} \\
P_t &= \left[\int_0^1 P_{ft}^{1-\theta} df\right]^{\frac{1}{1-\theta}}
\end{aligned}
$$

- Demand:

$$
Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\theta} Y_t \tag{1}
$$

- Costs: various assumptions: simply assume marginal cost is a function of the general price level P_t and the output of the firm

$$MC = P_t \left(Y_t(f)\right)^{\sigma} : \sigma > 1$$

- Flex price:

$$\max \left[P_{ft} Y_t(f) - TCY_t(f)\right]$$

subject to (1).

$$\begin{aligned} MR &= MC \\ P_f \left(1 - \frac{1}{\theta}\right) &= P_t \left(\frac{P_{ft}}{P_t}\right)^{-\theta\sigma} Y_t^{\sigma} \end{aligned} \tag{2}$$

Depends on P_t, Y_t. Log linearize around steady-state, assuming $P_{ft} = P_t$ in SS:

$$p_f = p_t - \theta\sigma \left(p_f - p_t\right) + \sigma y_t$$

Hence:

$$\begin{aligned} p_t^* &= p_t + \gamma y_t \\ \gamma &= \frac{\sigma}{1+\theta\sigma} \end{aligned} \tag{3}$$

- Fixed for two periods

$$(MR_1 - MC_1) + \beta\,(MR_2 - MC_2) = 0 \tag{4}$$

Hence from (2)

$$\begin{aligned} MR_t - MC_t &= P_f\left(\frac{\theta}{\theta-1}\right) - P_t\left(\frac{P_f}{P_t}\right)^{-\theta\sigma} Y_t^{\sigma} \\ &= \left(\frac{\theta}{\theta-1}\right) - \left(\frac{P_f}{P_t}\right)^{-(1+\theta\sigma)} Y_t^{\sigma} \end{aligned}$$

using (3) yields the intertemporal optimum (4) as

$$\left(\frac{\theta}{\theta-1}\right)-\left(\frac{P_f}{P_1}\right)^{-(1+\theta\sigma)}Y_1^{\sigma}+\beta\left(\left(\frac{\theta}{\theta-1}\right)-\left(\frac{P_f}{P_2}\right)^{-(1+\theta\sigma)}Y_2^{\sigma}\right)=0 \tag{5}$$

log-linearise (note you get the β because the structure is additive)

$$\begin{aligned}(1+\theta\sigma)\left(p_1-p_f\right)+\sigma y_1+\beta\left((1+\theta\sigma)\left(p_2-p_f\right)+\sigma y_2\right) &= 0\\ p_f-p_1^*+\beta\left(p_f-p_2^*\right) &= 0\end{aligned}$$

Hence

$$p_f=\frac{p_1^*+\beta p_2^*}{1+\beta}$$

over n periods you get the reset price

$$x_t=\frac{1}{\sum_{i=0}^{n-1}\beta^i}\sum_{i=0}^{n-1}\beta^i p_{t+i}^* \tag{6}$$

- The optimal price to set over n periods is a discounted average of the flex-prices over the duration of the price-spell.

$$MR - MC = \left(\frac{P_f}{P_t}\right)^{-(1+\theta\sigma)} Y_t^{\sigma}$$

log-linearising the RHS:

$$(1 + \theta\sigma)\left(p_t^* - p_f\right)$$

The difference between the log-deviation of the flex price and the actual price is proportional to the deviation of $MR - MC$.

2 Labour Supply as foundation of marginal cost.

Look deeper, relate MC to labour supply. Start by defining MC in terms of wages and the marginal product of labour

$$MC = \frac{W}{f_L}$$

Then use the fact that since the MRS between consumption and leisure equals the real wage to get

$$W = P\frac{U_\ell}{U_c}$$

Hence

$$MC = \frac{P}{U_c}\frac{U_\ell}{f_L}$$

ignoring capital let $\sigma > 1$ (Diminishing MPL)

$$\begin{aligned} Y_f &= L_f^{\nu} \\ f_L &= \nu L_f^{\nu-1} = \nu Y_f^{-\frac{1-\nu}{\nu}} \end{aligned}$$

so that $\sigma = \frac{1}{\nu}$, $\nu < 1$ ($\sigma > 1$ implies Diminishing MPL). Hence marginal cost is

$$MC = \frac{PU_\ell}{\nu U_c} \left(\left(\frac{P_{ft}}{P_t} \right)^{-\theta} Y_t \right)^{\frac{1-\nu}{\nu}}$$

