a unified framework for understanding and comparing dynamic wage and price setting models...

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1 Steady State Distributions of Durations across Firms.

1.1 Durations.

- there is a continuum agents $f \in [0, 1]$.

- time is discrete and infinite $t \in Z_+ = \{0...\infty\}$.

- price event: price set by firm f at time t: p_{ft}.

- "price spell" a duration, a sequence of consecutive periods that have the same price.

- $d(t, f)$ the price spell duration of which the price event is part of.

- reset price events, when firms set a new price:

$$R = \left\{(t, f) : p_{ft} \neq p_{ft-1}\right\} \subseteq [0, 1] \times Z_{+} \tag{1}$$

- define subsets of $R, i = 1...F$

$$R(i) = \{(t, f) \in R : d(t, f) = i\}$$

Thus $R(i)$ gives us the subset of durations of length i.

- distribution of durations is simply the proportions of all durations:

$$\boldsymbol{\alpha}^d = \left\{\alpha_i^d\right\}_{i=1}^{F} \in \Delta^{F-1}$$

In steady-state this simplifies,

$$\alpha_i^d = \alpha_i^d(t) = \frac{\int_0^1 I((f,t) \in R(i))df}{\int_0^1 I((f,t) \in R)df}$$

1.2 Ages.

- The age of a price-spell at time t

$$\begin{aligned} A(f,t) &= 1 + \min_s [t - s] \\ s.t\ (f,s) &\in R \\ s &\leq t \end{aligned}$$

we adopt the convention that the minimum age is 1.

- ubset of firms at time t that are of age $A = j$.

$$j(t) = \{f \in [0,1] : A(f,t) = j\}$$

Then the proportion of firms aged j at t is for all $t > F$

$$\alpha_j^A = \alpha_j^A(t) = \int_0^1 I\left((f,t) \in j(t)\right).df$$

- The steady-state distribution of ages is monotonic:

$$\mathbf{\Delta}_M^{F-1} = \left\{\boldsymbol{\alpha}^A \in \mathbf{\Delta}^{F-1} : \alpha_j^A \geq 0, \alpha_j^A \geq \alpha_{j+1}^A\right\}$$

M "(weak) monotonicity".

1.3 Hazard Rate.

- The hazard rate at a particular age is the proportion of price-spells at age i which do not last any longer

- given $\boldsymbol{\alpha}^A \in \Delta_M^{F-1}$, the corresponding vector of hazard rates $\boldsymbol{\omega} \in [0,1]^{F-1}$:

$$\omega_i = \frac{\alpha_i^A - \alpha_{i+1}^A}{\alpha_i^A}; i = 1 \ldots (F-1) \tag{2}$$

- *survival probability,* the probability at birth that the price survives for at least i periods, with $\Omega_1 = 1$ and for $i = 2 \ldots F$

$$\Omega_i = \Pi_{\kappa=1}^{i-1}(1 - \omega_\kappa)$$

$$\Sigma_\Omega = \sum_{i=1}^{F} \Omega_i \quad \bar{\omega} = \Sigma_\Omega^{-1}$$

Clearly, we can invert (2), hence:

Observation 1 given $\boldsymbol{\omega} \in [0,1]^{F-1}$, there exists a unique corresponding age profile $\boldsymbol{\alpha}^A \in \boldsymbol{\Delta}_M^{F-1}$ given by:

$$\alpha_i^A = \bar{\omega}\Omega_i \quad i = 1...F.$$

Observation 2 given $\boldsymbol{\omega} \in [0,1]^{F-1}$, there exists a unique corresponding distribution of durations $\boldsymbol{\alpha}^d \in \boldsymbol{\Delta}^{F-1}$ given by:

$$\alpha_i^d = \Omega_i \omega_i \quad i = 1...F.$$

observation 3. For any $\boldsymbol{\alpha}^d \in \boldsymbol{\Delta}_M^{F-1}$, the corresponding cross-section of ages $\boldsymbol{\alpha}^A \in \boldsymbol{\Delta}_M^{F-1}$ is given by

$$\alpha_i^A = \frac{\bar{\omega}}{\omega_i}\alpha_i^d$$

and vice-versa.