- $MC = MR$

$$P_f \left(1 - \frac{1}{\theta} \right) = \frac{PU_\ell}{\nu U_c} \left(\left(\frac{P_{ft}}{P_t} \right)^{-\theta} Y_t \right)^{\frac{1-\nu}{\nu}}$$

log-linearise

$$p_f = p + \eta_\ell n + \eta_c c + \frac{1-\nu}{\nu} y + \frac{1-\nu}{\nu} \theta \left(p - p_f\right)$$

where $\eta_\ell = -\left(\frac{U_{\ell\ell} L}{U_\ell}\right)$ and $\eta_c = -\left(\frac{U_{CC} C}{U_C}\right)$. Now, from production function $n = \frac{1}{\nu} y$, and if no capital $c = y$, hence

$$\left(p_f - p\right)(1 - \frac{1-\nu}{\nu}\theta) = \left(\frac{\eta_\ell}{\nu} + \eta_c + \frac{1-\nu}{\nu}\right) y$$

which yields the optimal flex price p^* :

$$\begin{aligned} p^* &= p + \gamma y \\ \gamma &= \frac{\eta_\ell + \nu \eta_c + 1 - \nu}{1 + (1-\nu)\,\theta} \end{aligned}$$

- As we can see, this approach relates the *marginal cost* to the *marginal rate of substitution* between leisure and consumption: in the log-linearized version, both of these become linked to aggregate output. The γ term captures the effect that as output rises, the marginal utility of leisure goes up (work more, play less), the marginal utility of consumption goes down as consumption increases. In addition there is the diminishing MPL; every extra unit of output uses up more labour so magnifying the effect as y increases.

- In the special case where $\nu = 1, \gamma = \eta_\ell + \eta_c$.

3 Calvo model.

- Can do in continuous or discrete time. We will do in discrete time.

- There is a reset probability (hazard rate) ω which does not depend on the duration. That is, once a firm sets a price, each period there is a probability of resetting the price, and the length the price remains in force (the "duration of the contract", or the "duration of the completed price-spell") will be random.

- Let us return to the two period case. The firm sets its price in period 1. With probability ω it will also be able to set its price in period 2. However, with probability $(1-\omega)$ the price will remain in force for the second period.

- Thus, the (expected) profits of the firm in period 1 are

$$\max\left[P_1 Y_1(f) - TCY_1(f)\right] + \beta\left[(1-\omega)\left(\left[P_1 Y_2(f) - TCY_2(f)\right]\right) + \omega \Pi_2^*\right]$$

In the case that the firm can reset its price, then the current price does not affect the second period's profits. However, with probability $1-\omega$ it does. Hence the first order condition for the price set in period 1 is

$$(MR_1 - MC_1) + \beta (1-\omega)(MR_2 - MC_2) = 0$$

The firm can "discount" the future when setting its current price more the larger is ω, because a bigger reset probability means the price it sets now is less likely to survive!

- So, the reset price x is in the log-linearised case:

$$x_t = \frac{1}{\sum_{i=0}^{n-1} (1-\omega)^i \beta^i} \sum_{i=0}^{n-1} (1-\omega)^i \beta^i p_{t+i}^* \tag{7}$$

3.0.1 Demographics of Calvo model

- Each period, a proportion ω of contracts (price-spells) "die". This gives an age distribution

age 1	age 2	age 3	age s
ω	$\omega(1-\omega)$	$\omega(1-\omega)^2$	$\omega(1-\omega)^{s-1}$

The mean age is:

$$
\begin{aligned}
\bar{s} &= \sum_{i=1}^{\infty} i\omega\left(1-\omega\right)^{i-1} \\
&= \omega + 2\omega\left(1-\omega\right) + 3\omega\left(1-\omega\right)^{2}\ldots \\
&= \omega\left[1 + 2\left(1-\omega\right) + 3\left(1-\omega\right)^{2}..\right] \\
&= \omega\left[\sum_{i=1}^{\infty}\left(1-\omega\right)^{i} + \left(1-\omega\right)\sum_{i=1}^{\infty}\left(1-\omega\right)^{i}\ldots\right] \\
&= \left[1 + \left(1-\omega\right) + \left(1-\omega\right)^{2}\right] \\
&= \omega^{-1}
\end{aligned}
$$

- The aggregate price is given by surviving prices (in log-linearised form).define the reset price at time t as x_t :

$$p_t = \sum_{i=1}^{\infty} \omega (1 - \omega)^{i-1} x_{t+1-i}$$

The current price is the average of all past reset prices, with geometrically declining weights.