2 The cross-sectional distribution of Completed Price-spells across Firms.

- $\boldsymbol{\alpha}^A \in \boldsymbol{\Delta}_M^{F-1}$, $\boldsymbol{\alpha}^d \in \boldsymbol{\Delta}^{F-1}$, $\boldsymbol{\omega} \in [0,1)^{F-1}$ are different ways of looking at the same object: a *panel* of price events.

- Each row of the panel is a trajectory of prices corresponding to a particular firm. Each column is a cross-section of all of the prices set by firms at a point in time.

- fourth distribution: cross-sectional distribution of *completed* durations or lifetimes across firms $\boldsymbol{\alpha} \in \boldsymbol{\Delta}^{F-1}$.

- define

$$R(i,t) = \{f \in [0,1] : d(t,f) = i\}$$

$$\alpha_i = \alpha_i(t) = \int_0^1 I((f,t) \in R(i,t))\, df$$

Proposition 1 *Consider a steady-state age distribution* $\boldsymbol{\alpha}^A \in \boldsymbol{\Delta}_M^{F-1}$. *There exists a unique distribution of lifetimes across firms* $\boldsymbol{\alpha} \in \boldsymbol{\Delta}^{F-1}$ *which corresponds to* $\boldsymbol{\alpha}^A$, *where*

$$\begin{aligned} \alpha_1 &= \alpha_1^A - \alpha_2^A \\ \alpha_i &= i\left(\alpha_i^A - \alpha_{i+1}^A\right) \\ &\,.. \\ \alpha_F &= F\alpha_F^A \end{aligned} \tag{3}$$

Corollary 1 Given $\boldsymbol{\alpha} \in \boldsymbol{\Delta}^{F-1}$, there exists a unique $\boldsymbol{\alpha}^A \in \boldsymbol{\Delta}_M^{F-1}$:

$$\alpha_j^A = \sum_{i=j}^{F} \frac{\alpha_i}{i} \qquad j = 1...F \tag{4}$$

- *Intuition* In a steady state, each period must look the same in terms of the distribution of ages This implies that if we look at the i period contracts, a proportion of i^{-1} must be renewed each period. Thus if we have 10 period contracts, 10% of these must come up for renewal each period. This implies that the proportion of contracts coming up for renewal each period (which have age 1) is:

$$\alpha_1^A = \sum_{i=1}^{\infty} \frac{\alpha_i}{i}$$

- The proportion of contracts aged 2 is the set of contracts that were reset last period (α_1^s), less the ones that only last one period (α_1) and so on.

- since 1-1 relation between Hazard rates Age distributions, also a 1-1 relationship between $\boldsymbol{\alpha} \in \boldsymbol{\Delta}^{F-1}$ and hazard rates.

Corollary 2 let $\boldsymbol{\omega} \in [0,1]^{F-1}$. The distribution of lifetimes across firms corresponding to $\boldsymbol{\omega}$ is:

$$\alpha_i = \bar{\omega}.i.\omega_i.\Omega_i\text{: } i = 1...F \tag{5}$$

- – The flow of new contracts is $\alpha_1^s = \bar{\omega}$ each period.
  - To survive exactly i periods, you have to survive to period i which happens with probability Ω_i, and then start a new contract which happens with probability ω_i.
  - Hence from a single cohort $\bar{\omega}.\omega_i.\Omega_i$ will have contracts that last for exactly i periods.
  - We then sum over the i cohorts (to include all of the contracts which are in the various stages moving towards the their final period i) to get the expression.

Corollary 3 Consider a distribution of contract lengths across firms given by $\boldsymbol{\alpha} \in \Delta^{F-1}$. The corresponding hazard profile that will generate this distribution in steady state is given by $\boldsymbol{\omega} \in [0,1]^{F-1}$ where:

$$\omega_i = \frac{\alpha_i}{i}\left(\sum_{j=i}^{F}\frac{a_j}{j}\right)^{-1}$$

Corollary 4. For completeness, we can also ask for a given cross-section DAF $\boldsymbol{\alpha} \in \Delta^{F-1}$, what is the corresponding distribution of durations $\boldsymbol{\alpha}^d \in \Delta^{F-1}$ is:

$$\alpha_i^d = \frac{\alpha_i}{i.\bar{\omega}} \tag{6}$$

2.1 A Comparison of the mean duration measures.

Mean duration

$$\bar{d} = \sum_{i=1}^{F} i.\alpha_i^d = \bar{\omega}^{-1}$$

Mean age

$$\bar{A} = \sum_{i=1}^{F} i.\alpha_i^A$$

Mean duration across firms.

$$\bar{T} = \sum_{i=1}^{F} i.\alpha_i$$

- First, the two cross-sectional measures $\bar{A}$ and $\bar{T}$, have length-biased sampling: we are more likely to observe longer price-spells* than in the duration measure $\bar{d}$.

- Duration measure, look at the start of price-spells: α_i^d is defined over the reset subset R of all price-events. we only look at the *subset of firms who reset their price*.

- In the two cross-sectional measures, we are in effect selecting over *all firms in the cross-section*.

*The probability of a price-spell being observed is proportional to its length: a 10 period spell is 10 times more likely to be observed than a 1 period contract.

- age distribution, there is an *interruption* bias: the age represents an interrupted duration, only a part of the completed lifetime of the price-spell, $A(f,t) \leq d(f,t)$.

- Hencetwo inequalities:

$$\bar{d} \leq \bar{T}; \bar{A} \leq \bar{T}$$

  $\bar{A} = \bar{T}$ only iff $F = 1$, otherwise $\bar{A} < \bar{T}$.

- Furthermore, since F is the longest contract for which $\alpha_F^d > 0$, we have

$$\begin{aligned} \bar{d} &= \bar{T} \text{ if } \alpha_F^d = 1 \\ \bar{d} &< \bar{T} \text{ if } \alpha_F^d < 1 \end{aligned}$$

  since there can be no length-bias if all price-spells have the same duration F.

- mean age and the mean duration: we can have $\bar{A} > \bar{d}$ and $\bar{A} < \bar{d}$ depending on whether the interruption bias is larger than the length bias.

- fort any $\bar{d}$ there are many distributions $\left\{\alpha_i^d\right\}$ that generate that mean: and each such distribution will have a corresponding DAF $\{\alpha_i\}$ and mean across firms $\bar{T}$.

- Since we know that $\bar{d} = \bar{\omega}^{-1}$, $H(\bar{\omega}) : [0,1] \rightarrow \mathbf{\Delta}^{F-1}$

$$H(\bar{\omega}) = \left\{ \boldsymbol{\alpha} \in \mathbf{\Delta}^{F-1} : \sum_{i=1}^{F} \frac{\alpha_i}{i} = \bar{\omega} \right\}$$

  $H(\bar{\omega})$ is the set of all $DAFs$ which are consistent with a given mean duration of price-spells $\bar{d}$ expressed in terms of the corresponding proportion of firms resetting prices $\bar{\omega}$.

- $\bar{T}(\boldsymbol{\alpha}) = \sum_{i=1}^{F} i.\alpha_i$.

$$\min \; \bar{T}(\boldsymbol{\alpha}) \quad s.t. \; \boldsymbol{\alpha} \in H(\bar{\omega}) \tag{7}$$

Proposition 2 Let $\boldsymbol{\alpha}^{\text{min}} \in \Delta^{F-1}$ solve (7) to give the shortest average contract length $\bar{T}^{\text{min}}$.

(a) No more than two sectors i have values greater than zero

(b) If there are two sectors $\alpha_i > 0$, $\alpha_j > 0$ then will be consecutive integers ($|i-j| = 1$).

(c) There is one solution iff $\bar{\omega}^{-1} = k \in Z_+$. In this case, $\alpha_k = 1$.

(d) The minimum is $\bar{T}^{\mathsf{min}} = \bar{\omega}^{-1} = \bar{d}$.

We can also ask what is the *maximum* average contract length consistent with a proportion of resetters $\bar{\omega}$:

$$\max\ \bar{T}(\boldsymbol{\alpha}) \quad s.t.\ \boldsymbol{\alpha} \in H(\bar{\omega}) \tag{8}$$

Proposition 3 Let $\boldsymbol{\alpha}^{\mathsf{max}} \in \boldsymbol{\Delta}^{F-1}$ solve (8). Given the longest contract duration F, the distribution of contracts that maximizes the average length of contract subject to a given proportion $\bar{\omega}$ of firms resetting price

$$\begin{aligned} \alpha_F^{\mathsf{max}} &= \frac{F}{F-1}(1-\bar{\omega}) \\ \alpha_1^{\mathsf{max}} &= \frac{F}{F-1}\bar{\omega} - \frac{1}{F-1} \end{aligned}$$

with $\alpha_i^{\text{max}} = 0$ for $i = 2...F-1$. The maximum average contract length is

$$\bar{T}^{\text{max}} = F(1-\bar{\omega}) + 1$$

- what is generating the mean duration $\bar{d}$ and the proportion of firms changing price each period $\bar{\omega}$?