- since

$$p_{t-1} = \sum_{i=2}^{\infty} \omega (1 - \omega)^{i-1} x_{t+1-i}$$

we have the simple formula:

$$p_t = \omega x_t + (1 - \omega) p_{t-1}$$

The current price level is a weighted sum of the preceding price level and the current reset wage, where the weight is given by the reset probability.

- The cross-section distribution of *completed spells* is given by

$$\alpha_i = \omega^2 i \left(1-\omega\right)^{i-1} : i = 1...\infty$$

with mean

$$\bar{T} = \frac{2-\omega}{\omega}$$

see Dixon and Kara *JMCB* 2006 for proof. Also, later lecture.

- The variance of Calvo prices in steady-state inflation. This is a goemetric age structure, so that the variance is

$$varC = \pi^2 \left(\frac{1-\omega}{\omega^2}\right)$$

where π is the inflation rate.

4 Taylor model and demographics.

- Firms set prices for N periods (usually assume $N = 2$ or 4). Unlike the Calvo model they know the length of price-spell ex ante. However, they still set a constant price over the duration of the contract. The firms are divided into N equal cohorts: each period a proportion N^{-1} of firms reset their prices.

- Hence, the age distribution is given by N equally sized cohorts going back from the current period.

- The average age of contracts is given by $(N + 1)/2$. e.g. 2 period contracts result in an average age of 1.5 periods; 4 period contracts an average age of 2.5 periods.

- The reset price is given by (6). Hence the current aggregate price is a moving average: it is the average of the N most recent reset prices

$$p_t = \frac{1}{N}\sum_{i=0}^{n-1} x_{t-i}$$

Hence

$$\begin{aligned} p_t &= p_{t-1} + \frac{1}{N}(x_t - x_{t-n}) \\ \pi_t &= \frac{1}{N}(x_t - x_{t-n}) \end{aligned}$$

The change in the price level at time t is caused by those firms resetting price, who last reset their prices n periods ago.

- If you compare the demographics of price-spells, in the Taylor model all price-spells have the same length. There are no contracts older than $N-1$

periods. In Calvo, there is a distribution of price-spell durations, with some very old reset prices still surviving. There are infinitely many prices, so that there is more price dispersion. Consider steady state inflation π, so that $x_t = x_{t-1} + \pi$. In this case with Taylor N you get a uniform distribution of prices with distance π between them. From the *variance of a uniform distribution* we get

$$VarT = \pi^2 \frac{(N-1)^2}{12}$$

in the Calvo case we have a geometric distribution, which has variance:

$$VarC = \pi^2 \frac{(1-\omega)}{\omega^2} \tag{8}$$

If we choose a Calvo age distribution so that its mean is the same mean age we have

$$\frac{N+1}{2} = \frac{1}{\omega}$$

Substituting this into (8) gives us

$$VarC = \pi^2 \frac{(N-1)^2}{4}$$

Hence the variance of prices with steady state inflation $\pi > 0$ is exactly three times that in the age-equivalent Taylor case. Thus for example with $N = 4$ we have the equivalent $\omega = 0.4$, if $\pi = 0.1$

5 Fischer contracts (Fischer JPE 1977).

- You have an $N-$period contract or planning horizon (possibly infinite). You choose a trajectory of prices over the planning horizon.

$$\left\{p_{t+i}^{t}\right\}_{i=0}^{N}$$

The price chosen for period $t+i$ is the expectation of the optimal price at that period conditional on the information available at time t (when the plan is made).

$$p_{t+i}^{t} = E_t p_{t+i}^{*}$$

- This, the current price level is made up of the what firms expectations were in the past of the current price! For example, suppose you have a Taylor structure: you update your price plan every N periods. In this case

$$p_t = \frac{1}{N} \sum_{i=0}^{n-1} E_{t-i} p_t^{*}$$

- Mankiw and Reis (QJE 2002). Calvo structure: you have a probability

ω of revising your price-plan. In this case:

$$p_t = \sum_{i=1}^{\infty} \omega \left(1 - \omega\right)^{i-1} E_{t-i} p_t$$

- Note: We can think of the Price-plan as infinite. The price you set now has no influence on the price you set in any future period (unlike the fixed nominal price contract). Hence the length of the contract has no influence on the price plan. In Mankiw and Reis's Calvo structure, the ω does not influence the pricing behaviour, but gives a distribution of contract lengths.

- "Sticky Information model". The price plans are perfectly flexible ex ante, but will get out of date if they are old.

6 Indexation.

- You set the price, and for the duration of the contract, it is updated by past inflation. The price you set at the beginning of the contract determines the level of the trajectory of prices. The expected path of the trajectory depends in inflationary expectations at the time the initial price is set. The ex post trajectory of prices depends on the out-turn.