- There is the unit interval of firms, divided into proportions with different price-spell durations $i = 1...F$.

- Firms with price-spell lengths i will set prices once every i^{-1} periods: the longer the price-spell, the more infrequently the firm will visit the price-setters club.

- Hence, we can have the same proportion of firms re-setting price (and hence same mean duration) and increase the mean duration across firms by more longer price-spells.

- The maximum T^{max} is reached when we have as many F period contracts as possible, consistent with $\bar{\omega}$.

- The minimum occurs when all firms have similar price-spells: if $\bar{d}$ happens to be an integer, then all price-spells have that length and the two distributions are the same: $\boldsymbol{\alpha}^d = \boldsymbol{\alpha}$.

2.2 Examples.

Example 3: Simple Taylor 4.

$$\begin{array}{lll} \omega_1 = 0 & \alpha_1^s = \frac{1}{4} & \alpha_1 = \alpha_1^d = 0 \\ \omega_2 = 0 & \alpha_2^s = \frac{1}{4} & \alpha_2 = \alpha_2^d = 0 \\ \omega_3 = 0 & \alpha_3^s = \frac{1}{4} & \alpha_3 = \alpha_3^d = 0 \\ \omega_4 = 1 & \alpha_4^s = \frac{1}{4} & \alpha_4 = \alpha_4^d = 1 \\ \bar{\omega} = \frac{1}{4} & \bar{A} = \frac{5}{2} & \bar{T} = \bar{d} = 4 \end{array}$$

Example 5: Simple Calvo The simple Calvo model has a constant reset probability ω (the hazard rate) in any period that the firm will be able to review and if so desired reset its price. The distribution of ages of price-spells is

$$\alpha_i^A = \omega\,(1-\omega)^{s-1} : \; s = 1...\infty$$

$\bar{A} = \sum_{i=1}^{\infty} i\alpha_i^A = \omega^{-1}$. Applying Proposition 1(a):

$$\alpha_i = \omega^2 i \left(1-\omega\right)^{i-1} : \; i = 1...\infty \tag{9}$$

$\bar{T} = 2\omega^{-1} - 1$ (see Dixon and Kara 2006). For the simple Calvo model, the distribution of ages is the same as the distribution of durations: from observation 3, since $\omega = \bar{\omega}$, $\alpha_i^A = \alpha_i^d$ $i = 1...\infty$. Hence the interruption and length bias are exactly offset. We illustrate the simple Calvo model with $\omega = 0.25$, to 4 d.p.

$\omega_1 = 0.25$	$\alpha_1^A = 0.25$	$\alpha_1 = 0.0625$	$\alpha_1^d = 0.25$
$\omega_2 = 0.25$	$\alpha_2^A = 0.1875$	$\alpha_2 = 0.09375$	$\alpha_2^d = 0.1875$
$\omega_3 = 0.25$	$\alpha_3^A = 0.1406$	$\alpha_3 = 0.1055$	$\alpha_3^d = 0.1406$
$\omega_4 = 0.25$	$\alpha_4^A = 0.1052$	$\alpha_4 = 0.1055$	$\alpha_4^d = 0.1052$
$\omega_i = 0.25$	$\alpha_i^A = 0.25\left(0.75\right)^{i-1}$	$\alpha_i = \left(0.25\right)^2 i \left(0.75\right)^{i-1}$	$\alpha_i^d = \alpha_i^A$
$\bar{\omega} = 0.25$	$\bar{A} = 4$	$\bar{T} = 7$	$\bar{d} = 4$

3 How to measure price stickiness.

- How should we think about nominal rigidity? Much of the recent literature has focussed on the distribution of durations, this has lead to a focus on the frequency of prices changing in a given period $\bar{\omega}$ to estimate $\bar{d}$. (Bils and Klenow 1994 etc.).

- Taylor with Calvo: 4 period Taylor $\bar{\omega} = 0.25$ is equated with a simple Calvo rest probability of $\omega = 0.25$ (see for example Kiley 2002).

- Since in most data sets there are a lot of short price durations, the mean duration estimated from the data seems quite small. Micro-data sets derived from CPI data, the proportion of firms changing price per month is in the 20-30% range: hence the mean duration will be around 3-5 months, or $1-2$ quarters.

- most important point to make is that nominal rigidity is a result of how *firms* set prices. If we want to look at an economy and evaluate the degree of nominal rigidity, we would want to look at the behavior of firms.

- Firms: essentially a cross-section perspective on price-spell durations, since at any one time, each extant price-spell is associated with one firm.

- example two firms two periods. Firm 1 sets its price in both periods (single period price-spells). Firm 2 sets the price for two periods.
  - firm based view, we would say that 50% of firms set $1-$period contracts, and 50% set two period contracts: the average contract is 1.5 periods.

– duration-based approach, we say that in the two periods there were 3 price-spells: two were 1−period, and 1 was 2−periods, so that the average duration is $1\frac{1}{3}$.

- 9 firms set prices for a year (all on January 1st), but 1 firm sets prices τ times per year of duration τ^{-1}. In a year $9+\tau$ price-spells. The averages across price-spells and across firms will be (in years):

$$\bar{d}(\tau) = \frac{9.1 + \tau.\tau^{-1}}{9+\tau} = \frac{10}{9+\tau}$$
$$\bar{T}(\tau) = \frac{9+\tau^{-1}}{10}.$$

- The economy consists of 90% firms who have rigid prices: 10% of the economy has flexible prices. In any plausible economic model, the behavior of the 90% of firms is going to dominate: our measure of nominal rigidity should pick this up.

- However, as $\tau \to \infty$ we have $\bar{d}(\tau) \to 0$! As price spells in the flexible firm get shorter and shorter, they drive the mean duration to zero. This is not a plausible measure of an economy in which most prices are rigid for the whole year.

- $\tau \to \infty$ $\bar{T}(\tau) \to 0.9$, is more reasonable.

- Baharad and Eden (2004). There is one firm. It keeps its price constant for 364 days of the year. On the 365th day it changes its price τ times with each duration τ^{-1} of a day:

$$\bar{d}(\tau) = \frac{364 + \tau.\tau^{-1}}{364 + \tau} = \frac{365}{364 + \tau}$$

Again: $\bar{d}$ goes to zero as $\tau \to \infty$.

- Baharad and Eden propose a measure of price rigidity in which the duration of the price spell is weighted according to its duration: longer price spells occupy more of the time:

$$BE(\tau) = \frac{364}{365}(364) + \frac{1}{365}\tau^{-1}$$

BE goes to 364 as $\tau \to \infty$:)

Observation 4. In steady-state $BE = \bar{T}$.

The reason that the two measures are equivalent is that they both weight the price-spells by their duration.

4 Micro data: prices are stickier than we thought.

- There are now several studies using micro data: in particular the *Inflation Persistence Network* (IPN) across the Eurozone.

- They also have *trajectories* for prices: this is the sequence of price spells for a product at an individual outlet.

- We can think of each trajectory as analogous to the sequence of price contracts for an individual firm in the context of this paper.

- These papers all provide estimates of the average length of a price spell: both across the population of all price spells (corresponding to $\bar{d}$)

- across trajectories, where a mean duration is calculated for each trajectory and then the average is taken across trajectories (corresponding to TR).

- However, because of within trajectory diversity of price-spells, taking an unweighted average of spell durations along a trajectory will tend to over-weight short spells, so that $TR \leq \bar{T}$.

- Italy: $\bar{d} = 8, TR = 13$.
- France: $\bar{d} = 5.28, TR = 7.24$,
- Spain: $\bar{d} = 6.2; TR = 14.7$.
- Dixon and Le Bihan (2010) For France $\bar{T} = 13.87$.
- Hungary 2002-6 $\bar{d} = 9$ and $\bar{T} = 16.4$ months.
- German micro-CPI data (1998-2004) $\bar{d} = 5.3$ and $\bar{T} = 26.8$.