- Assuming the initial price is set at t, the actual trajectory will be

$$p_{t+i} = p_t + \sum_{j=1}^{i-1} \pi_{t+j-1}$$

- The trajectory expected at time t will be

$$p_{t+i} = p_t + \sum_{j=1}^{i-1} E_t \pi_{t+j-1}$$

- Over 2 periods from (5) we have

$$\left(\frac{\theta}{\theta-1}\right) - \left(\frac{P_f}{P_1}\right)^{-(1+\theta\sigma)} Y_1^{\sigma} + \beta \left(\left(\frac{\theta}{\theta-1}\right) - \left(\frac{P_f\left(1+\pi_1\right)}{P_2}\right)^{-(1+\theta\sigma)} Y_2^{\sigma} \right)$$

which log linearises as

$$\begin{aligned} (1+\theta\sigma)\left(p_1 - p_f\right) + \sigma y_1 + \beta\left((1+\theta\sigma)\left(p_2 - p_f - \pi\right) + \sigma y_2\right) &= 0 \\ p_f - p_1^* + \beta\left(p_f + \pi - p_2^*\right) &= 0 \end{aligned}$$

Which gives us

$$p_f = \frac{p_1^* + \beta \left(p_2^* - \pi\right)}{1 + \beta}$$

Or in general over N periods the reset price is:

$$x_t = \frac{1}{\sum_{i=0}^{n-1} \beta^i} \sum_{i=0}^{n-1} \beta^i \left(p_{t+i}^* - \sum_{j=1}^{i-1} E_t \pi_{t+j-1} \right) \tag{9}$$

"Sticky information": the expectations are based on period t information.

- Woodford (2003, chapter 3). There is a Calvo setup with indexation when $\beta = 1$ particularly simple, since (9) simplifies to:

$$x_t = \omega \sum_{i=0}^{\infty} (1 - \omega)^i \left(p^*_{t+i} - \sum_{j=1}^{i-1} E_t \pi_{t+j-1} \right)$$

Now, note that if we look at the expected level of next periods price we have:

$$E_t x_{t+1} = \omega \sum_{i=1}^{\infty} (1 - \omega)^{i-1} \left(p^*_{t+i} - \sum_{j=1}^{i-1} E_t \pi_{t+j-1} \right)$$

$$x_t = \omega \left(p^*_t - \pi_t \right) + (1 - \omega) E_t x_{t+1} \tag{10}$$

We can also note that the current price can be expressed in a similar way

$$\begin{aligned} p_t &= \sum_{i=1}^{\infty} \omega (1-\omega)^{i-1} \left(x_{t+1-i} + \sum_{j=1}^{i-1} \pi_{t+j-1} \right) \\ p_{t-1} &= \sum_{i=2}^{\infty} \omega (1-\omega)^{i-1} \left(x_{t+1-i} + \sum_{j=1}^{i-1} \pi_{t+j-1} \right) \end{aligned}$$

Hence

$$p_t = \omega x_t + (1-\omega)(p_{t-1} + \pi_{t-1})$$

7 Generalised Calvo (duration dependent reset probabilities).

- In Calvo, the reset probability (hazard rate) is constant.

- Wolman (1999), Mash (2004), Dixon (2006), Sheedy (2006) allow for the hazard rate to vary with the duration of the price (how long the price has been at its current level). Suppose the longest length of price-spell is F, then $\omega_F = 1$ and we set $\omega_0 = 0$:

$$\boldsymbol{\omega} = \{\omega_i\}_{i=1}^{F} \tag{11}$$

- In this case you can define the survival rates:

$$\Omega_i = \Pi_{j=1}^{i}(1 - \omega_{j-1})$$

- The optimal reset price is then:

$$x_t = \frac{1}{\sum_{k=1}^{F} \Omega_k \beta^{k-1}} \sum_{k=1}^{F} \Omega_k \beta^{k-1} p^*_{t+k-1}$$

- The aggregate price is

$$p_t = \sum_{k=1}^{F} \Omega_k p^*_{t-k+1}$$

- When the reset probability is constant $\omega_i = \omega$ for $i = 1...F$, the GC becomes the C (Calvo) model.

8 Conclusion.

- In this paper, we have outlined the basics of the price-setting decision of firms and how this is modelled.

- We have NOT looked at state-sependent models. Another topic (BUT, see John and Wollman *JME* 2008).

- We have examined the log-linearised forms of the standard models: we have pstar (the optimal flex-price) and the reset price is some sort of wieghted average.

- Fischer contracts: "sticky information".

- Indexation: when you set the current price, you take into account the fact that your prices will be updated.