4.1 US data.

- several studies using BLS micro-CPI: Bils and Klenow (2004), Klenow Krystov (2008) and Nakamura and Steinsson (2008).

- These all adopt the frequency method: use proportion of firms changing price to predict $\bar{d}$. Do not compute TR or $\bar{T}$.

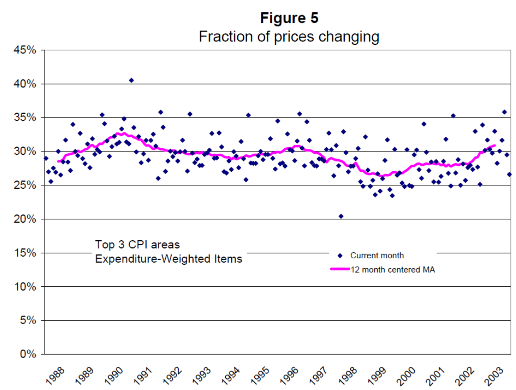
- Use continuous time formula:

$$\bar{d}^c = \frac{-1}{Ln(1-\bar{\omega})} < \frac{1}{\bar{\omega}}$$

- Bils and Klenow 1995-7: report $\bar{\omega}$ for each sector and "for all goods and services": $\bar{\omega} = 26\%$, so that $\bar{d} = 4$. This is shorter than Europe (US prices less sticky).

- Sales and forced substitution....reduces it a bit.

- Klenow and Krystov, Nakamura and Steinsson cover longer period 1988-2003.



- Nakamura and Steinsson try to get the average longer: leave out sales, use reference prices etc. See website, endless tables

- BUT only look at $\bar{d}$ and $\bar{\omega}$.

5 Pricing Models with steady state distributions of durations across firms.

5.1 The Generalized Taylor Economy GTE

- Dixon and Kara (2005), any steady-state distribution of completed durations across firms $\boldsymbol{\alpha} \in \boldsymbol{\Delta}^{F-1}$can be represented by the GTE with the sector shares given by $\boldsymbol{\alpha} \in \boldsymbol{\Delta}^{F-1}$: $GTE(\boldsymbol{\alpha})$.

- In each sector i there is an $i-$period Taylor contract, with i cohorts of equal size (since we are considering only uniform $GTEs$).

- The sector share α_i. Equal size cohorts $\alpha_i.i^{-1}$ prices reset each period

- In a GTE, the reset price at time t in sector i x_{it} is (in log-linearised form):

$$x_{it} = \sum_{k=0}^{i-1} \beta^k p^*_{t+k} \tag{10}$$

where p^*_t is the optimal flex-price at time t. There are F reset price equations, with $i = 1...F$. The F prices in each sector i are simply the average over the i cohorts in that sector:

$$p_{it} = \frac{1}{i} \sum_{k=0}^{i-1} x_{it-k} \tag{11}$$

The aggregate price level is simply:

$$p_t = \sum_{i=1}^{F} \alpha_i p_{it} \tag{12}$$

The GTE Taylor (1993), Coenen et al (2007), Dixon and Kara (2008), Kara (2008, 2009), Ascari et al (2010), Dixon and LeBihan (2010).

5.2 The Generalized Calvo model (GC): duration dependant reset probabilities.

- *Generalized Calvo Model* $GC\{$ sequence of reset probabilities: $\boldsymbol{\omega} \in [0,1)^{F-1}$ where F is the shortest contract length with $\omega_F = 1$.

- GTE and the GC are comprehensive and coextensive, both being consistent with any steady-state distribution of durations.

- GC differs from the GTE in that when they reset prices, firms do not know how long the price-spell is going to last. There is not a sector specific reset price, but one economy wide reset price x_t with $x_{it} = x_t$ for all $i = 1 \ldots F$.

- The log-linearised formula for the optimal reset price at t is

$$x_t = \frac{1}{\sum_{k=1}^{F} \Omega_k \beta^{k-1}} \sum_{k=1}^{F} \Omega_k \beta^{k-1} p^*_{t+k-1} \qquad (13)$$

The price in each sector i is then the average over the cohorts in that sector

$$p_{it} = \bar{\omega} \sum_{k=0}^{i-1} \Omega_k x_{t-k} \tag{14}$$

The GC model has been employed by Wolman (1999), Mash (2003,2004), Dotsey and King (2006), Guerrieri (2006), Sheedy (2007) and Paustian and von Hagen (2008).

5.3 The Multiple Calvo Model (MC).

We can define a multiple Calvo process MC as $MC(\boldsymbol{\omega}, \lambda)$ where $\boldsymbol{\omega} \in (0,1)^n$ gives a sector specific hazard rate† $\bar{\omega}_k$ for each sector $k = 1, ...n$ and $\boldsymbol{\lambda} \in \boldsymbol{\Delta}^{n-1}$ is the vector of shares λ_k (this might be expenditure or CPI weights). The reset price for each sector $k = 1...n$ is then:

$$x_{kt} = \frac{1}{\sum_{j=1}^{F} (1-\bar{\omega}_k)^{j-1} \beta^{j-1}} \sum_{j=1}^{F} (1-\bar{\omega}_k)^{j-1} \beta^{j-1} p^*_{t+j-1} \qquad (15)$$

†The notation here should not be confused: the substrcripts k are sectoral: none of the sectoral calvo reset probabiltities are duration dependent.

The average price in each sector k is then

$$p_{kt} = \sum_{j=1}^{F} (1 - \bar{\omega}_k)^{j-1} \beta^{j-1} x_{kt-j+1} \tag{16}$$

And the aggregate price is then

$$p_t = \sum_{k=1}^{n} \lambda_k p_{kt} \tag{17}$$

The Multiple Calvo model has been employed by Carvalho (2006) and Carvalho and Nechio (2008) and the earlier version of this paper (2006).

5.4 The Typology of Contracts.

In terms of contract structure, we can say that the following relationships hold:

- $GC = GTE = SS$. The set of all possible steady state distributions of durations is equivalent to the set of all possible GTEs and the set of all possible GCs.

- $C \subset MC \subset GC$. The set of distributions generated by the Simple Calvo is a special case of the set generated by MC which is a special case of GC.

- $ST \subset GTE = GC$ Simple Taylor is a special case of GTE, and hence also of GC.

- $ST \cap MC = \varnothing$. Simple Taylor contracts are a special case of GC, but not of MC.

6 Conclusion.

- In steady-state, there are identities linking together 4 different things: the distribution of durations, the hazard rate, and the two cross-sectional distributions of ages and completed price-spells.

- If you know one, you know them all: but need the information about the whole distribution.

- Empirical evidence has focussed on the distribution of durations, and used the frequency of price-changes to estimate it: get a short length (4-8 months)

- Some European studies have estimated the cross-sectional distribution. Get a long length (12 months plus).

- Can relate the micro-data directly to pricing models: GC, GTE, MC etc.....

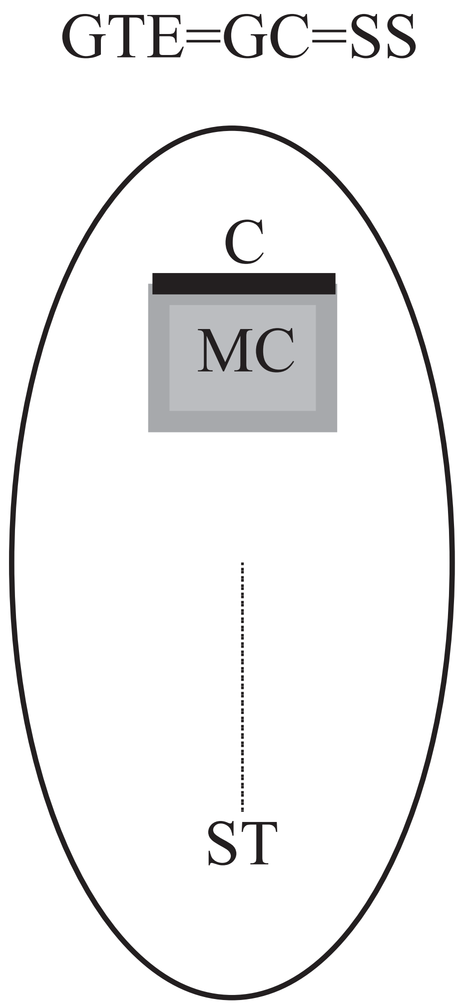


Fig.1. The Typology of contract types.